Lecture Notes - Mineralogy - Crystal Classes

- All patterns, including minerals, may be grouped according to the **symmetry** that they possess. Symmetry is defined as "invariance to an operation." The operations involved (rotation, reflection, translation, and their combinations) are called **symmetry operations**.

- Objects, such as single crystals, wooden blocks, people, etc., may be invariant to symmetry operations (rotation, reflection, and their combinations) that leave at least one point in space unmoved. The collection of such operations that characterize an object are termed the point symmetry, **point group**, or **crystal class** of the object.

- Objects that are invariant to a symmetry operation are said to contain a **symmetry element**, the collection of points that occupies a special position with respect to the symmetry operation. Objects that are invariant to a **rotation** contain a **rotation axis**. Objects invariant to a **reflection** contain a **mirror plane**. Objects invariant to the combined operation of rotation about an axis followed by reflection across a plane perpendicular to that axis contain a **rotoreflection axis**.

- Point symmetry operations have the property that repeating the operation will eventually return the object to its original position. For example, rotation of 180° followed by another rotation of 180° is equivalent to rotation of 360° (or 0°). Thus, the axis for rotation of 180° is called a "2-fold axis," because repeating the operation twice returns the object to its original position. Crystals can have only 1-fold, 2-fold, 3-fold, 4-fold, and 6-fold rotation or rotoreflection axes. 5-fold, 7-fold, etc. rotation axes cannot occur in patterns (we will prove this later), although objects that are not crystals can have these axes.

- If an object possesses 2-fold rotoreflection symmetry about one axis, it possesses 2-fold rotoreflection symmetry about all axes. For this reason, 2-fold rotoreflection symmetry is given a special name: **inversion**. Because the orientation of the axis doesn't matter, the symmetry element is termed an **inversion center** (the one point unmoved by this combined operation). Every rotoreflection operation may be represented instead by the combined operations of rotation followed by inversion (**rotoinversion**). For various reasons, our text (and most other texts) use **rotoinversion axes** rather than rotoreflection axes. Using a bar (¯) to represent rotoinversion and a tilde (˜) to represent rotoreflection we have:

$$\bar{1} = \tilde{2} = i = \bar{\mathbf{R}}_{360^\circ} = \tilde{\mathbf{R}}_{180^\circ}$$
$$\bar{2} = \tilde{1} = \mathrm{m} = \bar{\mathbf{R}}_{180^\circ} = \tilde{\mathbf{R}}_{360^\circ}$$
$$\bar{3} = \tilde{6} = 3{+}i = \bar{\mathbf{R}}_{120^\circ} = \tilde{\mathbf{R}}_{60^\circ}$$
$$\bar{4} = \tilde{4} = \bar{\mathbf{R}}_{90^\circ} = \tilde{\mathbf{R}}_{90^\circ}$$
$$\bar{6} = \tilde{3} = \frac{3}{\mathrm{m}} = \bar{\mathbf{R}}_{60^\circ} = \tilde{\mathbf{R}}_{120^\circ}$$

  where i is an inversion center, m is a mirror plane, 3/m is a 3-fold axis with a perpendicular mirror plane, and $3+i$ is a 3-fold axis plus a center of symmetry.

- Reflection, rotoreflection, inversion, and rotoinversion are all **enantiomorphous** operations: they change the "hand" of the object. This is to be contrasted with rotation, which is a **congruent** operation. It is possible to rotate an object physically to confirm rotation symmetry. Enantiomorphous symmetry can be verified "only with mirrors."

- A crystal may possess only certain combinations of symmetry elements. Only **32** possibilities exist and these are the 32 **crystal classes** or **crystallographic point groups**. Every mineral belongs to one of these crystal classes. Details of the 32 point groups are given in Klein and Hurlbut (p.60-103) and in the attached handout. This is reference material that will always be available to assist you.

- The 32 crystal classes may be grouped into seven **crystal systems** according to the following criteria:

System	**Identifying Criteria**	**Lattice Constraints**
Triclinic	One 1-fold or $\bar{1}$ axis	no constraints
Monoclinic	One 2-fold or $\bar{2}$ (=m) axis	$\alpha=\gamma=90°$ (2nd setting)
Orthorhombic	Three 2-fold or $\bar{2}$ axes	$\alpha=\beta=\gamma=90°$
Trigonal	One 3-fold or $\bar{3}$ axis	$a=b$, $\alpha=\beta=90°$, $\gamma=120°$
Hexagonal	One 6-fold or $\bar{6}$ axis	$a=b$, $\alpha=\beta=90°$, $\gamma=120°$
Tetragonal	One 4-fold or $\bar{4}$ axis	$a=b$, $\alpha=\beta=\gamma=90°$
Isometric	Four 3-fold or $\bar{3}$ axes	$a=b=c$, $\alpha=\beta=\gamma=90°$

- The crystal classes are identified in the textbook using **Hermann-Mauguin notation**. An alternative system was developed by Schöenflies and is used in some texts. The Hermann-Mauguin notation identifies rotation or rotoinversion axes (if present) along particular directions and mirror planes (if present) perpendicular to the same directions. The directions used by the notation are **different** for each of the six crystal systems as follows (see Klein and Hurlbut, Table 2.9):

Triclinic	no particular directions		
Monoclinic	**b**-axis	(2nd setting)	
Orthorhombic	**c**-axis	**a**-axis	**b**-axis
Tetragonal	**c**-axis	**a**-axis	**[110]**
Hexagonal	**c**-axis	**a**-axis	**[210]**
Isometric	**c**-axis	**[111]**	**[110]**

- Why are we talking about all this esoteric symmetry stuff? The main reason is "a fundamental postulate of crystal physics, known as **Neumann's Principle**. It may be stated as follows:

  The symmetry elements of any physical property of a crystal must include the symmetry elements of the point group of the crystal." (Nye, p.20)

  According to Neumann's principle, any physical property we may measure – *e.g.* hardness, thermal conductivity, refractive index, x-ray diffraction – must vary with direction in symmetrical ways such that all of the symmetry elements of the point group (and perhaps more) are present for that physical property. This can be of tremendous aid in identifying, characterizing, and understanding minerals.

Stereographic Projections of the Symmetry Elements in the 32 Crystal Classes