INSTRUCTIONS: This exam is DUE in my office by 3:30 PM on Friday, Oct 3. You may use any non-human resource you wish but may not consult with any person. You may use a computer. There is no time limit on this, but remember it is just a quiz.

1. (17 points) In this problem set $\hbar = 1$ and $m = 1$.
a. (3 pts) Find the ground state wave function for a particle on an infinite potential at $x$ less than zero and at $x$ greater than one. Between these limits, the potential is zero from $x = 0$ to $x = 0.5$, at which point it is given by the expression $V = 30x$. HINT: In Mathematica you can accomplish this with $\text{pot}[x, h] := h \cdot x /; x \geq 0.5; \text{pot}[x, h] := 0 /; x < 0.5$; where $h$ is then fixed at 30.
b. (2 pts) Find the first excited state wave function for a particle on an infinite potential with the same potential as in part a.
c. (5 pts) Show that the two functions in the parts above are orthogonal. HINT: Since this involves numerical integration, I would be happy with an answer within 0.001 of the expected answer; i.e., don’t beat your brains out.
d. (3 pts) Find the expectation value for $x$ for the ground state function from part b.
e. (4 pts) What is the expectation value of the energy for your wave function of part a using the Hamiltonian of that problem?

2. (5 points)
a. (5 pts) Let the operator $\mathcal{O}$ be
\[
q^2 - q \frac{\partial}{\partial q} + \frac{\partial}{\partial q} q - \frac{\partial^2}{\partial q^2}
\]
and let the function $f$ be $\text{Exp}[{-q^2/2}]$. Is this $f$ an eigenfunction of the operator $\mathcal{O}$? If so, what is the eigenvalue? HINT: There are four terms in the operator; pay attention to each.

3. (8 points)
a. (5 pts) Express your answer to part b of problem 1 in terms of the eigenfunctions of a particle on a pole of length 1.
b. (3 pts) What is the probability that a measurement of the energy of a large number of identically prepared systems in the state you found in part b of problem 1 if measured in a regular POP system will give an answer of $\frac{9\hbar^2\pi^2}{2m}$ (with $\hbar = 1$ and $m = 1$)?