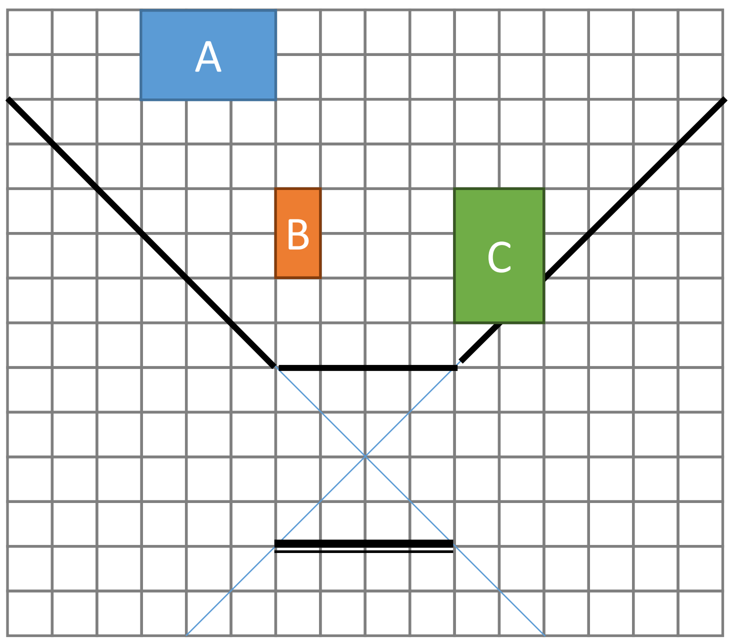
1. Projection to 1D: Just as one can do a perspective projection from 3D into 2D, it is also possible to perform a similar projection from 2D into 1D (a line). You can imagine this, if you want, as the view through a very narrow slit. The advantage of working in 2D and 1D is that they are easier to diagram on paper. We'll take advantage of this to work out a simple case of perspective by hand.

The diagram below shows a 2D scene. The boxes labeled A, B, and C are buildings. Each grid square is one unit, and the focal point is the origin. Under these assumptions, the corners of building A are (in 2D homogeneous coordinates):

$$\begin{bmatrix} -2 & -5 & -5 & -2 \\ 8 & 8 & 10 & 10 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



a.) What are the coordinates of the other two buildings?

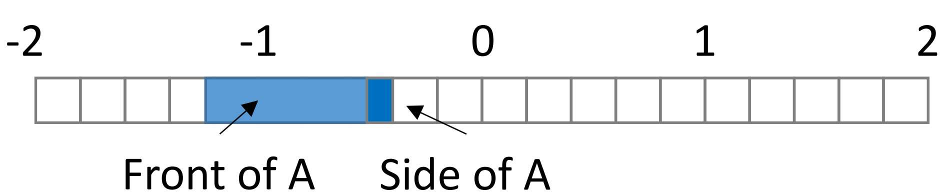
b.) The perspective projection matrix from 2D into 1D is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & {}^{1}/_{f} & 0 \end{bmatrix}$, where f is the focal distance (2 in this case).

After multiplying the points by the projection matrix, we must divide each vector by its last component to restore the homogeneous coordinates to the proper scale. For building A, that computation looks as follows:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} -2 & -5 & -5 & -2 \\ 8 & 8 & 10 & 10 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -5 & -5 & -2 \\ 4 & 4 & 5 & 5 \end{bmatrix} = \begin{bmatrix} -0.5 & -1.25 & -1 & -0.4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Compute the coordinates for the other two buildings.

c.) In this example, the visible region is from -2 to 2. If we render the building above, it looks like the diagram below. Add the other two buildings to the picture (you may cover part of building A).



2. Orthographic projection: Imagine a cylinder of radius 1 centered at the origin, with the circular faces lying on the planes $y = -2$ and $y = 2$. Draw the *back view* (looking towards the positive z axis) of an orthographic projection of this cylinder, labeling the positive and negative axis. Then do the same for the *top view* (looking towards the positive y axis).

3. Perspective projection: Given the following 8 vertices of a cube in world space, find the corresponding coordinates of each in viewport space, using a perspective camera at the origin with the viewport at $z = -1$. Then draw what the "viewer" would see.

World Coordinates	Projected Coordinates
(-2,1,-1)	
(-1,1,-1)	
(-1,2,-1)	
(-2,2,-1)	
(-2,1,-2)	
(-1,1,-2)	
(-1,2,-2)	
(-2,2,-2)	