Recap and Overview

• Review complex mathematics
• Phasors – more details
• Transformation between time and phasor domains
• Phasor form for circuit elements
  - Impedance: $Z = R + jX$

Phasors

• A phasor is a vector
• A phasor has:
  - A magnitude (amplitude), and
  - A phase relative to a reference phasor
    • This is similar to a node having a voltage value relative to the ground node

Time Domain Image of Phasors
Phasors: Euler’s Formula for the complex exponential

\[ re^{\pm j\phi} = r\cos \phi \pm j r\sin \phi \]

\[ = x + jy \]

So \( r = \) \( \theta = \)

\[ \cos \phi = \text{Re} \left( e^{\pm j\phi} \right) \]
\[ \sin \phi = \text{Im} \left( e^{\pm j\phi} \right) \]

Phasors: Using Euler

- Using Euler’s formula, express the following sinusoidal function as phasor

\[ v_1(t) = V_m \cos(\omega t + \varphi) \]
Phasors: Using Euler

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Define a phasor: \[ \textbf{V} = V_m e^{j\varphi} = V_m \angle \varphi \]
\[ v_1(t) = \text{Re}\left( V e^{j\omega t} \right) \]

- Note: temporarily stop writing the frequency term, \( e^{j\omega t} \)
- Note: use \( \cos() \) not \( \sin() \) (… to ensure real functions, or signals)

Polar Form ↔ Rectangular Form

- Express the complex number in
  - Rectangular form: \( x + jy \)
  - Polar form:
  - Euler’s formula:
- Important conversions (plot these numbers)
  - \( j^2 = -1 \) (degrees)
  - \( 1/j = \) ______
  - Note that \( 1/j = -j \)
Complex Math Examples

• Simplify and express in rectangular form

\[ \frac{2 + \frac{3 + j4}{5 - j8}}{} \]

Complex Math Examples

• Simplify and express in rectangular form

\[ (4 \angle -10^\circ) + \left(\frac{1 - j2}{3 \angle 6^\circ}\right) \]

Phasor Problem

• Write the sinusoid form for the following

\[ V = 60 \angle 15^\circ, \; \omega = 377 \]

\[ V = 2.8e^{-j\pi/3}, \; \omega = 10^3 \]

Phasor Problem

• Find a single sinusoid for the following

\[ V = -30 \angle 10^\circ + 50 \angle 60^\circ \]
Phasor Problem
• Find a single phasor for the following
  \[ 3\cos(20t + 10°) - 5\sin(20t - 30°) \]

The Process of Transformation
• If \( v(t) = V_m \cos(\omega t + \phi) \) (note ‘cos’ vs. ‘sin’)
  o Express \( \frac{dv(t)}{dt} \) as a phasor.

Phasors for Circuit Elements
• Transform voltages and currents to phasors
  o Plot these phasors in the complex plane
• \( v(t) = i(t)R \) transforms to…
Phasors for Circuit Elements

• Transform and plot the phasors in the complex plane
• \( v(t) = L \frac{di(t)}{dt} \) transforms to...

• \( i(t) = C \frac{dv(t)}{dt} \) transforms to...

(Also, \( v(t) = (1/C) \int i(t) \) transforms to...)

Summary of Phasors for Circuit Elements

- \( v(t) = i(t)R \) \( \Rightarrow \) \( V = \)
- \( v(t) = L \frac{di(t)}{dt} \) \( \Rightarrow \) \( V = \)
- \( i(t) = C \frac{dv(t)}{dt} \) \( \Rightarrow \) \( I = \)
- \( V \) source \( \Rightarrow \)
- \( I \) source \( \Rightarrow \)

Defining Impedance

- Impedance, \( Z \), is defined as the ratio -----
- As a complex number, \( Z = \)__________

- Resistor: \( Z_R = \)
- Inductor: \( Z_L = \)
- Capacitor: \( Z_C = \)
Today’s Concepts

- AC input signals for circuits & circuit behavior
- Sinusoids
  - Euler’s identity and sinusoids
- Complex numbers in all their forms
  - Converting between representations
- Phasors (~ vectors)
  - Simple representation of sinusoids
  - Representation of ‘Impedance’ = energy dissipation + energy storage
- Phasor ‘domain’ and transformation

For Reference: Properties page 371

- $\sin(A \pm B)$ and $\cos(A \pm B)$
- $\cos(\omega t \pm 90^\circ) = \mp \sin(\omega t)$
  - Or $\mp \sin(\omega t) = \cos(\omega t \pm 90^\circ)$
  - So $- \sin(\omega t) = \cos(\omega t + 90^\circ)$
  - $\sin(\omega t) = - \cos(\omega t + 90^\circ)$
  - $\cos(\omega t) = \cos(\omega t - 90^\circ)$
- Also $\sin(\omega t \pm 90^\circ) = \pm \cos(\omega t)$
- And $\cos(\omega t \pm 180^\circ) = - \cos(\omega t)$
Practice for Exam 2

• For each 1st and 2nd Order Circuit, set up the problem.
  a) Identify the type of circuit and write out the general final solution expression, $v(t)$ or $i(t)$
  b) Specify/identify what you need to find
  c) What are the initial conditions, and what does the circuit look like for $t < 0$?
  d) What are the final conditions, and what does the circuit look like for $t \to \infty$?
  e) What does the circuit look like during the transient period?
  f) What values do you need to find in order to fully characterize the circuit behavior for all $t > 0$?
  g) Sketch a graph of possible expected output behavior.
(add a source for t=0 to make analysis more interesting)