INSTRUCTIONS: This exam must be completed by 10:00 PM, Sunday, Oct 26. You are to take this exam in the library. You may NOT use a text, the web, or any other reference material; you may NOT talk to other people about the exam. You may NOT use a computer. You must complete this exam in two hour or less; it should take less time than that.

1. (6 points)
   a. (3 pts) What is Schrödinger’s equation for a parve in a cubic room?

   b. (3 pts) What procedure do you use to solve Schrödinger’s equation for a parve in a cubic room? How many steps are there in the procedure? What are they?
2. (10 points) This problem concerns the harmonic oscillator (in one dimension); $a_+$ and $a_-$ are the operators we defined for that system; the $|n\rangle$ are the harmonic oscillator eigenfunctions of the Hamiltonian operator, $\hat{H}$. Also, $|f\rangle$ is $a_-|n\rangle$. Evaluate each of the following:

\[ \hat{H} |3\rangle = \]

\[ a_+a_- |3\rangle = \]

\[ a_- |0\rangle = \]

\[ \hat{H} |f\rangle = \]

\[ \langle n|(a_+ + a_-)|n + 1\rangle = \]

3. (12 points) We found the following equation In the derivation of the wave function for a harmonic oscillator

\[ a_{n+2} = \frac{1 + 2n - K}{(n + 1)(n + 2)} a_n \]

For Schrödinger’s equation to work for this power series solution, we demand that at some point the value of a coefficient must go to zero. Find the relationship between $a_5$ and $a_7$ for the case where we demand that $a_{11} = 0$. 
4. (10 points) 

a. (4 pts) Let $O$ be a Hermitian operator and $|m\rangle$ and $|n\rangle$ represent eigenfunctions of it with eigenvalues of $o_m$ and $o_n$. What can you say about the following. Briefly indicate why. HINT: "Equal" or "Not equal" is an OK answer if there is no other relationship. If there is another relationship, use it.

$$\langle m|O|n\rangle \quad \text{and} \quad \langle m|O^\dagger|n\rangle$$

$$\frac{O|n\rangle}{|n\rangle} \quad \text{and} \quad \frac{\langle n|O^\dagger}{\langle n|}$$

$$\langle n|O^\dagger|n\rangle \quad \text{and} \quad \langle n|O|n\rangle$$

b. (6 pts) Assume $R$ is not a Hermitian operator. Let $|m\rangle$ and $|n\rangle$ still represent eigenfunctions of $O$ with eigenvalues of $o_m$ and $o_n$. What can you say about the following. Briefly indicate why. HINT: "Equal" or "Not equal" is an OK answer if there is no other relationship. If there is another relationship, use it.

$$\langle m|R|n\rangle \quad \text{and} \quad \langle Rn|m\rangle$$

$$\langle m|R|n\rangle \quad \text{and} \quad \langle n|R^\dagger|m\rangle$$

$$\langle Rm|n\rangle \quad \text{and} \quad \langle m|Rn\rangle$$

$$\langle n|R^\dagger|m\rangle \quad \text{and} \quad \langle Rn|m\rangle$$
5. (12 points) This problem deals with the rigid rotor wave functions in Dirac notation. The ket are therefore written $|\ell \ m\rangle$, etc. Evaluate each of the following:

\[ \hat{\ell}_z |3\ 2\rangle \]

\[ \hat{\ell}^2 |3\ 2\rangle \]

\[ \hat{\ell}_+ |2\ 2\rangle \]

\[ \hat{\ell}_- |1\ 1\rangle \]

\[ \hat{\ell}_z \hat{\ell}_+ |1\ 0\rangle \]

\[ \hat{\ell}^2 \hat{\ell}_+ |1\ 0\rangle \]