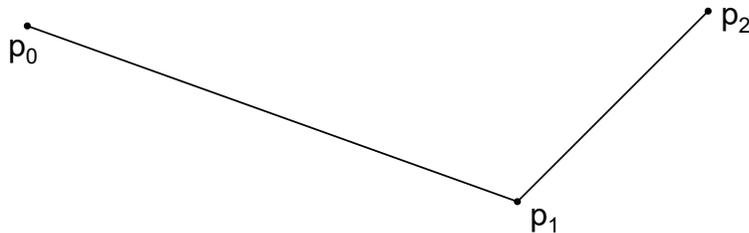


1. **2nd order (quadratic) Bézier curve:** Draw the Bézier curve with control points  $\vec{p}_0, \vec{p}_1, \vec{p}_2$  using guiding points with  $t = 0.25, 0.5, 0.75$ .



2. Here is the parametric equation of a quadratic Bézier curve

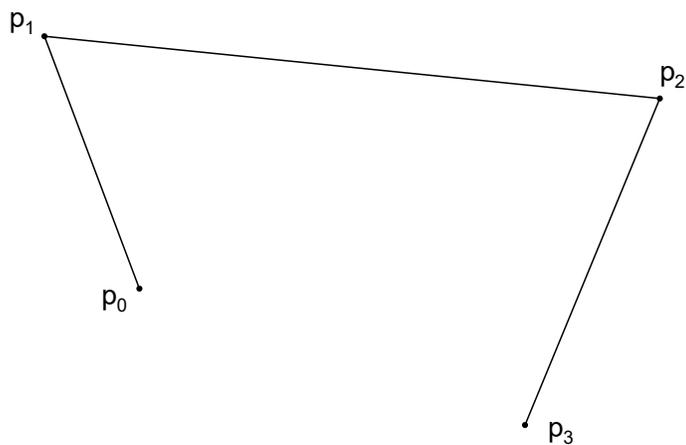
$$Q(t) = (1 - t)^2\vec{p}_0 + 2(1 - t)t\vec{p}_1 + t^2\vec{p}_2$$

Rearrange this function to make it look more like a quadratic in  $t$  (i.e.,  $Q(t) = at^2 + bt + c$ ).

- (a) Take the derivative of this rearranged function with respect to  $t$ .

- (b) What is the derivative at  $t = 0$ ?  $t = 1$ ? What can we say about the tangents at  $p_0$  and  $p_2$ ?

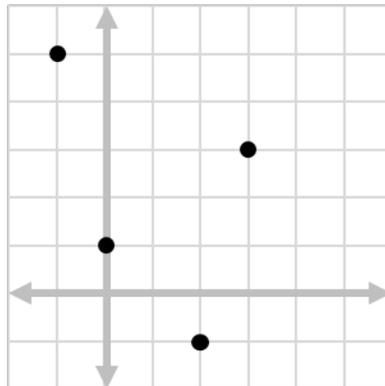
3. **3rd order (cubic) Bézier curve:** Draw the Bézier curve with control points  $\vec{p}_0, \vec{p}_1, \vec{p}_2, \vec{p}_3$  using guiding points with  $t = 0.25, 0.5, 0.75$ .



4. Cubic splines: We wish to find a cubic spline passing through the four control points (2,-1), (3,3), (0,1), and (-1,5). We will parameterize the curve as  $t=-1$  for the first point,  $t=0$  for the second,  $t=1$  for the third, and  $t=2$  for the fourth. Recall that we must solve the system of equations below:

$$\begin{bmatrix} t_1^3 & t_1^2 & t_1 & 1 \\ t_2^3 & t_2^2 & t_2 & 1 \\ t_3^3 & t_3^2 & t_3 & 1 \\ t_4^3 & t_4^2 & t_4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

- (a) Given the  $t$  values specified above, fill in the numeric values for the 4x4 matrix.
- (b) Using the  $x$  values of the points above and the definition of matrix multiplication, write the system of four equations that is represented by the matrix equation above.
- (c) Solve for the unknowns  $a$ ,  $b$ ,  $c$ , and  $d$ .
- (d) Use the formula  $x = at^3 + bt^2 + ct + d$  to compute  $x$  values at  $t = 0.25$ ,  $t = 0.5$ ,  $t = 0.75$ .
- (e) Carry out the same procedure to find  $a$ ,  $b$ ,  $c$ , and  $d$  parameters for computing the  $y$  component, and compute  $y$  values at  $t = 0.25$ ,  $t = 0.5$ ,  $t = 0.75$ .
- (f) Plot the curve between (3,3) and (0,1) using the computed coordinates.



In a cubic spline, filling in the remaining sections would require additional control points and calculation of new values of  $a$ ,  $b$ ,  $c$ , and  $d$ .