1. Matrix multiplication: Let

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 2 \\ -4 & 1 \end{bmatrix}$$

What is *AB*? What is *BA*?

- 2. What is the dimension of a 4 × 2 matrix times a 2 3 matrix? Could you multiply them the other way around?
- 3. **Rotate:** Let $\vec{y} = \begin{bmatrix} 0 \\ y \end{bmatrix}$

What are the new coordinates if \vec{y} is rotated θ (counter-clockwise)?

- 4. **Scale:** What scaling matrix could you use to get $[2 \ 3]^T$ from $[-5 \ 6]^T$?
- 5. **Composition I:** What transformations could we apply to the house below to rotate it 25 degrees about its <u>center</u>?



6. Composition II: Discuss how to use composition to build a matrix that reflects across an arbitrary line. Can you find a matrix that reflects across the line y = 2x + 4?

7. Transformations [From the fall 2018 midterm]

As you know, matrix transformations may be composed by multiplying two or more transformation matrices so as to form a new one. For this problem, your task is to construct a matrix that will transform the unit square with opposite corners at (0,0) and (1,1) into the shape shown. (Assume that the positive *y* axis points upwards in these figures.) The only catch is that you have to construct your transformation using only the limited set of building blocks shown below. You may use each of these as many times as you like, and multiply them in any order that you like. If you multiply by a matrix several times in a row, you may use exponents in your answer. For example, $A^4B^2 = AAABB$

Transformation matrices you may use to build your result:

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Target shapes:



c.)

