Picker Engineering Program

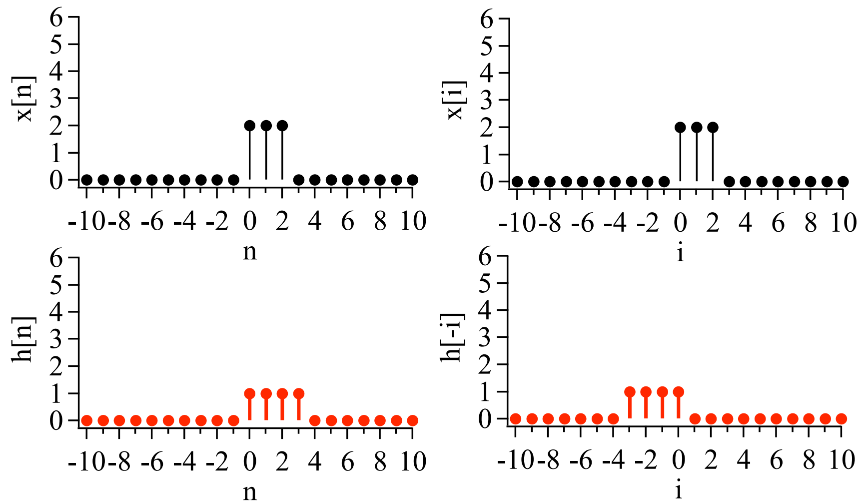
TIME-DOMAIN REPRESENATIONS OF LTI SYSTEMS: CONTINUOUS-TIME CONVOLUTION

EGR 320: Signals & Systems
Lecture 4: January 31, 2011

What is the DT sifting property?

CONVOLUTION: Example

$$y[n] = x[n] * h[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$$



CONVOLUTION: Example

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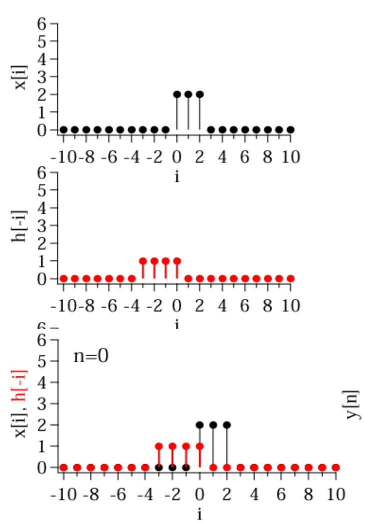


Determine y[n=0] Computationally

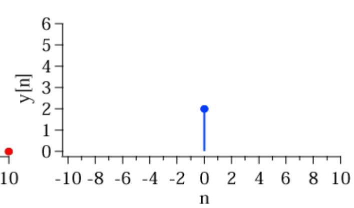
$$y[0] = \sum_{i=-\infty}^{\infty} x[i]h[0-i]$$

$$y[0] = \ldots + x[-1]h[1] + x[0]h[0] + x[1]h[-1] + x[2]h[-2] + x[3]h[-3] + \ldots$$

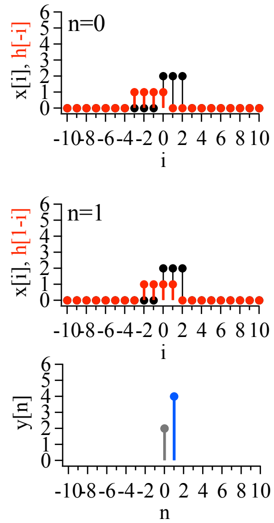
Determine y[n=0] Graphically

$$y[0] = (0)(0) + (2)(1) + (2)(0) + (2)(0) + (0)(0) + \ldots$$

$$y[0] = 2$$



CONVOLUTION: Example



$$y[n] = x[n] * h[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$$

Determine y[n=1] Computationally

$$y[1] = \sum_{i=-\infty}^{\infty} x[i]h[1-i]$$

$$y[1] = \ldots + x[-2]x[1-(-2)] + x[-1]h[1-(-1)] + x[0]h[1-0] + x[1]h[1-1] + x[2]h[1-2] + \ldots$$

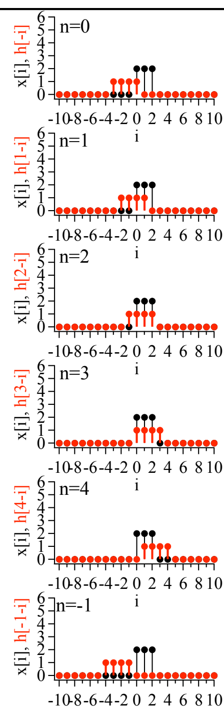
Determine y[n=1] Graphically

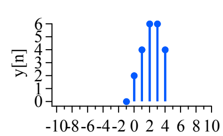
$$y[1] = (0)(0) + (0)(0) + (2)(1) + (2)(1) + (2)(0)$$

$$y[1] = 4$$

QUICK QUIZ

Demonstrate graphically how to compute y[5] and y[6].





CONTINUOUS TIME:

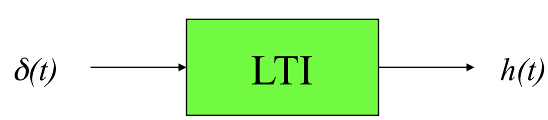
SIFTING PROPERTY OF THE IMPULSE *δ(t)*

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

See equation 2.27 of your textbook.

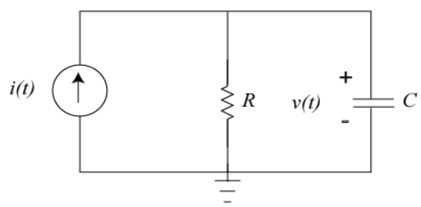
IMPULSE RESPONSE *h(t)*

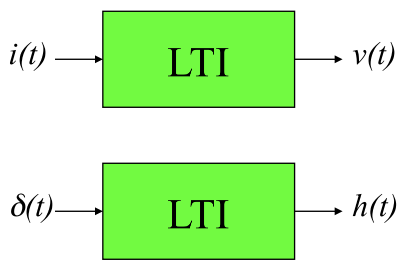
y(t) is the output of the continuous-time LTI system with input *x(t)* and no initial energy.



With the unit impulse as an input [i.e., $x(t)=\delta(t)$], the output is defined as the IMPULSE RESPONSE and is represented by *h(t)*.

Impulse response *h(t)* of the RC circuit: Example





$$\frac{dv(t)}{dt} + \frac{1}{RC}v(t) = \frac{1}{C}i(t)$$

With zero initial energy

$$v(t) = \int_{0^-}^{t} \frac{1}{C} e^{-(1/RC)(t-\lambda)} i(\lambda)d\lambda, \quad t \geq 0$$

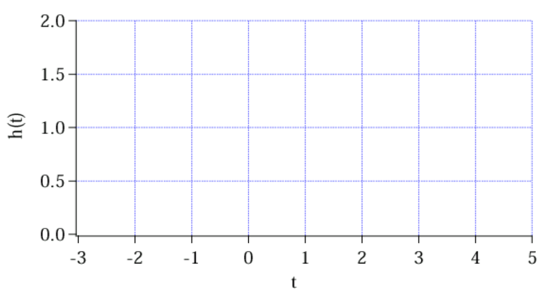
$$i(t) = \delta(t)$$

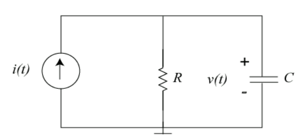
$$v(t) = \int_{0^-}^{t} \frac{1}{C} e^{-(1/RC)(t-\lambda)} \delta(\lambda)d\lambda, \quad t \geq 0$$

$$v(t) = h(t) = \frac{1}{C} e^{-(1/RC)t} u(t)$$

QUICK QUIZ

Plot the impulse response (i.e., system function) *h(t)* for the RC circuit on the previous slide. You may assume that *C=R=1*.

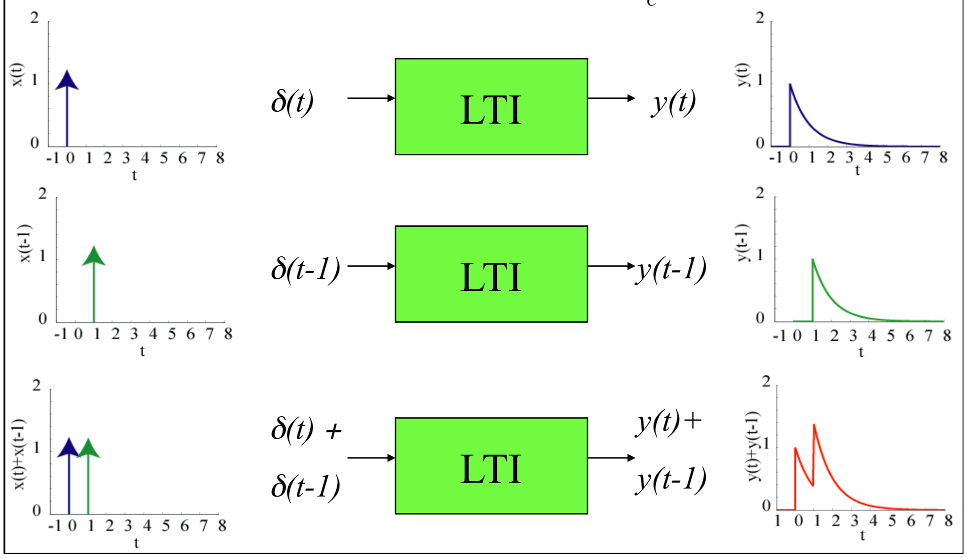




The output from any input can be determined via *h(t)*

What is *v(t)* in the RC circuit with input: $i(t) = \delta(t) + \delta(t-1)$

From an earlier slide: $h(t) = \frac{1}{C} e^{-(1/RC)t} u(t)$



A conceptual view of convolution

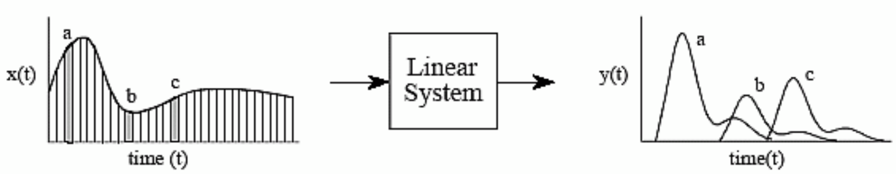


FIGURE 13-2
Convolution viewed from the input side. The input signal, *x(t)*, is divided into narrow segments, each acting as an impulse to the system. The output signal, *y(t)*, is the sum of the resulting scaled and shifted impulse responses. This illustration shows how three points in the input signal contribute to the output signal.

From http://www.dspguide.com/ch13/2.htm (downloaded 2/6/08)

SIFTING PROPERTY OF THE IMPULSE

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

Analogous to writing the input *x[n]* in discrete form as a sum of impulses.

$$x[n] = \sum_{i=0}^{\infty} x[i]\delta[n-i]$$

CONVOLUTION REPRESENTATION: Input Signal

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

"Express a CT signal as the weighted superposition of time-shifted impulses. Here, the superposition is an integral instead of a sum (as in DT), and the time shifts are given by the continuous variable τ. The weights $x(\tau)d\tau$ are derived from the value of the signal x(t) at the time τ at which each impulse occurs." (HVV text p. 115)

CT CONVOLUTION REPRESENTATION

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

For any fixed value of τ, let $y_\tau(t)$ denote the output response resulting from the input $x(\tau)\delta(t-\tau)$. Since $h(t-\tau)$ is the response to $\delta(t-\tau)$, the response $y_\tau(t)$ to $x(\tau)\delta(t-\tau)$ is:

$$y_\tau(t) = x(\tau)h(t-\tau)$$

What is the general expression for *y(t)?*

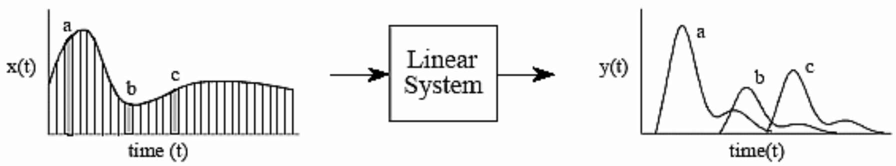
CT CONVOLUTION INTEGRAL OR SUPERPOSITION INTEGRAL

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Analogous to discrete-time convolution sum

$$y[n] = x[n] * h[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$$

A conceptual view of convolution



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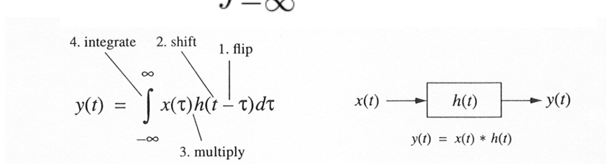
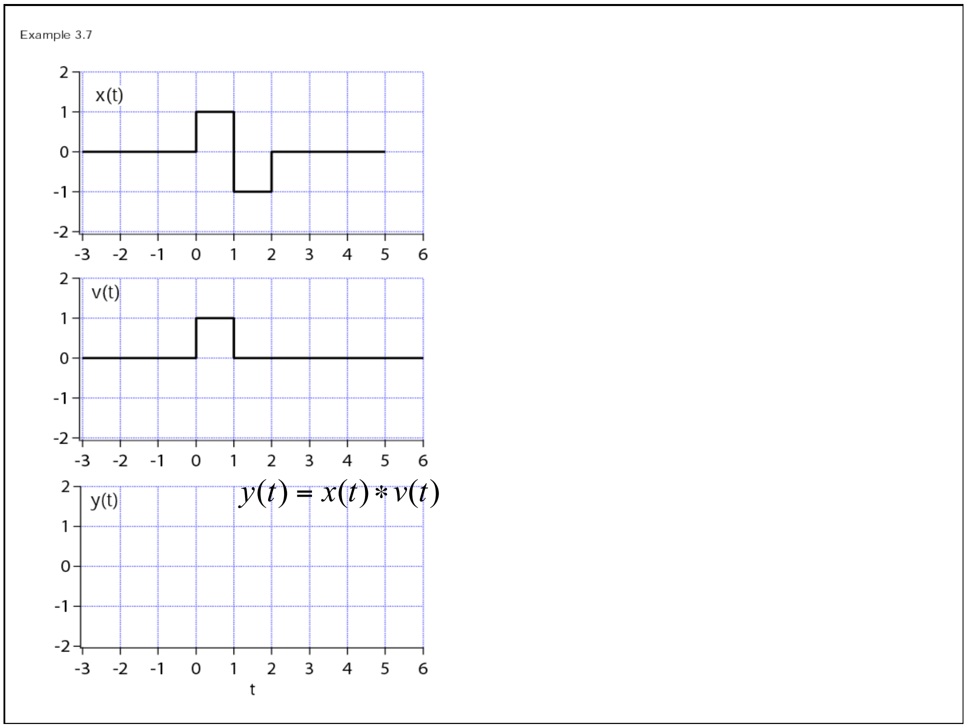
CONVOLUTION: A step by step procedure

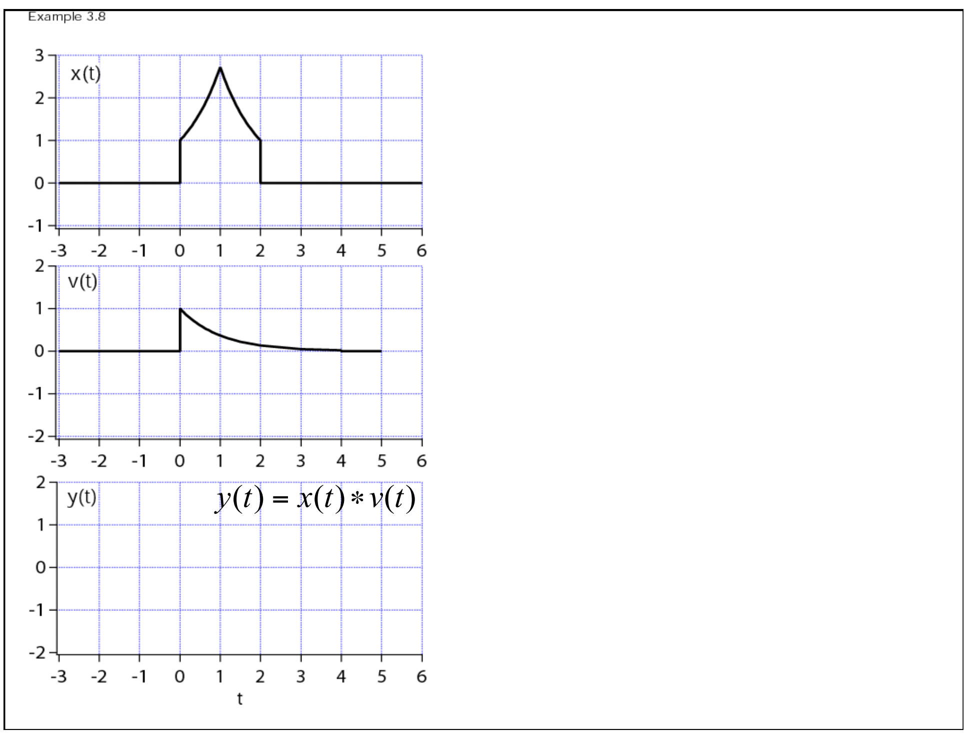
(from "Signals and Systems Made Ridiculously Simple" by Zoher Z. Karu p. 53)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$



1. Choose one signal to be *x(t),* the other is then *h(t);* draw them both on the τ axis.
2. FLIP *h(τ)* about τ *=0* and SHIFT signal to the right by *t*.
3. Identify the different regions of integration (look for breakpoints in the signals).
4. MULTIPLY *x(τ)* by flipped/shifted version of *h(- τ)* and INTEGRATE using correct limits on integral.
5. Step 4 produces the equation for *y(t)* over the specified region.
6. Repeat step 4 for all possible regions of interest.


Example 3.7
x(t)
v(t)
y(t)
$y(t) = x(t) * v(t)$
t


Example 3.8
x(t)
v(t)
y(t)
$y(t) = x(t) * v(t)$
t