1. **Matrix multiplication:** Let

\[ A = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 2 \\ -4 & 1 \end{bmatrix} \]

What is \( AB \)? What is \( BA \)?

\[ AB = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ -8 & 8 \end{bmatrix} \quad \text{and} \quad BA = \begin{bmatrix} 0 & 2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ -1 & 6 \end{bmatrix} \]

2. What is the dimension of a \( 4 \times 2 \) matrix times a \( 2 \times 3 \) matrix? Could you multiply them the other way around?

   The “inner” dimensions (both 2 here) must match for matrix multiplication to be possible. The “outer” dimensions will be the dimensions of the multiplication result. So in this case, the result would be a \( 4 \times 3 \) matrix. We cannot multiply them the other way around.

3. **Rotate:** Let \( \vec{y} = \begin{bmatrix} 0 \\ y \end{bmatrix} \)

   What are the new coordinates if \( \vec{y} \) is rotated \( \theta \) (counter-clockwise)?

   \[ \vec{y}' = \begin{bmatrix} -y \sin \theta \\ y \cos \theta \end{bmatrix} \]

4. **Scale:** What scaling matrix could you use to get \( [2 \ 3]^T \) from \( [-5 \ 6]^T \)?

   \[ \begin{bmatrix} -2/5 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} -5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \]

5. **Composition:** What transformations could we apply to the house below to rotate it 25 degrees about its center?

   ![House](image)
   ```
   translate(15,20);
   rotate(25 deg);
   translate(-15,-20);
   house();
   ```

6. **Composition II:** Discuss how to use composition to build a matrix that reflects across an arbitrary line. Can you find a matrix that reflects across the line \( y = 2x + 4 \)?

   Start with a reflection around a known line, like \( y = 0 \). Apply a transformation that maps \( y = 2x + 4 \) onto it, then apply the reflection, then invert the original transformation.
For the specified problem, here is the plan:

1. Translate the line so that it goes through the origin
2. Rotate the line so that it aligns with \( y = 0 \). Since the target line has slope 2, the rotation angle is \( \tan^{-1} 2 = 63.4^\circ \).
3. Reflect the line
4. Reverse the rotation from step 2
5. Reverse the translation from step 1.

Here are the matrices (listed right to left in order of application):

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 4 \\
0 & 0 & 1
\end{bmatrix} \cdot
\begin{bmatrix}
0.447 & -0.894 & 0 \\
0.894 & 0.447 & 0 \\
0 & 0 & 1
\end{bmatrix} \cdot
\begin{bmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{bmatrix} \cdot
\begin{bmatrix}
0.447 & 0.894 & 0 \\
-0.894 & 0.447 & 0 \\
0 & 0 & 1
\end{bmatrix} \cdot
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & -4 \\
0 & 0 & 1
\end{bmatrix} =
\begin{bmatrix}
-0.599 & 0.799 & -3.197 \\
0.799 & 0.599 & 1.602 \\
0 & 0 & 1
\end{bmatrix}
\]