Problem 1

for $t < 0$: Assume $L \cdot C$ are fully charged

find $I_L(0), V_C(0)$

\[
\begin{align*}
V_C(0) &= V_{4R} = 15A \cdot 4\Omega = 60V \\
I_L(0) &= 0A \quad \text{because no current can flow in that branch with the capacitor behaving as an open circuit}
\end{align*}
\]

For $t = 0$

\[
\begin{align*}
V_0 &= 0 \\
I_0 &= 0 \\
\text{because there is no forcing function}
\end{align*}
\]

For $t > 0$ (but $< \infty$): $C \& L$ are fully charged at time $t=0^+$ and immediately begin discharging

Find $\alpha, \omega_0, S_1, S_2$

For this TRANSIENT Time Period, the circuit is a series RLC circuit

This is a series RLC circuit for ALL time, not only $t>0$
(Problem 1, cont)

Find $\alpha + w_0$

Series RLC: $\alpha = \frac{R}{2L} = \frac{10}{2 \cdot \frac{1}{4}} = 20$

$w_0 = \frac{1}{\sqrt{LC}} = \sqrt{\frac{1}{4 \cdot \frac{1}{100}}} = 10$

$\alpha > w_0$

For $\alpha > w_0$, we have an overdamped circuit, with a natural response of the form:

$v(t) = A_1 e^{st} + A_2 e^{s_2t}$

Find $s_1 + s_2$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0^2} = -20 \pm \sqrt{20^2 - 10^2} = -37.3, -2.68$

Form for complete response = natural response

Because $V_{in} = 0$

$v(t) = A_1 e^{-37.3t} + A_2 e^{-2.68t}$

The general shape will decay from $V_0 = 60V$ to $V_0 = 0V$ with no oscillation since the system is overdamped.
Problem 2

for $t < 0$

$I_i(0) = 12A$
$V_c(0) = 0V$

$t > 0$

This is a parallel RLC circuit, so

$$\alpha = \frac{1}{2RC} = \frac{1}{2(10)(10\Omega)} = 5$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \cdot 10m}} = 20$$

$\omega_0 > \alpha$, so this circuit is underdamped.

$t \to \infty$

At $t > 0$ the 12A source is off and the 1A source is on. $I_i = 1A$, $V_c(0) = 0V$

For an underdamped circuit, the natural response is of the form

$$i(t) = e^{-\alpha t} (A_1 \cos \omega_0 t + A_2 \sin \omega_0 t)$$

so we must find $A_2 = \sqrt{\omega_0^2 - \alpha^2} = 19.4$

Since $I_0 = 0$, the complete response = natural response,

$$i(t) = e^{-5t} \left( A_1 \cos 19.4t + A_2 \sin 19.4t \right) + 1A$$

For an underdamped circuit, the waveform will oscillate and decay with the specified exponential fit, $e^{-5t}$.
Problem 3

for $t < 0$

both sources are connected, and with $L \rightarrow$ short circuit and $C \rightarrow$ open circuit, the circuit can be viewed as

Notice that the capacitor is shorted out by the inductor so $V_C(0) = 0V$

$\bar{I}_L(0) = \frac{12V}{4\Omega} + 3 = 3 + 3 = 6A = I_L(0)$

for $t \rightarrow \infty$ Again the capacitor is shorted out by the inductor, so $\bar{V}_C(\infty) = 0V$

Now only the 3A source is active, so $\bar{I}_L(\infty) = 3A$

For $t > 0$ (but $< \infty$)

the circuit appears as

This is a parallel RLC circuit

$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 5 \cdot \frac{1}{20}} = 2$ \hspace{1cm} \omega_0 = \frac{1}{\sqrt{LC}} = 2$
(problem 3 cont)

so with $\alpha = \omega_0$, we have a critically damped circuit

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\alpha = -2$$

The natural response is of the form $i(t) = e^{-\alpha t}(A_1 + A_2 t)$

* This circuit has a non-zero forcing function for $t > 0$, so to solve for $A_1 + A_2$ (next HW)
you MUST add $I_0$ to the natural response

BEFORE solving for $A_1 + A_2$

**Complete response**

$$i(t) = i_f(t) + i_n(t)$$

$$= I_0 + e^{-\alpha t}(A_1 + A_2 t)$$

**so**

$$i(t) = 3 + e^{-2t}(A_1 + A_2 t)$$

↑ MUST include the "3" for $I_0$ at this point before solving for $A_1 + A_2$

**Expected output waveform**

- There could be some "undershoot" The main thing is to have no repeated oscillation

$6A$ ← $I_0$

$3A$ ← $I_0$ (>)
Given \( s^2 + 100s + 10^6 = 0 \)

This 2nd order diff eq defines, or describes, the behavior of a 2nd order circuit, \( \rightarrow \) a circuit w/ \( 1-R, 1-L + 1-C \)

Assume \( R = 2k \Omega \). Find \( L + C \)

**SOLUTION**

from §8.6, eq'n (8.47) we see that

\[
\frac{d^2i}{dt^2} + \frac{1}{RC} \frac{di}{dt} + \frac{1}{LC} i = \frac{I_s}{LC}
\]

substitute \( i(t) = Ae^{st} \)

OR from §8.4 eq'n (8.29)

\[
\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{1}{LC} v = 0 \quad \Rightarrow \quad Ae^{st} \left( s^2 + \frac{1}{RC} s + \frac{1}{LC} \right) = 0
\]

Matching the coefficients from either of these equations to the eq'n for our design circuit we see

\[ \frac{1}{RC} = 100 \quad \text{and} \quad 10^6 \leq \frac{1}{LC} \]

\[ R = 2k \quad \text{so} \quad C = \frac{1}{100 \cdot 2k} = 5 \mu F \quad + \quad L = \frac{1}{10^6 \cdot 5 \times 10^{-6}} = 0.2 H \]

**Circuit** - This circuit is a source-free circuit. To make it not have zero-energy for all time, assume there is a charging source (for \( t < 0 \))

\[ \text{Is} u(t) \quad \text{or} \quad V_s u(t) \]
Now for plotting → again we need to assume some initial stored energy, which is provided by the source, $I_s u(t)$, that charged the E-field for the capacitor and the B-field for the inductor. This is so we can have a circuit response that is not simply zero for all time.

Now you have 2 decisions

1. Will the circuit response expression you develop be for $i(t)$ or $v(t)$? Either will work, §8.4 uses $v(t)$. §8.6 uses $i(t)$

2. What source value (for charging $C + L$) will you select, and will it be a current source or a V-source?

I will select $I_s = 2A$. Note that choosing a voltage source will be more problematic, so don’t make things harder than they need to be.

$t < 0$: L is a short circuit so all 2A flow through it ($i_L(0) = 2A$)

Since L shorts out C, $v_C(0) = 0V$
Find roots \( s_1 + s_2 \), so find \( \alpha + \omega_0 \)

for parallel RLC \( \alpha = \frac{1}{2RC} = 500 \)

\[
\omega_0 = \frac{1}{\sqrt{LC}} = 10^3
\]

\( \omega_0 > \alpha \) so this is an underdamped system

\[
\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 998.7
\]

For current response, note source-free so \( i(\infty) = 0 \)

\[
i(t) = i(\infty) + e^{-\alpha t} (A_1 \cos \omega_d t + A_2 \sin \omega_d t)
\]

with \( i(\infty) = 2 \)
Problem 4 Graphs: More detailed than you need to be

For these two graphs notice:

- The frequency of oscillation is the same for both $i(t)$ and $v(t)$.
- The initial values and amplitudes for $i(t)$ and $v(t)$ are 2 orders of magnitude different!!
- One of these graphs is a sine wave and the other is a cosine wave.
- The outer 'envelope' of the exponential decay of the oscillations is the same for both current and voltage, and is equal to $e^{-at} = e^{-50t}$.