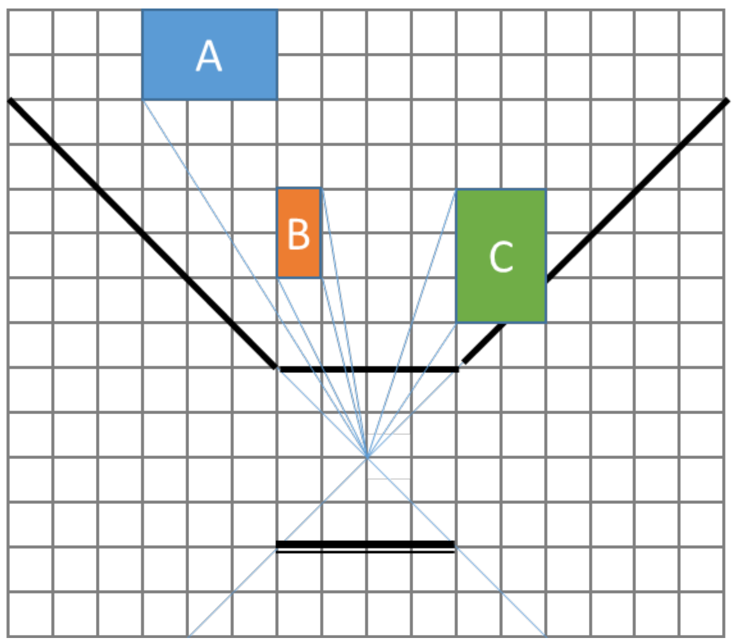
CS 240: Computer Graphics
Handout: Perspective Projection

1. Just as one can do a perspective projection from 3D into 2D, it is also possible to perform a similar projection from 2D into 1D (a line). You can imagine this, if you want, as the view through a very narrow slit. The advantage of working in 2D and 1D is that they are easier to diagram on paper. We'll take advantage of this to work out a simple case of perspective by hand.

The diagram below shows a 2D scene. The boxes labeled A, B, and C are buildings. Each grid square is one unit, and the focal point is the origin. Under these assumptions, the corners of building A are (in 2D homogeneous coordinates):

$$\begin{bmatrix} -2 & -5 & -5 & -2 \\ 8 & 8 & 10 & 10 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



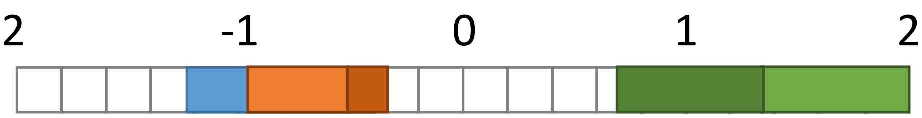
a.) What are the coordinates of the other two buildings? $\begin{bmatrix} -2 & -2 & -1 & -1 \\ 4 & 6 & 6 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 & 4 & 4 \\ 3 & 6 & 6 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

b.) Compute the coordinates for the other two buildings.

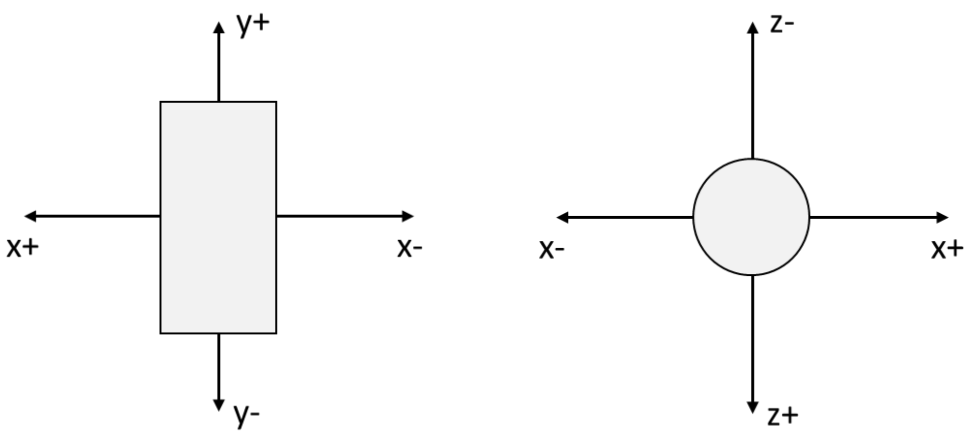
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} -2 & -2 & -1 & -1 \\ 4 & 6 & 6 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -2 & -1 & -1 \\ 2 & 3 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & -2/3 & -1/3 & -1/2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 & 4 & 4 \\ 3 & 6 & 6 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 & 4 \\ 1.5 & 3 & 3 & 1.5 \end{bmatrix} \rightarrow \begin{bmatrix} 4/3 & 2/3 & 4/3 & 8/3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

c.) In this example, the visible region is from -2 to 2. If we render the building above, it looks like the diagram below. Add the other two buildings to the picture (you may cover part of building A).



2. Orthographic projection: Imagine a cylinder of radius 1 centered at the origin, with the circular faces lying on the planes $y = -2$ and $y = 2$. Draw the view looking towards the positive z axis of an orthographic projection of this cylinder, labeling the positive and negative axis. Then do the same for the view looking towards the positive y axis.



3. Perspective projection: Given the following 8 vertices of a cube in world space, find the corresponding coordinates of each in viewport space, using a perspective camera at the origin with the viewport at $z = -1$. Then draw what the "viewer" would see.

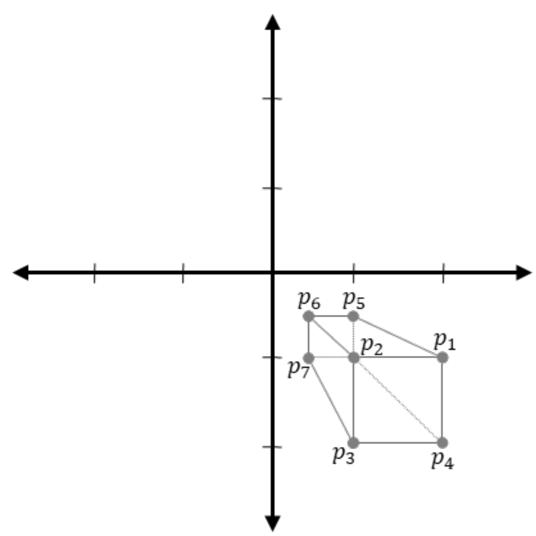
World Coordinates	Projected Coordinates
(-2,1,-1)	(-2,1)
(-1,1,-1)	(-1,1)
(-1,2,-1)	(-1,2)
(-2,2,-1)	(-2,2)
(-2,1,-2)	(-1,1/2)
(-1,1,-2)	(-1/2,1/2)
(-1,2,-2)	(-1/2,1)
(-2,2,-2)	(-1,1)

The focal length is 1, since the focal point is at the origin and the view plane is at z=-1.[1] *So the projection matrix is:*

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Multiplying by the points matrix we get:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -2 & -1 & -1 & -2 & -2 & -1 & -1 & -2 \\ 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \\ -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & -1 & -1 & -2 & -2 & -1 & -1 & -2 \\ 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \\ -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} 2 & 1 & 1 & 2 & 1 & 1/2 & 1/2 & 1 \\ -1 & -1 & -2 & -2 & -1/2 & -1/2 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



[1] It is a bit strange to have the projection in the negative direction because it seems to flip the other axes. Normally, instead of projecting directly from world coordinates we would apply a transformation to camera coordinates with a positive z.