CSC 240 Computer Graphics
Video 10: Introduction to 3D Graphics

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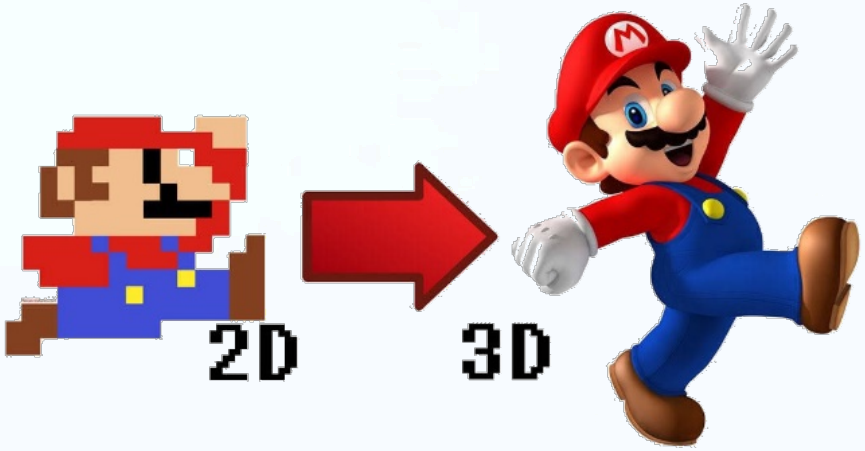
Some slides & content courtesy Sara Mathieson

3D Graphics

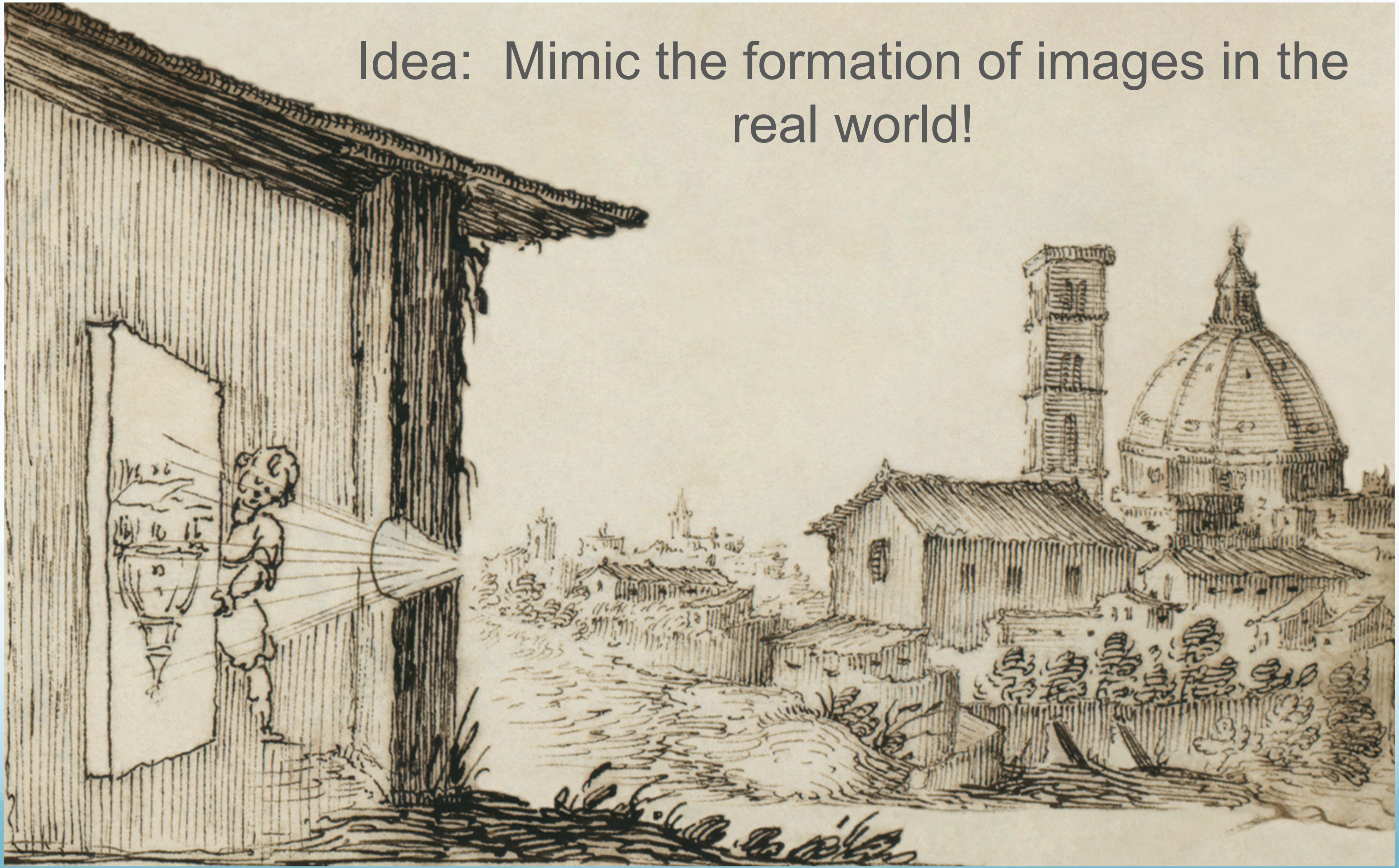
How do we get from 2D to 3D?

- Just add a z? $p = (x, y)$, now $p = (x, y, z)$?
- But we still need to view in 2D!
- Solution: $p_W = (x_W, y_W, z_W)$ in 3D world coordinates
  Rendering transformation converts into a 2D $P_V = (x_V, y_V)$



http://bestanimations.com/Earth&Space/Earth/Earth.html, https://desenio.co.uk/en/artiklar/world-map-grey-poster.html

Image Formation



Idea: Mimic the formation of images in the real world!

Image Formation: Photography

- Light from 3D observed scene enters camera
- Recorded by 2D grid of sensor pixels
- Position of feature on sensor grid depends on angle of incoming light ray
- Image formation happens automatically thanks to physics & geometry

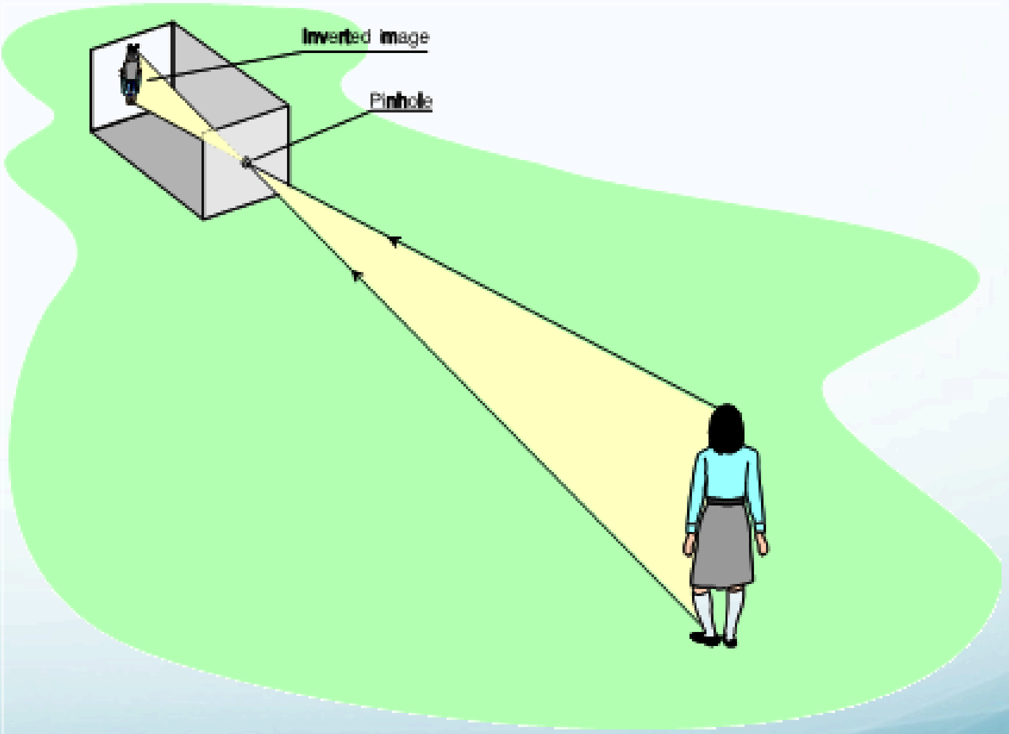
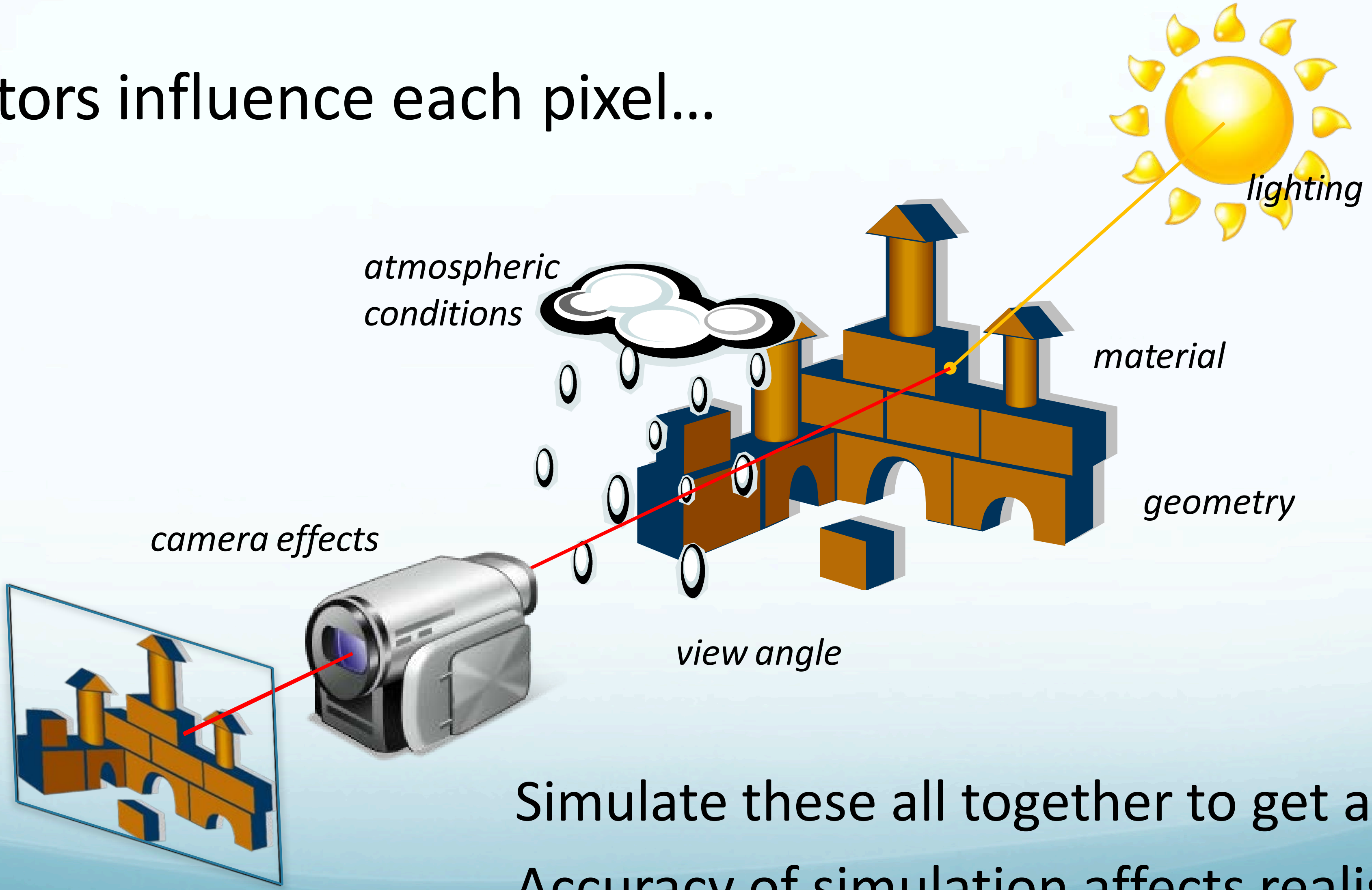


Image: http://exchange.smarttech.com/details.html?id=5a0a0131-be42-4f08-992f-71733a7c1c67

Image Formation: Simulation

Many factors influence each pixel…



Simulate these all together to get an image

Accuracy of simulation affects realism of result

Coordinate Systems

- **World coordinates**: where all objects live, fully 3D
- **Model or object coordinates**: one for each object (choose origin to be the center of each object), 3D
- **Camera coordinates**: 3D coordinate system that moves with the point of view

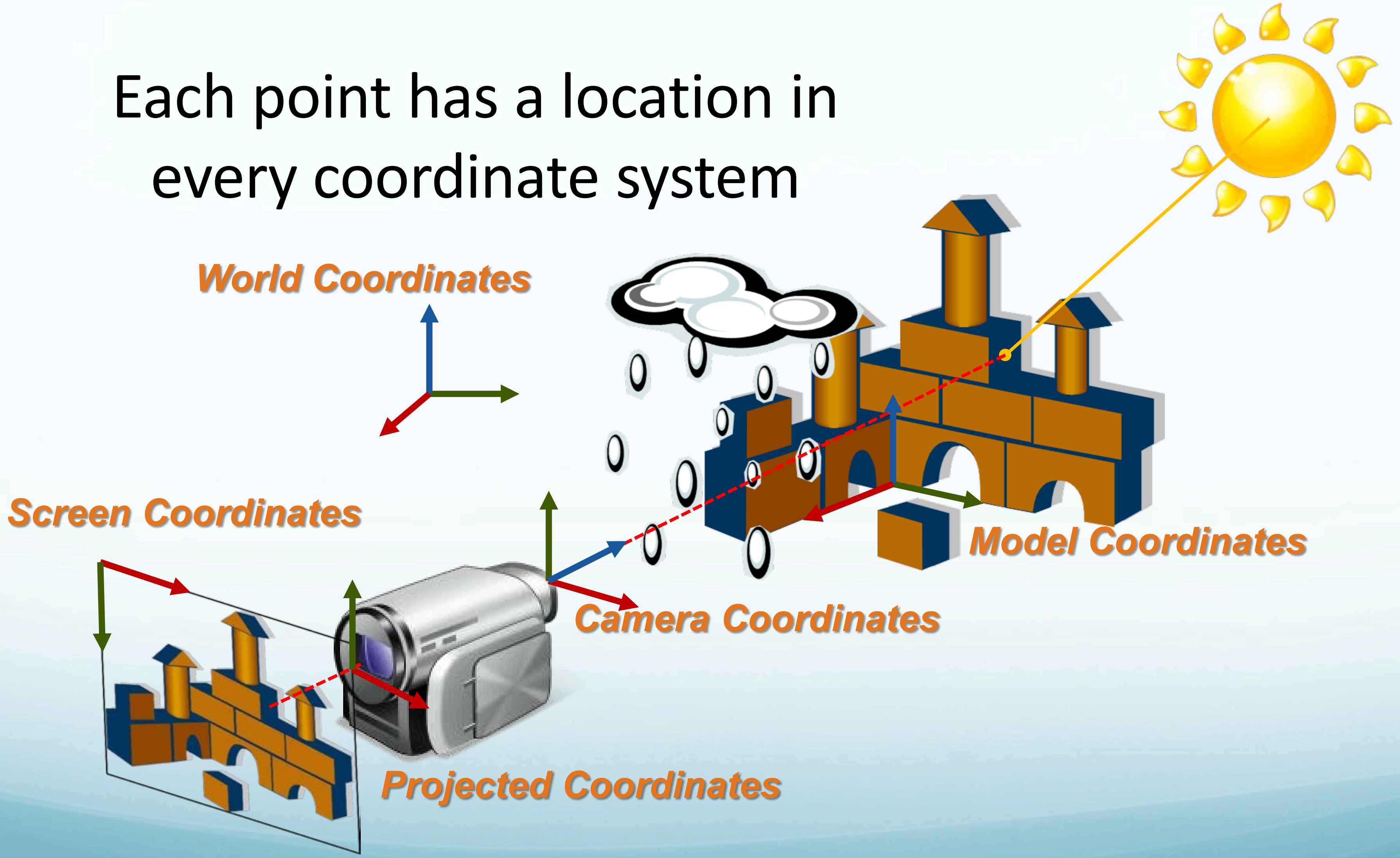
  "Eye" coordinates
- **Projected coordinates**: coordinates after projection (either orthographic or perspective), 2D

  "Clip" coordinates
- **Screen coordinates**: pixel locations (i.e. 300x400 window), all viewport coordinates need to be mapped to pixel locations

  "Device" coordinates

Coordinate Systems & Rendering

Each point has a location in every coordinate system



Questions

PAUSE NOW & ANSWER

1. List five factors that contribute to the appearance of a pixel in an image.

   Materials, geometry, lighting, atmospherics, angle of view, camera effects.

2. Match each coordinate system with its description

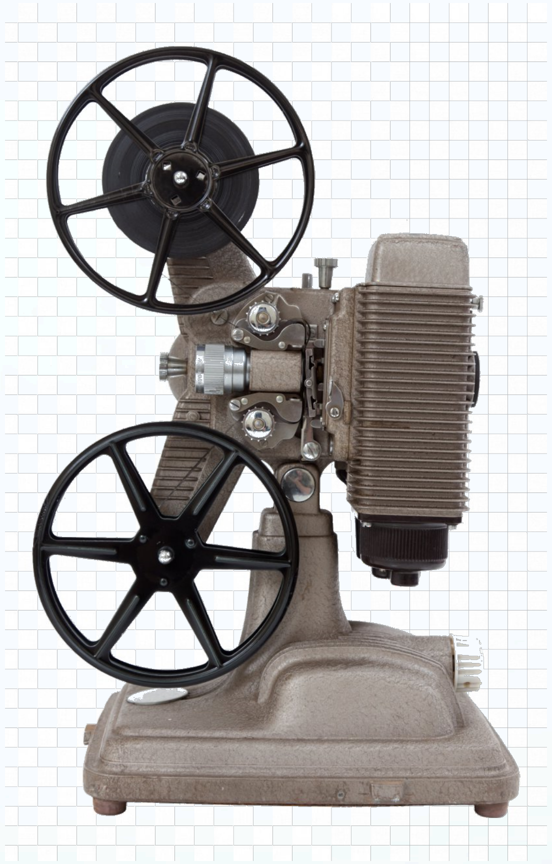
   A. Projected coordinates
   B. World coordinates
   C. Camera coordinates
   D. Model coordinates
   E. Screen coordinates

   U. 3D, moves with point of view
   V. 2D, with pixel-sized units
   W. 3D, encompasses entire scene
   X. 2D, version of 3D scene
   Y. 3D, specific to a single object

3. How can the different coordinate systems be related to one another?

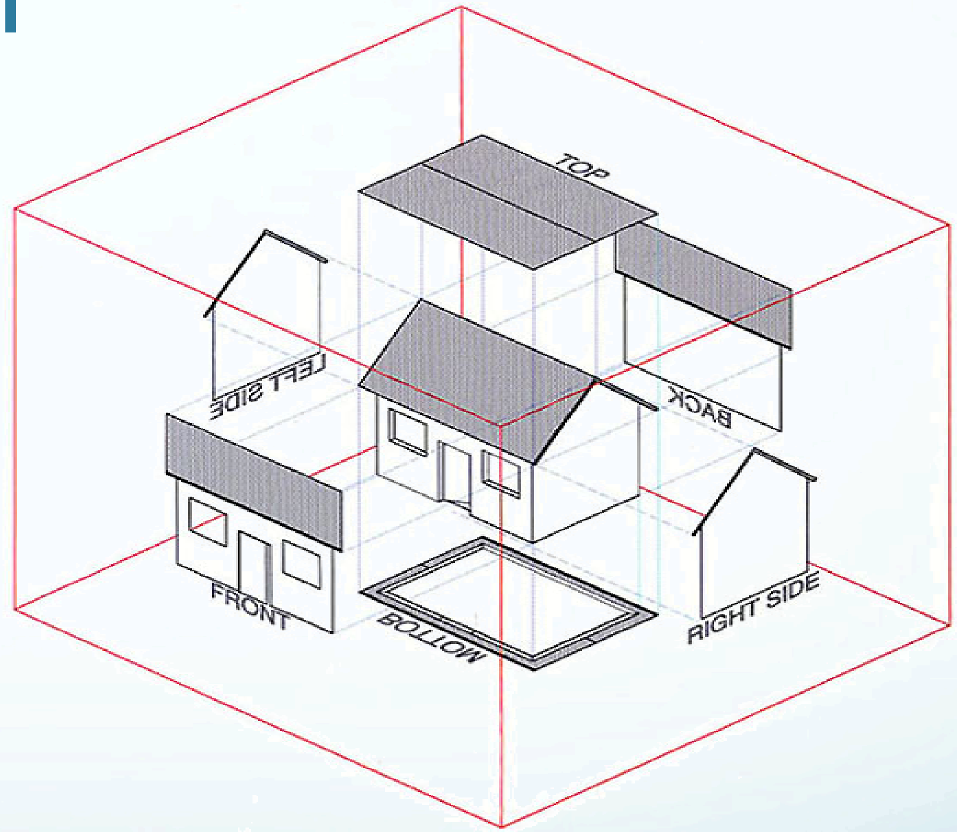
   By a transformation that converts points from one to the other

Projection

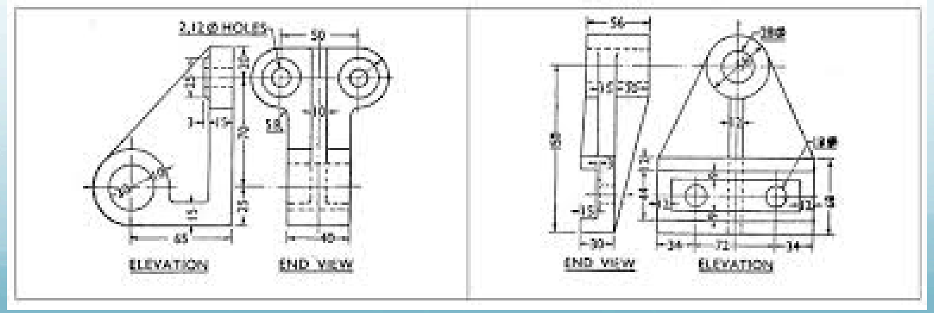


Projection

- Easiest way to go from 3 to 2 dimensions is to drop one.
- This is called **orthographic projection**.
- Used in plan views of buildings
  - Floor plan
  - Side elevations
- Also used in mechanical drawings



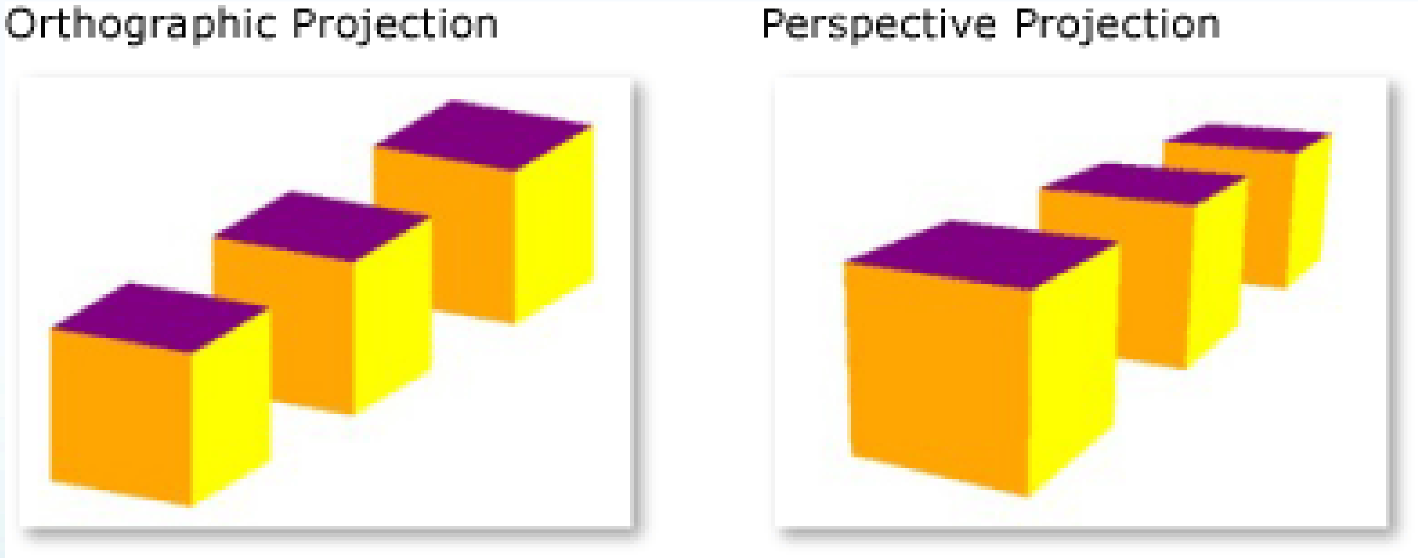
Robert Gardiner, the University of British Columbia



http://ecoursesonline.iasri.res.in/mod/page/view.php?id=2472

Projection

Orthographic projection is useful for drafting diagrams, but doesn't match the way we see the world

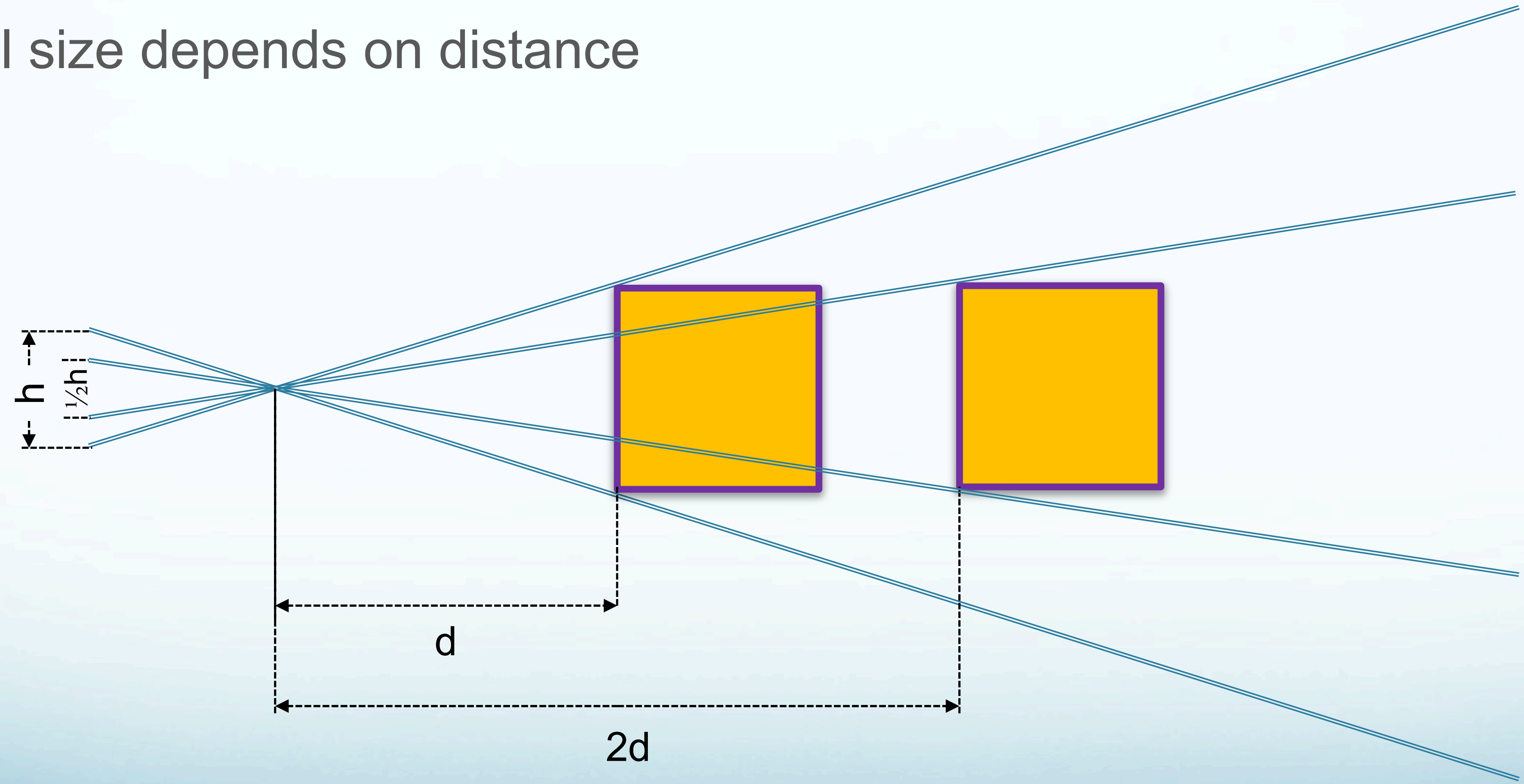


Under orthographic projection, objects look the same size no matter how far away they are

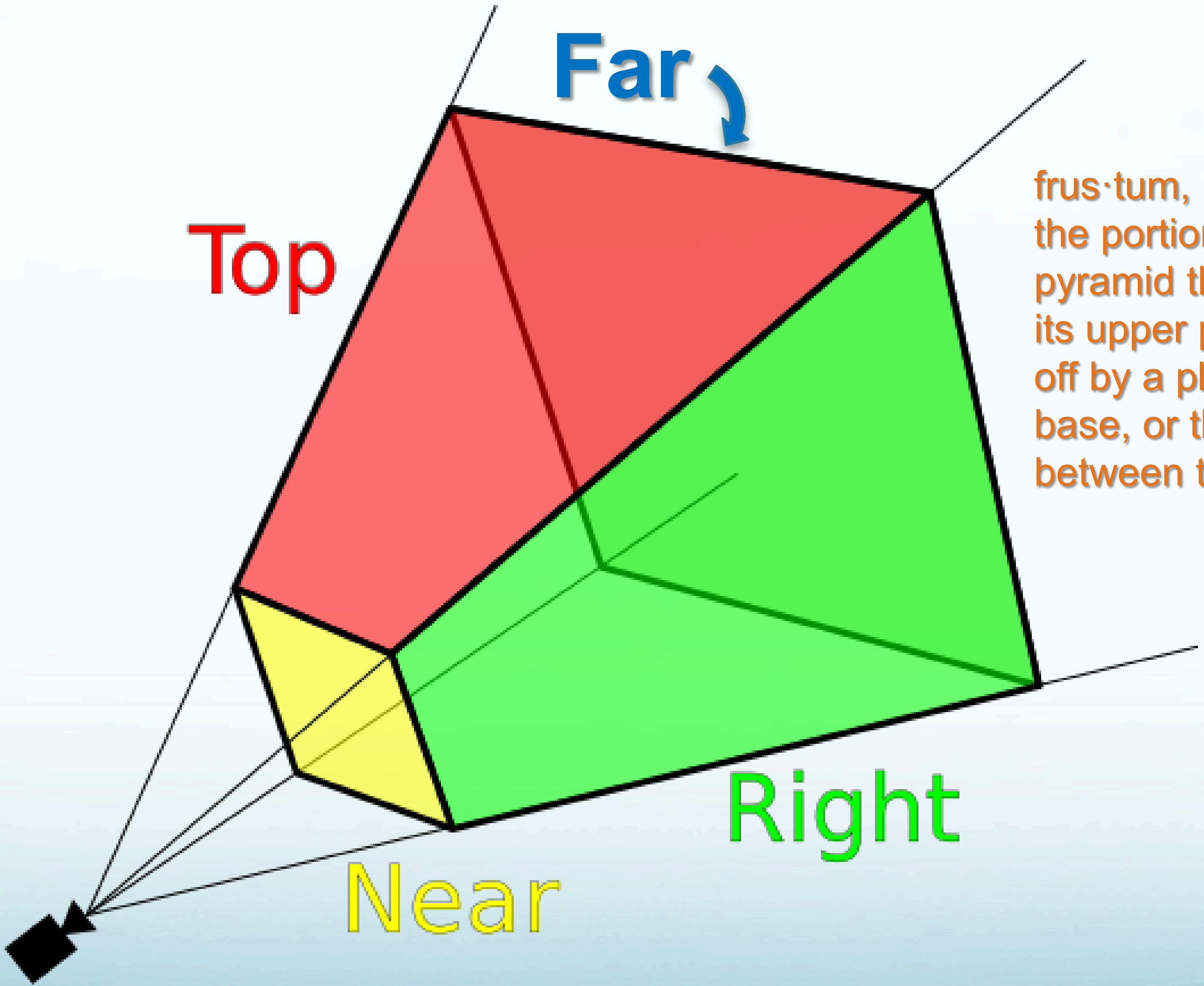
In our lived experience, the visual size of an object depends upon its distance

Perspective Projection

Visual size depends on distance



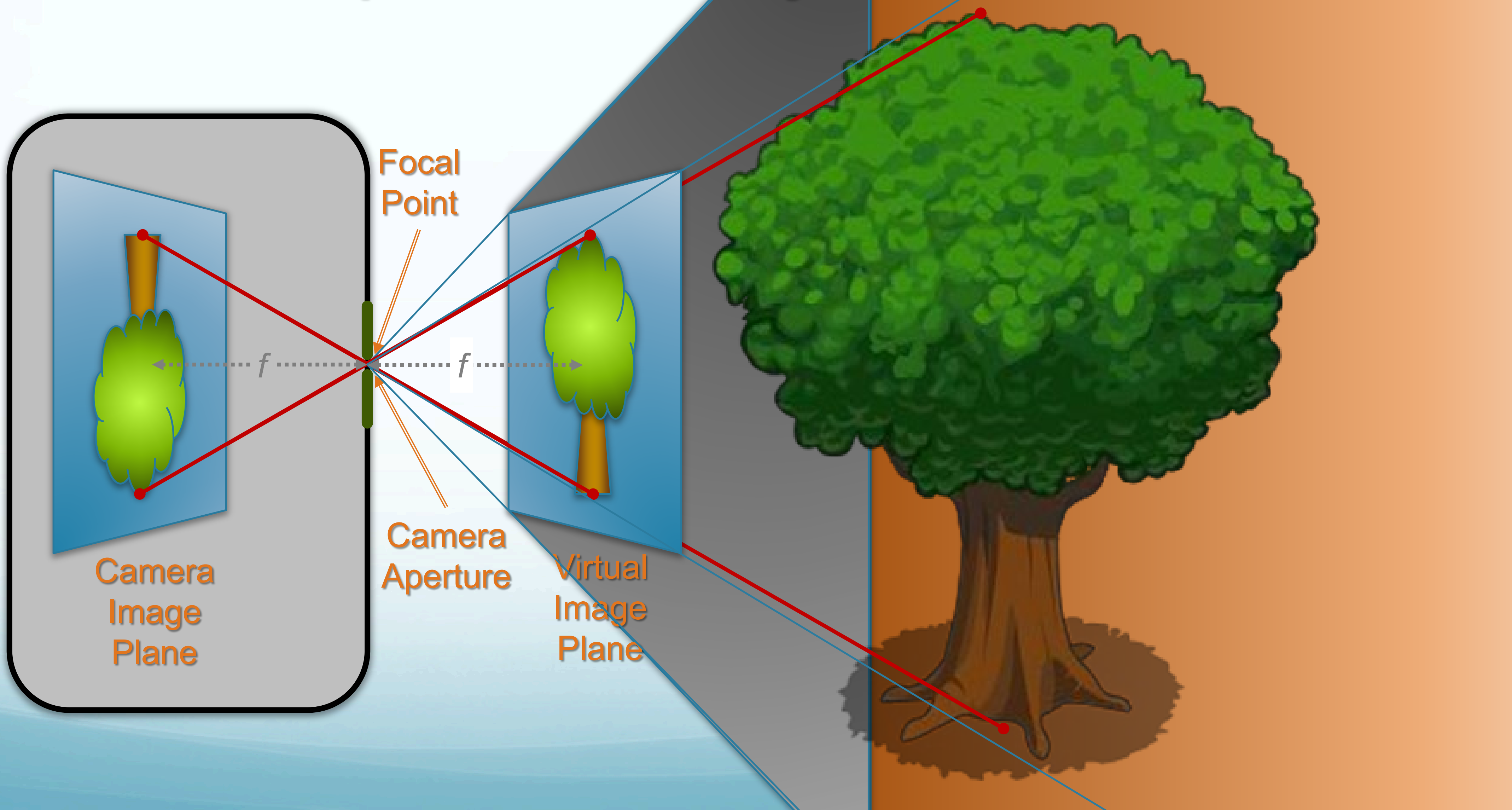
Perspective Projection: Frustum



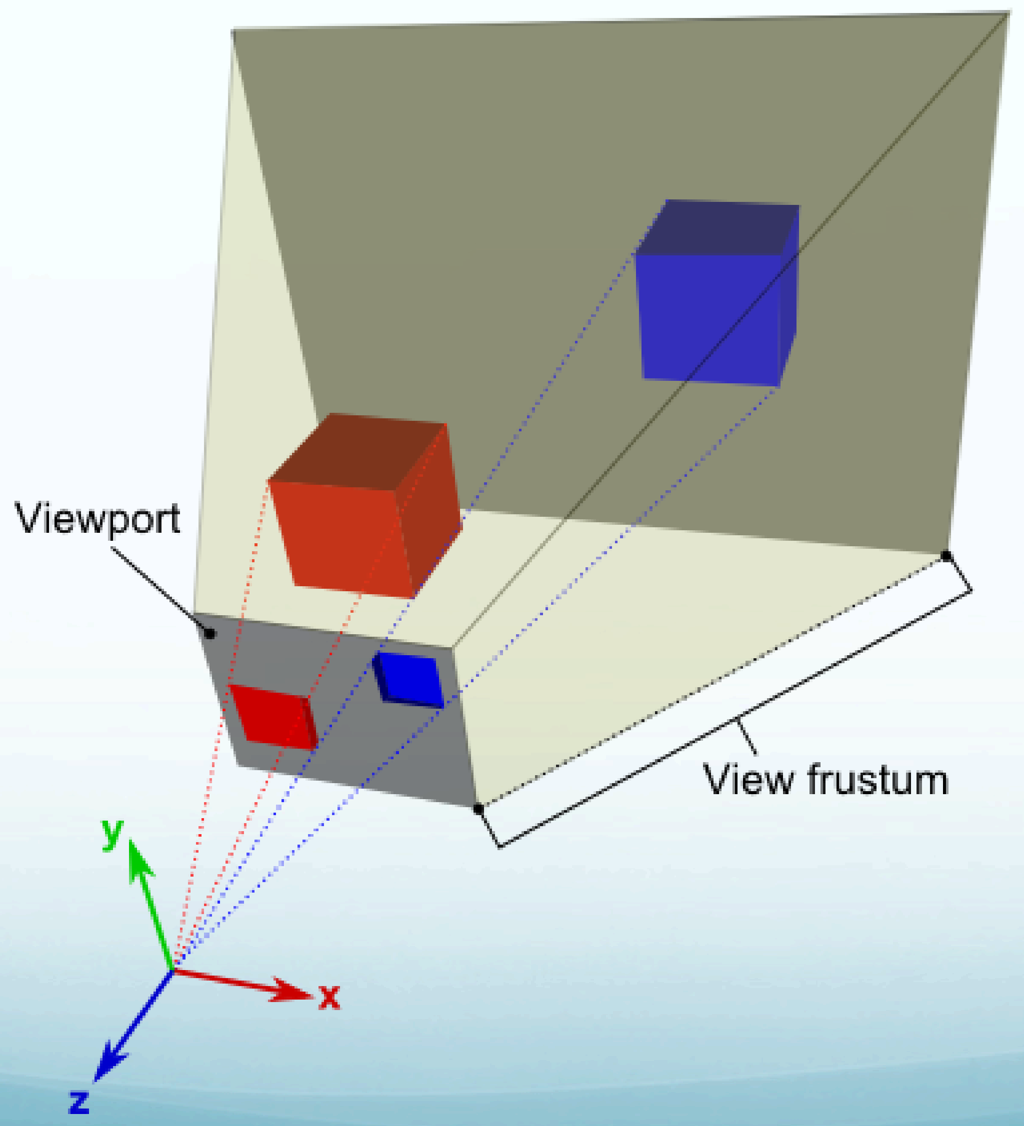
frus·tum, *noun*
the portion of a cone or pyramid that remains after its upper part has been cut off by a plane parallel to its base, or that is intercepted between two such planes.

"ViewFrustum" by MithrandirMage

Perspective Projection



Perspective Projection



XOJO (3D)

Projection

How does projection work mathematically?

- 2x3 matrix takes 3D points down to 2D
- (In homogeneous coordinates, matrix would be 3x4)
  - Ex: Orthographic projection is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
  - Perspective projection scales x and y by z component

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix}$$

f is the focal length

  - to restore homogeneous coordinates, 3rd coordinate must be normalized back to 1 after doing the projection

Not a matrix multiplication!

Perspective Projection

Example: Project the points (1,0,1), (1,0,2), and (2,0,2) using focal length of 1

Perspective projection matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 2 \end{bmatrix}$$

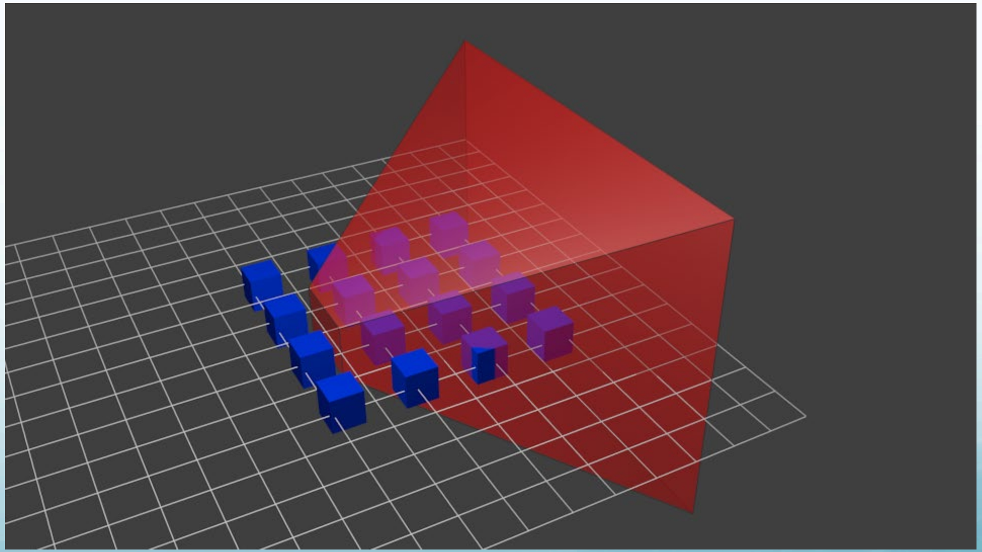
Matrix of points

The homogeneous coordinates must be normalized to 1!

Divide each column by the value in the bottom row

$$= \begin{bmatrix} 1 & 0.5 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Result is 2D homogeneous form of the original 3D points under perspective projection



Questions

PAUSE NOW & ANSWER

1. Which projection draws objects at the same size no matter how far away?

   Orthographic

2. Why do you think the view frustum has a far clipping plane?

   To avoid drawing many objects that are too small to see clearly.

3. Compute the **orthographic** projection of the points (6,4,4), and (2,8,2).

   (6,4) and (2,8)

4. Compute the **perspective** projection of the points (6,4,4), and (2,8,2) using focal length of 2

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 6 & 2 \\ 4 & 8 \\ 4 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 4 & 8 \\ 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 \\ 2 & 8 \\ 1 & 1 \end{bmatrix}$$

Review

After watching this video, you should be able to…

- List factors that influence the formation of images
- Identify different coordinate systems used during 3D rendering
- Define two types of projections & express as projection matrices
- Describe the view frustum and its purpose
- Project points numerically from 3D to 2D under both orthographic and perspective projection