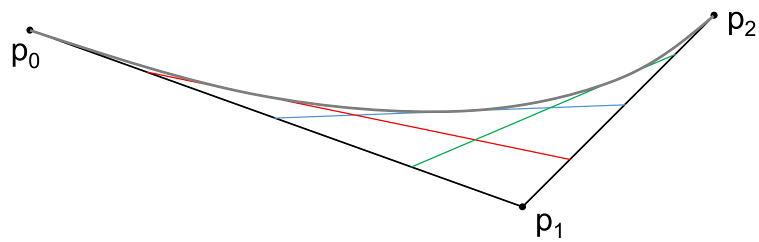
1. **2nd order (quadratic) Bézier curve:** Draw the Bézier curve with control points p_0,p_1,p_2, using guiding points with t = 0.25,0.5,0.75.



2. Here is the parametric equation of a quadratic Bézier curve

$$Q(t) = (1 - t)^2 p_0 + 2(1 - t)t\, p_1 + t^2 p_2$$

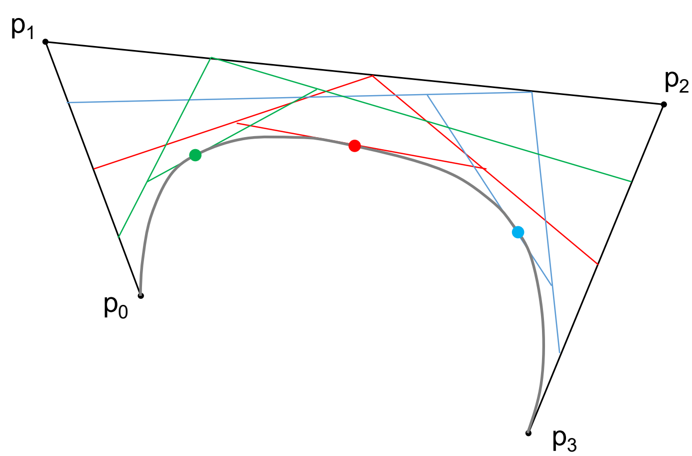
(a) Rearrange this function to make it look more like a quadratic in t (i.e. $Q(t) = at^2 + bt + c$).
$Q(t) = p_0 - 2\,t\,p_0 + t^2 p_0 + 2\,t\,p_1 - 2t^2 p_1 + t^2 p_2$
$Q(t) = t^2 p_0 - 2t^2 p_1 + t^2 p_2 - 2\,t\,p_0 + 2\,t\,p_1 + p_0$
$Q(t) = (p_0 - 2p_1 + p_2)\,t^2 + (2\,p_1 - 2\,p_0)\,t + p_0$

(b) Take the derivative of this rearranged function with respect to t.

$$\frac{dQ}{dt} = 2(p_0 - 2p_1 + p_2)t + 2(p_1 - p_0)$$

(c) What is the derivative at $t = 0$? $t = 1$? What can we say about the tangents at p_0 and p_2?
At t = 0, $\frac{dQ}{dt} = 2(p_1 - p_0)$, *in other words it points from p_0 towards p_1.*
At t = 1, $\frac{dQ}{dt} = 2(p_0 - 2p_1 + p_2) + 2(p_1 - p_0) = 2(p_2 - p_1)$, *so it points from p_1 towards p_2.*

3. **3rd order (cubic) Bézier curve:** Draw the Bézier curve with control points p_0,p_1,p_2,p_3, using guiding points with t = 0.25,0.5,0.75.



Cubic splines: We wish to find a cubic spline passing through the four control points (2,-1), (3,3), (0,1), and (-1,5). We will parameterize the curve as $t=-1$ for the first point, $t=0$ for the second, $t=1$ for the third, and $t=2$ for the fourth. Recall that we must solve the system of equations below:

$$\begin{bmatrix} t_1^3 & t_1^2 & t_1 & 1 \\ t_2^3 & t_2^2 & t_2 & 1 \\ t_3^3 & t_3^2 & t_3 & 1 \\ t_4^3 & t_4^2 & t_4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

(a) Given the t values specified above, fill in the numeric values for the 4x4 matrix.

$$\begin{bmatrix} -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}$$

(b) Using the x values of the points above and the definition of matrix multiplication, write the system of four equations that is represented by the matrix equation above.

$$-a + b - c + d = 2$$
$$d = 3$$
$$a + b + c + d = 0$$
$$8a + 4b + 2c + d = -1$$

(c) Solve for the unknowns a, b, c, and d.

$$a = 1, b = -2, c = -2, d = 3$$

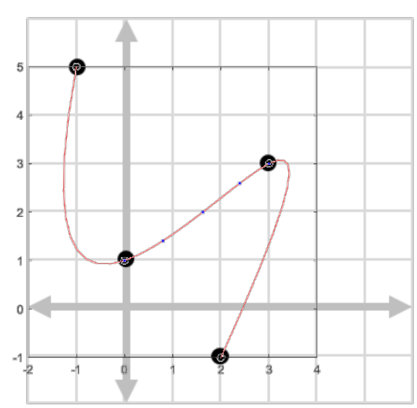
(d) Use the formula $x = at^3 + bt^2 + ct + d$ to compute x values at $t = 0.25$, $t = 0.5$, $t = 0.75$.

$$x(0.25) = 2.390625, x(0.5) = 1.625000, x(0.75) = 0.796875$$

(e) Carry out the same procedure to find a, b, c, and d parameters for computing the y component, and compute y values at $t = 0.25$, $t = 0.5$, $t = 0.75$.

$$a = 2, b = -3, c = -1, d = 3$$
$$y(0.25) = 2.593750, y(0.5) = 2.000000, y(0.75) = 1.406250$$

(f) Plot the curve between (3,3) and (0,1) using the computed coordinates.



In a cubic spline, filling in the remaining sections would require additional control points and calculation of new values of a, b, c, and d.