ACADEMIC HONOR CODE

Students and faculty at Smith are part of an academic community defined by its commitment to scholarship, which depends on scrupulous and attentive acknowledgement of all sources of information and honest and respectful use of college resources.

Smith College expects all students to be honest and committed to the principles of academic and intellectual integrity in their preparation and submission of course work and examinations. All submitted work of any kind must be the original work of the student who must cite all the sources used in its preparation.

Student Signature: ________________________________

EXAM INSTRUCTIONS

ALL ANSWERS SHOULD BE SCANNED AND SUBMITTED ON MOODLE. PLEASE INCLUDE THE SIGNED AND SCANNED HONOR CODE STATEMENT FROM THE FRONT PAGE OF THIS EXAM.

THIS IS AN OPEN-BOOK ASSESSMENT. YOU MAY USE THE TEXTBOOK, CLASS NOTES, AND (WITH A CITATION) INTERNET REFERENCES. YOU MAY NOT DISCUSS ANY PART OF THIS EXAM WITH ANY PERSON OTHER THAN THE INSTRUCTOR, ELECTRONICALLY OR OTHERWISE, EXCEPT AS BELOW.

Option A. Take the exam on your own, without any assistance from others. Under this option, the exam is due on May 21 at 4:00 PM. (All deadlines for this assignment are EDT.)

Option B. Participate in a group exam process. Under this option, you must turn in your individual effort on the exam by Wednesday, May 19 at 4:00 PM. I will then assign you to a group, and groups will have until Thursday, May 20 at 4:00 PM to submit a single revised solution. Finally, the entire set of individuals who have chosen Option B will collaborate to produce their best combined solution by Friday, May 21 at 4:00 PM.
Consider the following three algorithms.

```javascript
function line1(p1, p2) {
    m = (p2.y - p1.y) / (p2.x - p1.x);
    b = ((p2.x * p1.y - p1.x * p2.y) / (p2.x - p1.x));
    for (x = p1.x; x <= p2.x; x++) {
        y = m * x + b;
        colorPixel(x, y);
    }
}

function line2(p1, p2) {
    y = Math.round(p1.y);
    x = Math.round(p1.x);
    while (x <= p2.x) {
        colorPixel(x, y);
        F = (p1.y - p2.y) * x + (p2.x - p1.x) * y + p1.x * p2.y - p2.x * p1.y + dx / 2 + dd;
        if (F < 0) {
            y++;
            d += dd;
        } else {
            d += dx;
        }
        x++;
    }
}

function line3(p1, p2) {
    y = Math.round(p1.y);
    x = Math.round(p1.x);
    dx = p1.y - p2.y;
    dy = p2.x - p1.x;
    dd = dx + dy;
    d = dx * p1.x + dy * y + p1.x * p2.y - p2.x * p1.y + dx / 2 + dd;
    while (x <= Math.round(p2.x)) {
        colorPixel(x, y);
        if (d < 0) {
            y++;
            d += dd;
        } else {
            d += dx;
        }
        x++;
    }
}
```

a.) Will these algorithms color the same pixels for a line between \( p_1 \) and \( p_2 \), assuming the slope is 0 < \( m < 1 \)? If there are differences, describe them. Which is closest to the true line?

b.) Comment on any advantages of each algorithm relative to the others (aside from the pixels colored).
Transformations (16 points)

Match each of the following matrices with all the descriptions from the list that correspond. If none of the descriptions applies then select “No match.” Assume the Y axis is oriented upwards, and we are applying each of the steps in the description to the graphics transform in the order shown. An example has been done for you.

Descriptions:

1. Rotate by 45 degrees, then scale by a factor of two in both X and Y
2. Rotate by -90 degrees, then translate by two units in the positive Y direction
3. Translate by two units in the negative X direction, then rotate by 90 degrees
4. Reflect across the line Y = X, then rotate by 45 degrees clockwise
5. Rotate by -315 degrees, then reflect across the X axis
6. Scale by a factor of 2 in X, then rotate 90 degrees
7. Rotate 90 degrees, then scale by a factor of 2 in X
8. Shear by 1 in the X direction, then rotate 90 degrees
9. Shear by 1 in the X direction, then rotate 45 degrees, then shear by 1 in the X direction, then rotate 45 degrees
10. Scale by 2 in the X dimension and 1/2 in the Y, then shear by 3 in the Y direction.
11. Rotate by -45 degrees and scale by 2 in both dimensions.
12. Reflect across the Y axis, then shear by 1 in the X direction
13. Reflect across the X axis, then shear by 1 in the Y direction
14. Translate 3 units in the positive X direction, rotate by 180 degrees, then translate 3 units in the negative X direction

\[
\begin{array}{ccc}
\text{a.) } & \begin{bmatrix}
1.4 & -1.4 & 0 \\
1.4 & 1.4 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
\text{b.) } & \begin{bmatrix}
2 & 0 & 0 \\
1.5 & 0.5 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
\text{c.) } & \begin{bmatrix}
1 & 0 & 0 \\
-1 & -1 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
\text{d.) } & \begin{bmatrix}
0 & -2 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
\text{e.) } & \begin{bmatrix}
0 & -1 & 2 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
\text{f.) } & \begin{bmatrix}
0.7 & 0.7 & 0 \\
0.7 & -0.7 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
\text{g.) } & \begin{bmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
\text{h.) } & \begin{bmatrix}
1 & 2 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
\text{i.) } & \begin{bmatrix}
0 & -1 & -2 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\end{array}
\]

\(\text{This is description #1}\)
**Line Clipping (16 points)**

Suppose that we have a 100x100 viewport, and want to draw a line segment from (-50,-80) to (150,120). If the line is clipped using the algorithm we studied in class, indicate for each time the recursive function is called (i) which case is triggered, and (ii) new clipped endpoints computed, if any.

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**Rendering Pipeline (12 points)**

Suppose we create a camera using the following Three.js commands:

```javascript
    camera = new THREE.PerspectiveCamera(120, 2, 10, 100);
    camera.position.set(0,0,5);
```

a.) The camera’s focal length is 5. What is the perspective projection matrix?
b.) If the canvas is 200 pixels wide, how high is it?
c.) Compute 3D coordinates for the eight corners of the view frustum. (Round to the nearest tenth.) *Hint: follow the rays from the corner pixels of the screen*
Texture Mapping (16 points)

Suppose that we want to map a cube as shown in the images below, using the texture image at right. What UV coordinates should we use for the vertices of the specified polygons? Hint: all the UV coordinates are multiples of 0.5.

a.) ABC
b.) BDC
c.) GEC
d.) CAE
e.) EAF
f.) FBA
g.) GHC
h.) CDH
**Shading (12 points)**

A surface rotates on an axis at point F. Material A has 100% diffuse reflectance and 0% specular reflectance, while material B has 0% diffuse reflectance and 100% specular reflectance, with $\alpha = 10$. Both the materials and the light source are white. The light is a directional light slanting downwards at a 20 degree angle away from the camera, as shown. Use this information to answer the questions below.

a.) For what angle $\theta_A$ will the shading at point F show the brightest shading with material A?

b.) For what angle $\theta_B$ will the shading at point F show the brightest shading with material B?

c.) How will the shading at point G compare to that at point F for angle $\theta_A$ and material A? (e.g., is it brighter, darker, or the same)

d.) How will the shading at point G compare to that at point F for angle $\theta_B$ and material B?

e.) Suppose $\theta_1 = 55^\circ$ with material A gives shading intensity $s$ at point F. Compute the angle $\theta_2$ that would result in shading intensity $s/2$ at point F.

f.) [Hard] Suppose $\theta_1 = 55^\circ$ with material B gives shading intensity $s$ at point F. Compute the angle $\theta_2$ that would result in shading intensity $s/2$ at point F.
Ray Tracing (16 points)

Hint for this problem: all the light travels in the XY plane. Draw a careful diagram and use trigonometry to compute the necessary angles. You can also use vector math.

While rendering an image, a computed ray passes through the air and strikes a semi-transparent planar surface. The ray equation is $R = (0,0,0) + (1,0,0)t$, and the surface normal vector of the surface is $(-0.87,0.50,0)$. If the point of contact is $(10,0,0)$, compute the following:

a.) The equation for the reflected light ray.
b.) The equation for the light ray transmitted through the material, if its index of refraction is 1.5.
c.) If there is a light source at $(0,25,0)$, what is the diffuse illumination at the point of contact as a fraction of the maximum strength of the light?
d.) Suppose there is a sphere of radius 2 centered at $(4,10,0)$. Determine whether it will block the light coming from the source mentioned in part c and provide the justification for your answer.