# List of Abstracts for Student Talks

## Morning Talks

## Combinatorics and Graph Theory

11:10-11:25 AM *Planar Seperating Graphs* Catherine Jacobs (Wellesley College)

In this presentation, we focus our attention on certain classes of planar graphs. Let $H$ and $J$ be planar multigraphs. A loopless planar multigraph $G$ is $(H, J)$-separating if there exists a cycle $C$ such that for every planar embedding of $G$, $H$ is in one component of $\mathbb{R}^2$ - $C$ and $J$ is in the other component. In 2019, Dehkordi and Farr found the complete minor-minimal (roughly speaking, the simplest) set of graphs that are $(H, J)$-separating where $H$ and $J$ are both vertices. We have characterized some of the other complete minor-minimal sets of graphs that separate other more complicated subgraphs such as edges both with and without their endpoints. We note the partial ordering of graphs that separate subgraphs from each other: for example, a graph that separates an edge from a vertex will also separate a vertex from another vertex. Thus, all elements of a minor-minimal set that separates graphs higher in the partial order will also separate all lower subgraphs. We also briefly discuss future possibilities for research in this topic, which includes finding the cardinality of the minor-minimal set of graphs that separate larger subgraphs.

11:30-11:45 AM *Repetitions of Pak-Stanley Labels in the G-Shi Arrangement: Playing Games on Paths, Trees, and More* Ava Mock (Wellesley College)

11:50 AM - 12:05 PM *Packing Edge-Colorings of Graphs with Maximum Degree At Most 4* Cicely Henderson (Wellesley College)

A $(1^j, 2^k)$-packing edge-coloring of a graph G is an assignment of the colors $\{1_1, 1_2, ..., 1_j\}$ and $\{2_1, 2_2, ..., 2_k\}$ to the edges of $G$ such that any two edges that receive the same color are not incident to each other, and furthermore, if two edges are both colored $2_i$ for the same $i$ where $1 \leq i \leq k$, there cannot be a third edge incident to both. Note that when $k = 0$, this notion is equivalent to a proper $j$-edge-coloring, and when $j = 0$ this is equivalent to a strong $k$-edge-coloring. In 1985, Erdös and Nešetřil posed a conjecture regarding strong edge-colorings of graphs based on their maximum degree $\Delta$ that has been proven for $\Delta \leq 3$. In the language of packing edge-colorings, this conjecture posits that every graph with $\Delta \leq 4$ has a $(1^0, 2^{20})$-packing edge-coloring. It was recently shown that every graph with $\Delta \leq 4$ has a $(1^0, 2^{21})$-packing edge-coloring. In this talk, we approach this conjecture from a different direction by showing that every graph $G$ with $\Delta \leq 4$ has a $(1^1, 2^{19})$-packing edge-coloring.

## Biomathematics and Modeling

11:10-11:25 AM *Asymptotic Analysis of a System of Reaction-Diffusion Equations for Modeling Gene Drives* Riuji Sato (Worcester Polytechnic Institute)

We consider a system of reaction-diffusion equations modeling the spatial spread of Wolbachia infection within a mosquito population. Wolbachia is a maternally inherited bacterium that has been shown to inhibit the replication of dengue in its primary vectof, Aedes aegypti. The dynamics of such systems are complex and can be difficult to describe qualitatively. We show that the system can be reduced to fewer equations in the limit of high fecundity. In particular, we show that the frequency of Wolbachia-infected females converge to a function that satisfies a simpler reaction-diffusion equation, and that the total female population can be well-approximated by an explicit function of the proportion of infected females and infected males. We discuss these results in the context of viability of Wolbachia and other gene drives relevant to the mitigation of dengue.

11:30-11:45 AM *Identifying Relationships Between Migrant Status and Rifampin Resistance in Ukraine Tuberculosis Cases* Marley DeSimone (Wellesley College)

With the current war in Ukraine causing increased migration, understanding how Tuberculosis affects migrants is critical to improving treatment. Drug resistant TB is particularly prevalent in Ukraine, with resistance to rifampin being the most common. This talk will focus on applying biostatistical methods to a large observational data set of TB cases in Ukraine from 2015-2018 in order to analyze the relationship between migrant status and rifampin drug resistance. A multiple logistic regression model is used to determine the odds ratio of rifampin resistance in migrants, adjusting for confounding variables. I will explore the limitations of observational data sets, the considerations for multiple logistic regression models, and different methods for determining confounding variables. Our research concludes that migrants have a 35% increased risk of developing a rifampin resistant tuberculosis infection compared to non-migrants after adjusting for other variables. I will discuss possible reasons as to why we see an increased risk of developing rifampin resistance in migrants, as well as possible areas for future study.

11:50 AM - 12:05 PM *Biological Mathematics* Mobina Golmohammadi (Kansas State University) Remote Talk

## Statistics, Data Science, and Optimization

11:10-11:25 AM *Adaptive Tests for Mixed Paired and Two-Sample Designs* Thomas Yacovone (College of the Holy Cross)

A common situation in statistics is the desire to measure the impact on a population before and after some treatment. Traditionally, this is solved using a paired test-a measure is taken of a sample, a treatment is applied, then a second measurement is taken of the same exact sample. Inference can then be performed on the difference of locations. Consider a teacher who wants to determine whether a particular lesson improves student knowledge on a topic; an exam can be administered to the students prior to the lesson, then the same exam can be administered to the students after the lesson. But what if, at the time of the second test, some students are home sick? Or have moved districts? Alternatively, experiments may be designed wherein investigators intentionally have both paired and unpaired observations. We may be left with a combination of data that is paired and unpaired. While we could simply throw out either the paired or unpaired data, and perform the correspondingly appropriate analysis, this risks losing important information. We propose adaptive tests for mixed paired and two-sample designs using t and Wilcoxon-based tests. Previous simulation studies have found that t-based tests perform better for normal data, while Wilcoxon-based tests perform better for nonnormal data. The proposed adaptive tests use two different tail index combination schemes to distinguish between normal and nonnormal mixed pairs data to select the situationally more powerful test. A simulation study is conducted to estimate the power and Type I error rate of the proposed adaptive tests, compared to using their constituent tests uniformly. The proposed adaptive tests tended to have power comparable to the test that performed better under the particular distribution, and provide a distribution-free approach when there are no assumptions or knowledge of the underlying distribution.

11:30-11:45 AM *Text Simplification Using Graph Algorithms* Tina Giorgadze (Bard College)

Text simplification is beneficial for communities such as non-native English speakers, children, people with autism or dyslexia, and so on. The text simplification problem can be reduced to sentence simplification, which is what we focus on in this talk. We used machine learning and a sentence fusion graph to generate simplifications of some original sentences.

11:50 AM - 12:05 PM *Optimizing Efficiencies in an Electromagnetic Heat Exchanger* Ye Chen and Kiersten Greico (Worcester Polytechnic Institute and King's College)

An electromagnetic heat exchanger is a device whose energy source comes from the conversion of radiation to thermal energy within a specialized material system. In this research,

we investigate the energy conversion efficiency in two different planar material systems where the radiation propagates normal to the system from one side, while the other is electrically grounded. In each, the loss factor of one or more materials is dependent on the local temperature. The first system consists of two infinite planar slabs with different electrical and thermal properties, each with its own finite thickness. The second system is a single slab which is composed of many thin pairs of materials; the thickness of these material pairs is much smaller than the wavelength of the applied electromagnetic radiation. Using these models, we then apply optimization methods to find the material properties of the systems which results in minimizing the reflection coefficient while maximizing energy absorption or energy transport. Results and implications of our findings will be discussed.

## Combinatorics and Linear Algebra

11:10-11:25 AM *Counting Whittaker Functions Using Modular Linear Algebra*
Veronica Lang (Smith College)

Whittaker functions are special functions that arise in $p$-adic number theory and representation theory. They may be defined on representations of reductive groups as well as their metaplectic covering groups: fascinatingly, many of their number theoretic applications survive the transition between the reductive and metaplectic cases. However, one notable difference is that the space of Whittaker functions on a reductive group over a nonarchimedean local field is one-dimensional, whereas this is no longer true in the metaplectic case. Frechette showed that $\dim(\mathfrak{W})$, the dimension of the space of Whittaker function on $n$-fold metaplectic covers of $GL_r(F)$, can be counted in terms of the number of solutions to a particular set of linear equations modulo $n$. In this talk, I describe how my collaborators and I calculated the number of solutions, yielding a formula for $\dim(\mathfrak{W})$, at the Twin Cities Combinatorics and Algebra REU.

11:30-11:45 PM *Triangular Patterns in Springer Fiber Flags* Emily Hafken and Yunxi Yan (Smith College)

During the summer of 2022, I joined Professor Julianna Tymoczko's research group to study the pattern of entries in flags of Springer Fibers. We explored ways to find entries in flags using linear algebra. However, as the matrix grows, the computation behind the matrix becomes increasingly tedious. Therefore, we tried to identify potential patterns in all matrices and simplify the computation as much as possible. After collecting data based on calculations of $9 \times 9$ Springer flags, I found triangular patterns in the entries and proved some of the reasons why and when these triangular patterns show up.

# Afternoon Talks

## Algebra

2:20-2:35 PM *Arboreal Galois Groups of Cubics with Colliding Critical Points* William Degroot, Xinyu Ni, Jesse Seid, Annie Wei, and Min Winton (Amherst College)

2:40-2:55 PM *The Saxl Conjecture for (4,4) Hooks* Devin Brown (Northeastern University)

The Saxl conjecture states that the tensor square of the irreducible representation associated with the staircase partition, $\rho_k = (k, k-1, \ldots, 1)$, contains every irreducible representation of $S_n$ where $n = \binom{k+1}{2}$. We show the tensor square of the associated representation, $V^{\rho_k}$, contains some families of hook-like partition shapes, including partitions of Durfee size 4 for sufficiently large $k$. Additionally, we conjecture that each irreducible appears linearly in its dimension, and we present some computational data supporting this observation.

## Geometry and Topology

2:20-2:35 PM *Pivoting to Positive Links* Lizzie Buchanan (Dartmouth College)

We set out to produce an infinite family of knots that have a minimal (with respect to crossing number) almost-alternating diagram. While working on this problem, we found a new upper bound on the maximum degree of the Jones polynomial of a fibered positive link. In particular, the maximum degree of the Jones polynomial of a fibered positive knot is at most four times the minimum degree. With this result, we complete the classification of all knots of crossing number less than or equal to 12 as positive or not positive.

2:40-2:55 PM *Linking Invariants in Simple Branched Covers* Annika Gonzalez-Zugasti and Silvia Heng Song (Smith College)

A mathematical knot is a positioning of a circle in space. Two knots are the same if it is possible to gradually deform one into the position of the other. Topologists want to find tools to tell knots apart. We studied one such tool, called branched covers of knots, in order to facilitate the process of telling knots apart.

The branched cover of a knot is a new three dimensional space that you can build from a labeled knot. The knot corresponds to a collection of knots in the branched cover which

we call branch curves. The linking number of two knots measures how many times one knot wraps around another. A powerful knot invariant is the linking number of two branch curves in the branched cover of the original knot. This number being an invariant means that it is a valuable tool in helping us distinguish one knot from another.

We created a program which computes this invariant of a knot whose arc labelings are transpositions in $S_4$, the symmetric group on 4 letters. This program automates the lengthy process of finding the number by hand. To calculate the number, the program requires four inputs for each crossing of the knot: the number of the crossing (which we will refer to as i), the transposition label assigned to the ith arc, the numbers of the arcs that cross over the crossing, and the sign of the ith crossing. In addition, the total number of crossings is needed for the program to run.

This program represents an algorithm which could previously only be done by hand. An automated algorithm of this sort has only existed for knots that are three-colorable or, in other terms, have arcs labeled with transpositions in $S_3$. Thus, our work takes the next step toward automating this process for knots with any permutation labeling. The automation of this calculation allows topologists the ability to quickly and reliably tell knots apart.

3:00-3:15 PM *The Case of the Impossible Triangles* Zachary Steinberg (Amherst College)

A simple-sounding problem about finding right triangles with a given area whose sides are rational numbers ends up diving into a rabbit hole of these things called elliptic curves, one of applied math's biggest success stories, and an unsolved problem with a million-dollar prize. Unlike most math presentations, this one's animated.

## Combinatorics and Number Theory

2:20-2:35 PM *Investigating Boundary Generating Curves of Finite Field Numerical Ranges* Nataya Tucker (Smith College)

We define the numerical range of a $2 \times 2$ matrix as a set of vectors in $\mathbb{Z}_p[i]$ given by $W(A) = \{x^*Ax : x \in \mathbb{Z}_p[i], x^*x = 1\}$, with $p = 4k + 3$, which can be plotted as ordered pairs onto a small section of the complex plane. The boundary generating curve borders the numerical range over the infinite complex plane and, due to the looping nature of $\mathbb{Z}_p[i]$, plots itself as a conic in disordered segments across the $\mathbb{Z}_p[i]$ plane. Previously, Camenga, et al. used the boundary generating curve to study the geometry of finite field numerical ranges of matrices in $M(\mathbb{Z}_7[i])$. For my research, I investigated the shapes of boundary generating curves of $2 \times 2$ matrices in $M(\mathbb{Z}_7[i])$. In this talk, I will present the highlights of this investigation.

2:40-2:55 PM *Almost All Wreath Product Character Values are Divisible by Given Primes* Skye Rothstein (Bard College)

For a group $G$ with an integer-valued character table and a prime $p$, we show that almost every entry in the character tables of $G \wr S_N$ is divisible by $p$ as $N \to \infty$. These results generalize the work of Peluse and Soundararajan on the character table of $S_N$.

3:00-3:15 PM *The Sperner Property for* 132*-Avoiding Intervals in the Weak Order* Katherine Tung (Harvard College)

A well-known result of Stanley from 1980 implies that the weak order on a maximal parabolic quotient of the symmetric group $S_n$ has the Sperner property; this same property was recently established for the weak order on all of $S_n$ by Gaetz and Gao, resolving a long-open problem. In our research, we interpolate between these results by showing that the weak order on any 132-avoiding interval has the Sperner property.

This result is proven by exhibiting an action of $\mathfrak{sl}_2$ respecting the weak order on these intervals. As a corollary we obtain a new formula for principal specializations of Schubert polynomials. Our formula can be seen as a strong Bruhat order analogue of Macdonald's reduced word formula.