INSTRUCTIONS: This exam must be completed by 10:00 PM, Sunday, Nov 16. You are to take this exam in the library. You may NOT use a text, the web, or any other reference material; you may NOT talk to other people about the exam. You may NOT use a computer. You may use a calculator. You must complete this exam in two hours or less; it should take less time than that.

Time checked out: ___________  Time checked in: ___________

1. (10 points) You have a system with a Hamiltonian operator as given below. In general terms, what do you need to do in order to determine the energy of the system (for which this operator is valid) by first order perturbation theory. HINT: This question is giving you the opportunity to explain how first order perturbation theory works in general terms.

\[ \hat{H} = \frac{-\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + V(x, y, z) \]
2. (15 points)
a. (5 pts) Make a rough plot of the radial wave function for a hydrogen atom in the
state \(|3 \ 0 \ 0⟩\) (\(|n \ \ell \ m⟩\)).

b. (5 pts) Make a rough plot of the total hydrogen atom wave function for the state
\(|3 \ 1 \ 0⟩\) as you move out the z axis (that is, as r changes with \(\theta = 0\)). HINT: You can
make a plot of the total wave function because I have fixed some of the variables.

c. (5 pts) Make a rough plot of the total hydrogen atom wave function for the state
\(|3 \ 1 \ 0⟩\) as you move out the x axis (that is, as r changes with \(\theta = \frac{\pi}{2}\) and \(\phi = 0\)). HINT:
Same as part b.
3. (15 points)
   a. (10 pts) In the derivation of first order perturbation theory we multiply the following equation from the left with $|n^{(0)}\rangle$. Give the simplified result of that multiplication.

   $$\hat{H}'|n^{(0)}\rangle + \hat{H}_0 \sum_k a_k |k^{(0)}\rangle = E_n^{(1)}|n^{(0)}\rangle + E_n^{(0)} \sum_k a_k |k^{(0)}\rangle$$

   b. (5 pts) Explain what the symbol $|n^{(0)}\rangle$ represents in the first part of this problem.

4. (10 points) Given the generalized angular momentum operators $j^2$, $j_z$, and $j_+$, which you will recall is equal to $j_x + i j_y$, establish the following:

   $$j^2 = j_+ j_- + j_z^2 - \hbar j_z$$