TALKS

Sarah Brauner, University of Minnesota
Card shuffling, q-analogues and derangements

How many times do you need to shuffle a deck of cards to ensure it is adequately mixed? This is a question in probability theory, but for many methods of card shuffling, the answer relies on combinatorics and representation theory. In this talk, I will discuss several classical card-shuffling processes and introduce their q-analogues, which can be understood as random walks on the space of complete flags of a vector space over a finite field. Motivated by questions of mixing times, I will present recent results and conjectures concerning the eigenvalues and eigenspaces of these (q-)shuffling operators. Along the way we will see derangements, desarrangements, and tableau combinatorics. This is joint work with Commins and Reiner, as well as Axelrod-Freed, Chiang, Commins and Lang.

Theo Douvropoulos, UMass Amherst
Recursions and Proofs in Coxeter-Catalan combinatorics

The collection of parking functions under a natural Sn-action (which has Catalan-many orbits) has been a central object in Algebraic Combinatorics since the work of Haiman more than 30 years ago. One of the lines of research spawned around it was towards defining and studying analogous objects for real and complex reflection groups W; the main candidates are known as the W-non-nesting and W-non-crossing parking functions.

The W-non-nesting parking functions are relatively well understood; they form the so called algebraic W-parking-space which has a concrete interpretation as a quotient ring. The W-non-crossing ones on the other hand have defied unified explanations while simultaneously proving themselves central in the study of Coxeter and Artin groups (their geometric group theory, representation theory, and combinatorics). One of the main open problems in the field since the early 2000's has been to give a type-independent proof of the W-isomorphism between the algebraic and the non-crossing W-parking spaces. In this talk, I will present the first such proof, solving the more general Fuss version of the problem, that proceeds by comparing a collection of recursions that are shown to be satisfied by both objects. This relies on a variety of recent techniques we introduced, in particular the enumeration of certain flats of full support via Crapo's beta invariant, the W-Laplacian matrices for reflection arrangements and, in collaboration with Matthieu Josuat-Verges, the refined f- to h- transformation between the cluster complex and the non-crossing lattice of W.

Tamar Friedmann, Colby College
Counting conjugacy classes of elements of finite order in Lie groups

Dan Johnston, Trinity College
Rainbow Saturation

A graph G is rainbow H-saturated if there is some proper edge coloring of G which is rainbow H-free (that is, it has no copy of H whose edges are all colored distinctly), but where the addition of any edge makes such a rainbow H-free coloring impossible. Taking the maximum number of edges in a rainbow H-saturated graph recovers the rainbow Turan numbers whose systematic study was begun by Keevash, Mubayi, Sudakov, and Verstraete. In this talk, we introduce and examine the corresponding *rainbow saturation number* -- the minimum number of edges among all rainbow H-free graphs.

Nadia Lafreniere, Dartmouth College
A study of homomesies on permutations using the FindStat database

We performed a systematic study of permutation statistics and bijective maps on permutations, looking for the homomesy phenomenon. Homomesy occurs when the average value of a statistic is the same on each orbit of a given map. The maps that exhibit homomesy include the Lehmer code rotation, the reverse, the complement, and the Kreweras complement, many of which have some geometric interpretations. The statistics studied relate to familiar notions such as inversions, descents, and permutation patterns, among others. Beside the many new homomesy results, I'll discuss our research method, in which we used SageMath to search the FindStat combinatorial statistics database to identify potential homomesies. I'll briefly discuss how this technique could be extended.

This is joint work with Jennifer Elder, Erin McNicholas, Jessica Striker and Amanda Welch.

Goran Malic, Smith College
Circuit polynomials in algebraic matroids, computation and algorithms

The collection of circuits in an algebraic matroid M (over a prime ideal P) is in a 1-1 correspondence with irreducible polynomials in P with minimal support. These polynomials are called circuit polynomials. Ileana Streinu and I have developed an algorithm to compute these polynomials when M is the algebraic rigidity matroid. Our algorithm requires computation of only one determinant, whereas previously a Grobner Basis computation was required, which would crash even on small examples. Our algorithm can be generalized to certain classes of matroids in which all circuits that are not of minimal size have what we call a combinatorial resultant decomposition. In this talk I will present our algorithm in the case of the rigidity matroid, as well as its generalization, and mention a number of open questions. This is joint work with Ileana Streinu.

Lauren Rose, Bard College
EvenQuads, Finite Geometry, and Sidon Sets

In this talk we introduce the card game Quads, published by the AWM as EvenQuads. Quads is a SET-like game that can be viewed as a model for the affine geometry AG(6,2), and can be generalized to AG(n, 2) for any n. We are most interested in collections of cards that don't contain a Quad, which turn out to be Sidon Sets. We describe an analog of the "Cap Set problem" for SET, providing results and work in progress.

Tom Tucker, Colgate University
Revisiting the Torus: Geometry and Algebra

POSTERS

Pablo Castilla (UMass)
Perturbed Ehrhart Polynomials and the Non-negativity of its h^*-polynomial

William Dugan (UMass)
Faces of Generalized Pitman-Stanley Polytopes

The Pitman-Stanley polytope is a convex polytope whose integer lattice points biject onto the set of plane partitions of a certain shape with entries in { 0 , 1 }. In their original paper, Pitman and Stanley further suggest a generalization of their construction depending on a natural number m whose integer lattice points biject onto the set of plane partitions of the same shape having entries in { 0 , 1, ... , m }. In this poster, we give further details of this **generalized Pitman-Stanley polytope**, $PS_n^m(\mathbf{a})$, demonstrating that it can be realized as the flow polytope of a certain graph. We then use the theory of flow polytopes to describe the faces of these polytopes and produce a recurrence for their f-vectors.

Melanie Ferreri (Dartmouth)
Bijections for generalized Wilf equivalences

An inversion sequence is an integer sequence $e = e_1e_2 \cdots e_n$ such that $0 \leq e_i < i$ for all $1 \leq i \leq n$. A pattern is a sequence $p = p_1p_2 \cdots p_r$ with $p_i \in 0, 1, \ldots, r-1$ for all $1 \leq i \leq r$ such that j can only appear in p if j-1 also appears. An inversion sequence e is said to contain the consecutive pattern p if there is a consecutive subsequence of e whose reduction is p; otherwise, e is said to avoid p. We say two patterns p and p' are Wilf equivalent if the number of inversion sequences of length n which avoid p is equal to the number of inversion sequences of length n which avoid p'. In his thesis, Auli characterizes generalized Wilf equivalences for all 75 consecutive patterns of length 4, proving some with direct bijections and some via inclusion-exclusion arguments. For those proved via inclusion-exclusion, we derive direct bijections proving these equivalences.

Felicia Flores (Bard)
2-Caps in the Game of EvenQUADS

We studied EvenQUADS, a variant of the popular card game SET®. A Quad is 4 cards that satisfy a certain pattern. Our goal was to find and classify collections of cards that don't contain a Quad, called 2-caps. In particular, for each k, we classified 2-caps that contain k distinct triples of cards in the 2-cap that determine the same fourth card. This game is modeled by the affine geometry AG(n,2), allowing us to study this problem in higher dimensions.

Josef Lazar (Bard)
A Closer Look at Projective SET

Kathy Lin (Dartmouth)
The first occurrence of a pattern in a random sequence

We consider the permutation analogue of Penney's game for words. Two players, in order, each choose a permutation of length k where k is at least 3; then a sequence of independent random values from a continuous distribution is generated, until the relative order of the last k numbers coincides with one of the chosen permutations, making that player the winner.

We compute the winning probabilities for all pairs of permutations of length 3, showing that, like the original version for words, the game is non-transitive. We give some formulas to compute the winning probabilities more generally, and we conjecture a winning strategy for the second player when k is arbitrary.

Alejandro Morales (UMass)
Enumeration of vertices of generalized permutahedra for max-pooling responses

Generalized permutahedra is an important family of polytopes defined by Postnikov with very interesting face structure and volume formulas. The combinatorics of their faces were further studied by Postnikov, Reiner and Williams. We study a generalized permutahedra that is the Minkowski sum of certain simplices motivated from a problem in machine learning. We give generating functions and asymptotics for the number of vertices (arxiv:2209.14978).

Guillermo Nunez Ponasso (WPI)
Hadamard's Maximal Determinant Problem and Generalizations

Hadamard's inequality gives an upper bound for the determinant of a matrix with entries in the complex unit disk. If the matrix has entries in {+1,-1} then this bound is achieved only if its order is 1, 2 or a multiple of 4. Hadamard's maxdet problem asks about the maximum value of the determinant for {+1,-1} matrices at all orders. We consider this problem together with some natural generalizations motivated by a refinement of Hadamard's inequality.