Texture Mapping Practice

Texture Mapping a Cone

When texture mapping a cone, there are several ways one could “unwrap” the cone onto the texture. Here, we’ll imagine that the entire top line of the texture ($v = 1$) is mapped to the tip of the cone, and the bottom line of the texture ($v = 0$) is mapped to the bottom rim of the cone. For a cone with height $h$ and radius $r$, create a texture mapping method that will map each point $(x, y, z)$ on the surface (not the base) of the cone to a point $(u, v)$ on a rectangular texture.

Establish the XYZ coordinate system as shown, with origin at the center of the cone’s base. Imagine the surface of the cone unfolding as shown above to get the UV plane. There is a seam along the positive x axis.

The $z$ value on the cone’s surface ranges from 0 at the base to $h$ at the top. To convert to $v$, all we do is scale by $h$.

$$v(x, y, z) = \frac{z}{h}$$

The angle in the XY plane ranges from 0 at one edge of the sheet to $2\pi$ at the other. So we compute $\theta$ from $x$ and $y$, and scale the result by $2\pi$.

$$u(x, y, z) = \frac{\tan^{-1} \frac{y}{x}}{2\pi}$$

Suppose that the cone is approximated using a geometry consisting of twelve triangular faces to form the sides. Could the mapping described above serve as the basis for choosing UV coordinates for these panels? If not, propose an alternative.

There is no way to map a vertex to the entire top line in Three.js. Instead, one can arrange the triangular sections according to several schemes. The first shown maintains the contiguity of neighboring faces, so that a pattern can transfer continuously onto the surface. The second option shown introduces gaps between the segments, but is more regular. Either is possible, as are other configurations.