List of Abstract for Student's Talks

Morning Talks

11:00-11:15 PM *Number Theoretic Properties of a Discrete Discontinuous Dynamical System* Anh Nguyen (Smith College) Live Talk

We study a discrete dynamical system with a countably infinite number of discontinuities, focusing on the number theoretic properties of special orbits. This system is generated by a function that is related to Sturmian sequences if seeded with an irrational number. The special orbits we are interested in come from an infinite family of rational numbers. We show that the orbit of an element of the sequence contains the orbits generated by smaller numbers in the family. Moreover, all the orbits in the family stabilize at 1 and have symmetry. Furthermore, these orbits seem to have both fractal-like structure, and quasi-periodicity.

11:20-11:35 AM *Bridge trisections of (un)knotted surfaces in $\mathbf{S}^4$: combinatorial methods on broken surface diagrams* Zoe Dillon-Davidson (Smith College) Live Talk

Of the many diagrammatic methods for representing knotted surfaces in $\mathbf{S}^4$, triplane diagrams, a recently introduced method by Meier and Zupan, have shown to be particularly useful. Triplane diagrams arise from bridge trisections, which are decompositions of knotted surfaces into three trivial disk systems. It is natural to consider whether a surface is unknotted based on simple combinatorial properties of its triplane diagram. In this talk, we will show that if S is an orientable surface which admits a triplane diagram with 1 crossing, S is unknotted. Furthermore, if S is a collection of 2-spheres and admits a triplane diagram with 2 - 4 crossings, S is unknotted. We also prove some results about sets of self-intersection on broken surface diagrams; namely, certain sets of intersection imply unknottedness.

11:20-11:35 AM *Critical subsets of infinite Latin Squares on $\mathbb{Z}$* Yolanda Zhu (Bard College at Simon's Rock) Live Talk

A critical subset is a partial Latin Square with the following two properties: 1) It is uniquely completable to a Latin Square. 2) If one entry is deleted, the partial Latin square is no longer uniquely completable. A partial Latin Square is said to be uniquely completable if it only has one possible completion. In this talk, I will discuss examples of critical sets for infinite Latin Squares and their properties. In addition, I will show some uniquely completable infinite Latin Squares that have interesting behaviors in its infinite structure, which might offer some insights for how Latin Squares behave differently in the infinite case.

11:40-11:55 AM *Splines on dual graphs to triangulations* Jessie Cai and Joy Mahoney (Smith College) Pre-Recorded Talk

Splines are a fundamental tool across applied mathematics and analysis, used in areas such as computer graphics, engineering models, and data interpolation. Our research considers a more abstract idea of splines in which we work with an algebraic-combinatorial generalization of splines on an edge-labeled graph dual to a triangulation. A spline on a graph is a way of labeling the vertices such that if two vertices share an edge, then their vertex-labels differ by a multiple of the edge-label. As part of a longstanding open problem sometimes called the "upper-bound conjecture," we are developing approaches to find a basis of the space of splines when using polynomial labels of the form $(x + cy)^2$. Motivated by graphs dual to triangulations, we introduce parachute graphs and present some results describing splines on parachute graphs.

11:40-11:55 AM *p-Colorability of Surfaces Bounded by Knots* Kate Bernklau Halvor and Annika Gonzalez-Zugasti (Smith College) Pre-Recorded Talk

A knot in mathematics is similar to a knot in real life. Imagine that you take a piece of string and tangle it up. Then, glue the two ends of the string together. When we draw a knot on paper, we draw a diagram with over-under crossing information. However, there are a couple more rules to these theoretical knots. A knot is an embedding of the circle in 3D space. An embedding simply means that the circle does not touch itself anywhere.

A knot diagram can be deformed to another knot diagram of the same knot. In order to deform a knot, we use the three Reidemeister moves to assure that our knot does not pass through itself while being deformed, because that would result in a completely different knot.

There are knot invariants which are numbers that can be used to tell different knots apart. One of the most important invariants for our purposes is the 3-coloring and, by extension, the p-coloring. The 3-coloring assigns 3 different colors to a knot, using the following rules: each arc (meaning if you follow a strand of the knot, the arc will end at a crossing where that strand is under) is assigned a color, at each crossing the 3 strands either have to be all the same color or three different colors, and at least 2 of the colors must be used in the entire diagram. If the determinant is divisible by 3, then it is 3-colorable. The p-coloring is a similar invariant where the "3" is replaced by any prime number p. However, for the p-coloring, at each crossing, the sum of the two bottom strands must be equivalent to twice the upper strand mod p.

We studied surfaces bounded by knots, where the surface lies in the 4th dimension. The 4-genus of the knot is the minimum number of holes in any such surface. These surfaces are defined by the amount of births, deaths, and saddles they have, which can be used to compute the 4-genus. A p-coloring of the knot will extend over the surface when the saddle moves can be applied using only strands that share the same color. We then explored for which knots the minimal 4-genus and the minimal p-colored 4-genus are the same.

Afternoon Talks

2:00-2:15 PM *Gender-specific experiences of women undergraduate STEM students in online mathematics courses during a pandemic* Erika Fiore (University of Rhode Island) Live Talk

This presentation will give an overview of the preliminary findings of the research I have conducted for my dissertation. I have collected and began to analyze data for a qualitative interview study seeking to investigate the gender-specific experiences of women undergraduate STEM students taking online mathematics courses during a pandemic. This study aims to provide recommendations to instructors of mathematics who seek to create an equitable online learning experience for their students. Introductory math classes have proved to be a significant barrier to degree completion for students in STEM. In particular, women are more likely to leave STEM fields after failing a math course than their male peers. After the switch to remote instruction in March 2020, most students and instructors across the country were forced to learn and teach in an online format. Many of these students and instructors were dissatisfied with their online experience, while others began to realize the opportunities that online learning has to offer, especially for some marginalized populations. I interviewed 9 women in STEM about their experiences taking math courses online during a pandemic. Participants identified the ability to work at their own pace and flexibility of online courses as benefits of online learning. On the other hand, some students struggled staying focused and craved more interaction with other students and attention from instructors. After further analysis, a more comprehensive description of the lived experiences of my participants will be developed.

2:20-2:35 PM *Gödel, Kafka, and the Incompleteness of Ethics* Anna Lowery (Smith College) Live Talk

In this talk, I utilize Gödel's Incompleteness Theorems to discuss ethics, framing the discussion with Kafka's novel The Castle. In the novel, the titular Castle controlls the nearby village with a complex system of rules and laws. We can look at this ethical system through the lens of Gödel's theorems. A consistent axiomatic system is considered complete if every proposition in the system can be proven true or false within the system; a consistent ethical system is "complete" if every ethical situation can be proven right or wrong within the system. We see that although the Castle acts as though its system is complete, it is riddled with issues; for example, the protagonist's role in the village is undecided and cannot be resolved by the Castle's rules. Using Gödel's result, we can see that the Castle's system will not ultimately be able to resolve every issue. This enables us to think critically about what to do when an ethical system presents a situation as right when it is evidently harmful. If an ethical system is incomplete to begin with, should we cohere to its rules when the rules have a negative effect on us or the people around us? Or should we allow ourselves to look outside the system for a solution to an ethical problem?

2:20-2:35 AM *Disentangling a Triangle* Kathryn Anderson (Keene State College) Live Talk

The basic functions in triangle trigonometry are the sine, cosine, tangent, cotangent, secant, and cosecant. There are also many relationships between them such as, for any pair of angles x and y, then $\sin(x+y) = sin(x)cos(y)+sin(y)cos(x)$. This presentation will show how many of the relationships between the trigonometric functions can be illustrated using a triangle that is inside a circle and that has its vertices on a circle.

2:40-2:55 PM *Minimal Presentation Sizes of Numerical Semigroups* Hannah Park-Kaufmann (Bard College) Live Talk

A numerical semigroup is a subset of integers closed under addition, while a minimal presentation is a choice of minimal relations between generators of the numerical semigroup. It is a well-known fact that if m is the smallest positive element, then the size of the minimal presentation is at most $\binom{m}{2}$. Finding the possible minimal presentation sizes of numerical semigroups whose smallest positive element, or multiplicity, is m has been a long-standing open problem. In this talk, we introduce the role of embedding dimension in determining the attainable minimal presentation sizes. For each pairing of multiplicity and embedding dimension, we present multiple classes of numerical semigroups and pose upper and lower bounds. Our methods are not only combinatorial, but also involve posets and Betti elements.

2:40-2:55 PM *Prime geodesics on compact hyperbolic 3-manifolds* Lindsay Dever (Bryn Mawr College) Live Talk

The study of hyperbolic 3-manifolds draws deep connections between number theory, geometry, and topology. An important geometric invariant of a hyperbolic 3-manifold is the set of its closed geodesics, which are parametrized by their length and holonomy. It turns out that for geodesics of increasing lengths, holonomy is equidistributed throughout the circle; it is equally likely to land in any interval of a given size. In this talk, I'll introduce compact hyperbolic 3-manifolds and present new results on the distribution of holonomy, including equidistribution in shrinking intervals.