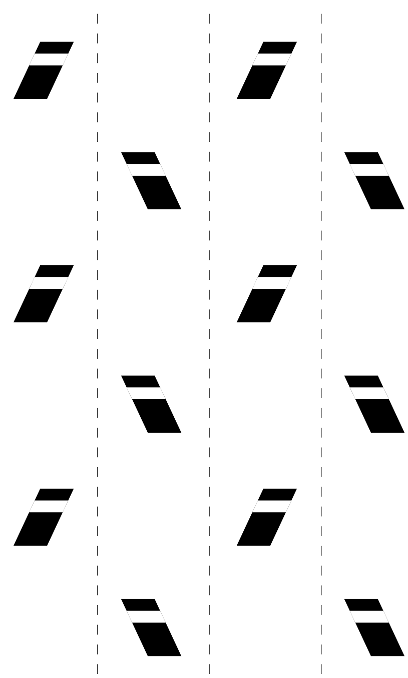
Lecture Notes - Mineralogy - Plane Patterns

- Back in September we observed the constancy of interfacial angles in all crystals of the same mineral. Based on this observation we concluded that minerals are built according to a pattern. We identified vectors called **translations** that represent the directions and magnitudes of offset that are characteristic of the repetition in the pattern. Three non-coplanar translations were used to define a **unit cell**, a parallelpiped that contains all the essential element of the pattern. In the context of a pattern of infinite extent, the operation of repeating the unit cell by offset (**translation**) is a symmetry operation: the pattern is unchanged by the operation of translation. We defined the **lattice** of a crystal as a set of points that are equivalent by translation.

- Because patterns (as opposed to objects) have translational symmetry, they may also posses symmetry elements that are combinations of translation and reflection (**glide planes**) or translation and rotation (**screw axes**). Two-dimensional patterns can contain glide planes and many popular wallpaper patterns repeat motifs by glide reflection. Three dimensions are needed for the screw rotation that leads, for example, to the helix popular among organic molecules.

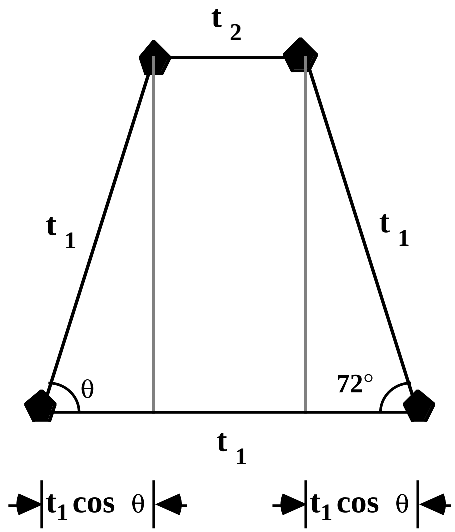
- Glide planes repeat motifs by (a) translation parallel to the glide plane, followed by (b) reflection across the glide plane. Typically, the magnitude of the translation part of the glide reflection is one half the lattice translation parallel to the glide translation. In three dimensions, the glide plane is identified by the unit cell translation that is parallel to the glide translation (*e.g.* **a-**, **b-**, or **c-glide**). Diagonal glides are also possible. Glide reflection is an **enantiomorphous** operation.



- Screw axes repeat motifs by (a) translation parallel to the screw axis, followed by (b) rotation about the screw axis. Typically, the magnitude of the translation part of the screw rotation is equal to the lattice translation parallel to the screw axis divided by the "fold" of the rotation part of the screw rotation. Thus, a 6-fold screw axis would translate a motif 1/6 of the lattice translation parallel to the screw axis and rotate the motif 60° about the screw axis. Screw rotation is a **congruent** operation. However, screw axes may be either right-handed or left-handed.

- Because all symmetry elements of a pattern repeat one another, only certain collections of symmetry elements are possible for patterns. These collections are the **17 plane groups** in two dimensions and the **230 space groups** in three dimensions. **All** two-dimensional patterns belong to one of the 17 plane groups described in the handout. All minerals have crystal structures that belong to one of the 230 space groups described in the *International Tables for X-ray Crystallography***.** This means that it is not necessary to identify all the symmetry elements of a two-dimensional pattern or crystal structure from scratch. Only enough symmetry elements need be idenified to place the pattern or structure in one of the possible groups.

- As an example of the interaction of symmetry elements that limits the number of possible space groups, consider the possibility of a 5-fold axis in a two-dimensional pattern. The 5-fold axis repeats the translation $\mathbf{t}_1$ and the translation repeats the 5-fold axis. Note that these combinations lead to a new translation $\mathbf{t}_2$ that is parallel to $\mathbf{t}_1$. If the result is to be a pattern, then $\mathbf{t}_2$ must be an integer multiple of $\mathbf{t}_1$. From the figure and a bit of trigonometry, it follows that $\mathbf{t}_2 = \mathbf{t}_1 - 2\,\mathbf{t}_1 \cos\theta = \mathbf{t}_1(1 - 2\cos\theta)$. Thus, $(1 - 2\cos\theta)$ must be an integer. The possibilities are as follows:



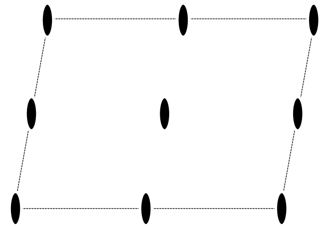
Integer	**Cos θ**	**θ**	**Axis**
0	.5	60°	6-fold
1	0	90°	4-fold
-1	1	360°	1-fold
2	-.5	120°	3-fold
-2	1.5	Does not exist	
3	-1	180°	2-fold
All others		Do not exist	

This is the reason that **only** 1-, 2-, 3-, 4-, and 6-fold rotation axes are observed in minerals.

- The symmetry elements of a pattern operate on every point in the pattern, every atom in the crystal. The number of times a motif is repeated by the symmetry elements in one unit cell of a pattern (its **multiplicity**) depends on the coordinates of the motif. A motif with no symmetry of its own must be located in a **general position** in the pattern. A motif with symmetry of its own may be located **on** a symmetry element, *i.e.* on a **special position**. The multiplicity of a special position will be less than the multiplicity of a general position. A pattern may have a number of different special positions with different multiplicities located on non-equivalent symmetry elements. A knowledge of special positions, which are tabulated in the *International Tables for X-ray Crystallography*, may be used to help locate the positions of atoms in the unit cell of a crystal of known symmetry. Determining the multiplicity of a special or general position should remind you of determining the number of faces in a crystal **form** {hkl}. For example, only six faces are required for the cube form {100}, whereas twelve faces are required for the dodecahedron from {110} in the crystal class 4/m $\bar{3}$2/m.

The 17 Plane Patterns

p1



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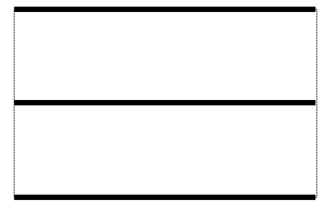

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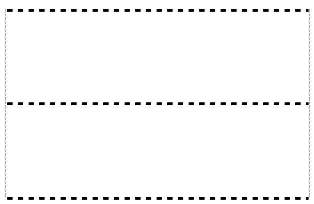

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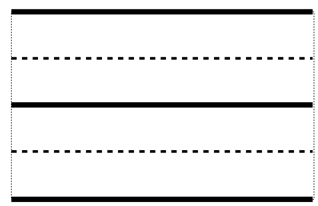

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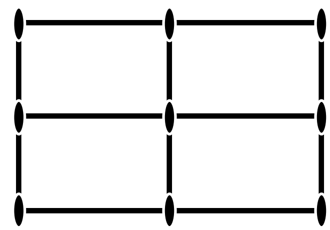
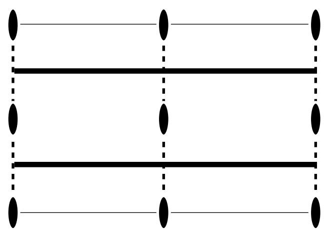


p2mm



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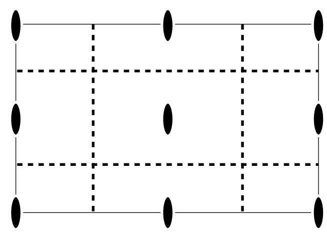

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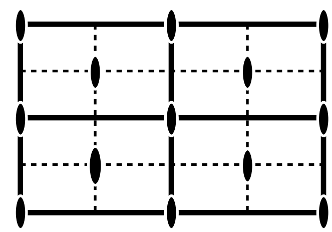

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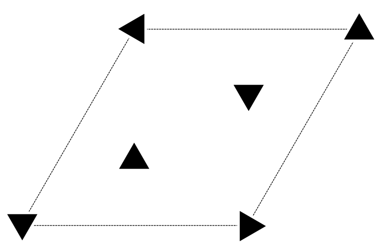

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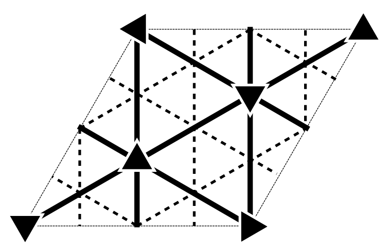
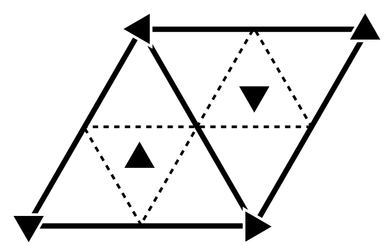


p3m1



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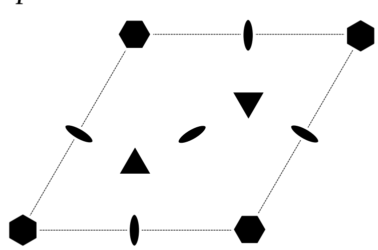

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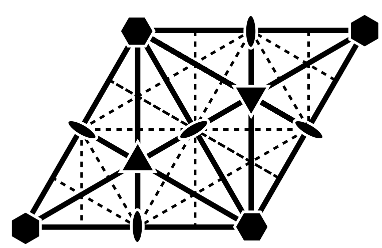

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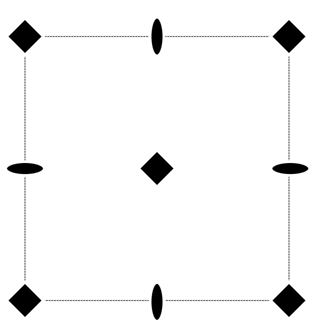
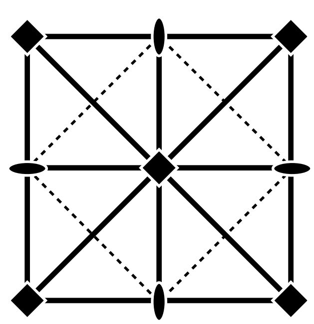
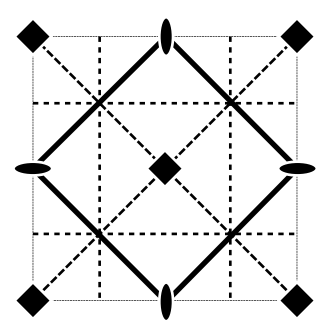


p4



p4mm



p4gm

2-fold axis 4-fold axis mirror plane

3-fold axis 6-fold axis glide plane