Sinusoidal Input: Phasors & Impedance ...

... and RLC Review

EGR 220, Chapter 9
November 6, 2018

Overview

- Introduction to phasors
  - Review on own polar notation and rectangular coordinates, and the conversion back and forth
- Review of RLC circuits and Lab 6a

Recap: Unit Step Function

$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$

$u(-t) = \begin{cases} \_\_\_\_\_, & t < 0 \\ \_\_\_\_, & t > 0 \end{cases}$

- “Unit” means “1”
- The value is always either ‘1’ or ‘0’
- Which $u(.)$ above is a switch turning “on” and which is a switch turning “off”?

Switch/Step input

Second Order Circuit

$x(t) = Ae^{\alpha t} + Be^{\beta t}$

$x(t) = (A + Bt)e^{-\alpha t}$

$x(t) = e^{\alpha t} (A\cos \omega t + B\sin \omega t)$
**AC input Sinusoidal Steady State**

- A phasor is a vector
- A phasor has:
  - A magnitude (amplitude), and
  - A phase relative to a reference phasor
    - This is similar to a node having a voltage value relative to the ground node

**Phasors**

- Magnitude and phase are represented mathematically with complex numbers
  - Sinusoidal voltage and current
  - Energy dissipation and storage elements

**Third Part of Course**

- AC (alternating current) circuits
  - Voltage and current sources alternate the polarity (sine and cosine input signals)
  - Introduce frequency and time as considerations in circuit analysis
- Review complex math (QLC or PHY 210)
  - Transform to and from ‘phasor domain’
Third Part of Course

• AC (alternating current) circuits
  o Voltage and current sources alternate the polarity (sine and cosine input signals)
  o Introduce frequency and time as considerations in circuit analysis

• Review complex math (QLC or PHY 210)
  o Transform to and from ‘phasor domain’

• Single frequency: Electric power systems
  o Sinusoidal Steady-State & Phasors

• Multiple frequencies: audio, etc.

Sinusoids

• Amplitude?
• Frequency?
• Phase angle?
  o (always relative to a reference signal)
• $v_2 = ?$
• Which signal “leads” the other?

• Which are the most useful mathematical tools to add, subtract, multiply and divide sinusoids?
Phasors: Euler’s Formula for the complex exponential

\[ re^{\pm j\phi} = r \cos \phi \pm jr \sin \phi \]
\[ \cos \phi = \text{Re}\left(e^{\pm j\phi}\right) \]
\[ \sin \phi = \text{Im}\left(e^{\pm j\phi}\right) \]

(Note: see page 321, chapter 8, for “Where is the ‘j’ in constants A1 & A2?”)

Dynamic Circuits Review
**RLC Concept Discussion**

Discuss with your neighbors:

- What are the main concepts for dynamic circuits?
- What behavior is expected from a dynamic circuit?
  - 1st order vs. 2nd order, vs higher order
  - Sketch the behavior options
- Identify role, or impact, of each parameter in the solution expression on each graph
  - $\tau$, $\alpha$, $\omega_0$, $\omega_d$, initial & final state (or condition)
- What is resonance?

**RLC Concept Review**

- What are the differences between:
  - Series RLC circuit
  - Parallel RLC circuit
  - General RLC circuit
- What is meant by
  - The *behavior* of a circuit
  - The *response* of a circuit

**RLC Concept Review**

- In a series RLC circuit, setting $R = 0$ will produce
  - a) an overdamped response
  - b) a critically damped response
  - c) an underdamped response
  - d) an undamped response
  - e) none of the above
- What happens in a parallel RLC circuit with a very small $R$?

**RLC Concept Review**

- The response of the circuit will be
  - a) overdamped
  - b) underdamped
  - c) critically damped
  - d) none of the above
**RLC Concept Review**

- If $v_L(0) = 5V$, what do we know about $dv_L/dt$?
  - a) $dv_L/dt = 5 \text{ V/s}$
  - b) nothing
  - c) $dv_L/dt = 0 \text{ V/s}$
  - d) none of the above

- Think of the notation $\frac{dv_L(t)}{dt}|_{t=0}$

**2nd Order Circuit Review**

- Dynamic circuits are either in
  - Steady-state, or...
  - Transition from one steady-state to the next

- Our steady-states
  - For $t < 0$ and $t = \infty$
  - All energy storage elements are fully charged
  - All ‘C’ are open ckts, all ‘L’ are short ckts

- Transient behavior (in ‘transition’)
  - Underdamped $\rightarrow$ ‘ringing’ behavior
  - Critically- or over-damped behavior
Resonance

• A vibration of large amplitude in a mechanical or electrical system caused by a relatively small periodic stimulus of the same or nearly the same period as the natural vibration period of the system.
• (Merriam–Webster)

Resonant Circuits

• Characteristics are:
  o Sharp peak in amplitude of output at $f_0$.
  o Oscillation of energy from one form to another
    • So we must have two different types of storage elements
    • Electric and magnetic fields charging and discharging 180° out of phase
  o More to come in chapter 14

Images for the Series RLC Step-Response Lab
(Lab 6, Part 1)
Notes for Series RLC lab circuit

- Series RLC circuit
  - $L$ becomes a short circuit so in steady-state $V_L = 0$
  - $C$ becomes an open circuit so $V_C = V_S$
  - $R$ becomes what? $V_R = $ what?

![RLC Circuit Diagram]
Examples of truncating the natural response in different places, which leads to interesting shapes as the superposition of the natural response + next step input add together.
Given position of truncating the natural response, compare the overshoot of this versus next slide.
Note at resonance, the ‘overshoot’ is all we see, and with the
+/- summing of VC + VL, can get huge amplitude. Need to rescale V/div
on vertical scale
VL – settling to 0V, since L becomes an open circuit

Note: $V_L$ settles to 0V since L becomes a short circuit and C becomes an open circuit
Phasors: Euler’s Formula for the complex exponential

\[ re^{\pm j\phi} = r \cos \phi \pm jr \sin \phi \]

\[ \cos \phi = \text{Re}\left(e^{\pm j\phi}\right) \]
\[ \sin \phi = \text{Im}\left(e^{\pm j\phi}\right) \]

(Note: see page 321, chapter 8, for “Where is the ‘j’ in constants A_1 & A_2?”)

Phasors: Using Euler

• Using Euler’s formula, express the following sinusoidal function as phasor

\[ v_1(t) = V_m \cos(\omega t + \phi) \]

\[ v_1(t) = \text{Re}\left(V_m e^{j(\omega t + \phi)}\right) = \text{Re}\left(V_m e^{j\omega t} e^{j\phi}\right) \]

Phasors: Using Euler

• Using Euler’s formula, express the following sinusoidal function as phasor

\[ v_1(t) = V_m \cos(\omega t + \phi) \]

\[ v_1(t) = \text{Re}\left(V_m e^{j(\omega t + \phi)}\right) = \text{Re}\left(V_m e^{j\omega t} e^{j\phi}\right) \]

Define a phasor: \[ \underline{V} = V_m e^{j\phi} = V_m \angle \phi \]

\[ v_1(t) = \text{Re}\left(\underline{V}e^{j\omega t}\right) \]
Phasors: Using Euler

- Using Euler's formula, express the following sinusoidal function as phasor
  \[ v_1(t) = V_m \cos(\omega t + \phi) \]
  \[ v_1(t) = \text{Re}(V_m e^{j(\omega t + \phi)}) = \text{Re}(V_m e^{j\omega t} e^{j\phi}) \]
  Define a phasor: \( \mathbf{V} = V_m e^{j\phi} = V_m \angle \phi \)
  \[ v_1(t) = \text{Re}(V e^{j\omega t}) \]

- Note: temporarily stop writing the frequency term, \( e^{j\omega t} \)
- Note: use \( \cos() \) not \( \sin() \) (… to ensure real functions, or signals)

Simplify math ... with complex numbers

- Express the complex number in
  - Rectangular form: \( x + jy \)
  - Polar form:
    - Exponential form:
      - Using Euler's formula:
  - Important conversions (plot these numbers)
    - \( j = \theta \) (degrees)
    - \( 1/j = \theta \)
    - Note that \( 1/j = -j \)

Complex Math Examples

- Simplify and express in rectangular form
  \[ \frac{2 + \frac{3 + j4}{5 - j8}}{} \]

Complex Math Examples

- Simplify and express in rectangular form
  \[ (4 \angle -10^\circ) + \left(\frac{1 - j2}{3 \angle 6^\circ}\right) \]
Phasor Problem

• Write the sinusoid form for the following
  \[ V = 60\angle15^\circ \ \text{V}, \ \omega = 377 \]

  \[ V = 2.8e^{-j\pi/3} \ \text{A}, \ \omega = 10^3 \]

Phasor Problem

• Find a single sinusoid for the following
  \[ V = -30\angle10^\circ + 50\angle60^\circ \]

Phasor Problem

• Find a single phasor for the following
  \[ 3\cos(20t + 10^\circ) - 5\sin(20t - 30^\circ) \]

Leading & lagging phasors

• Plot the voltage and current phasors in the complex plane
  a) \[ V = V_m \angle -\varphi; \ \ I = I_m \angle \delta \]
  b) \[ V = 10\angle30; \ \ I = 5\angle -90 \]
Time v. Phasor Domain

• A phasor is a special vector
  o Be familiar with Euler’s formula: the basis for converting between time and phasor domains
  • \( v(t) = (A_1e^{s_1t} + A_2e^{s_2t}) \) V
    o Is time dependent
    o Is always a real number
• \( V \) (phasor)
  o IS NOT time dependent
  o Typically is a complex number

Complex Algebra (for Phasors)

• Polar form
  o Easy to multiply and divide
• Rectangular form
  o Easy to add and subtract
  o Tedious to multiply and divide
• Complex conjugates
  o If \( z = x + jy \), which also \( = r \angle \phi = re^{j\phi} \)
  o Then \( z^* = \)

Today’s Concepts

• AC input signals for circuits & circuit behavior
• Sinusoids
  o Euler’s identity and sinusoids
• Complex numbers in all their forms
  o Converting between representations
• Phasors \( (~ \text{vectors}) \)
  o Simple representation of sinusoids
  o Representation of ‘Impedance’ = energy dissipation + energy storage
• Phasor ‘domain’ and transformation

For Reference: Properties page 371

• \( \sin(A \pm B) \) and \( \cos(A \pm B) \)
  • \( \cos(\omega t \pm 90^\circ) = +/- \sin(\omega t) \)
    o Or \( = \pm \sin(\omega t) = \cos(\omega t \pm 90^\circ) \)
    o So \( - \sin(\omega t) = \cos(\omega t + 90^\circ) \)
    o \( \sin(\omega t) = - \cos(\omega t + 90^\circ) \)
    o \( + \sin(\omega t) = \cos(\omega t - 90^\circ) \)
  • Also \( \sin(\omega t \pm 90^\circ) = \pm \cos(\omega t) \)
• And \( \cos(\omega t \pm 180^\circ) = - \cos(\omega t) \)