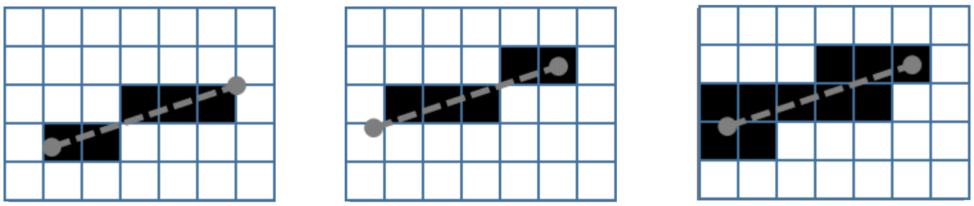
Line Drawing (16 pts)

A line needs to be drawn from (0.7,3.1) to (5.5,1.5) on each of the very small canvases below. (Each square is one pixel.) Show the pixels that would be filled in for each case.

a.) Simple line algorithm using pixel-center origin.
b.) Incremental midpoint algorithm using corner origin.
c.) Antialiased line using corner origin. (In the diagram, indicate all pixels that would be shaded even partially. You do not need to show the amount of shading.)
d.) For the line in part (b), what is the value of the discriminator $F(x, y)$ at the midpoint $x = 4.5$?

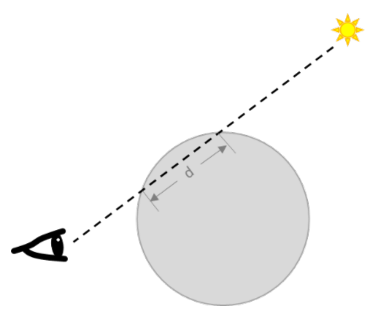


(a) (b) (c)

The line equation is $y = \frac{y_2 - y_1}{x_2 - x_1} x + \frac{x_2 y_1 - x_1 y_2}{x_2 - x_1} = -\frac{1}{3} x + \frac{10}{3}.$
The discriminator is $F(x, y) = (y_1 - y_2)x + (x_2 - x_1)y + x_1 y_2 - x_2 y_1 = 1.6x + 4.8y - 16$
At the midpoint when x=4.5, the evaluation point between the two possible next pixels is y = 2.
Thus $F(x, y) = F(4.5,2) = 1.6(4.5) + 4.8(2) - 16 = 0.8$

Subsurface Scattering & Raycasting (20 pts)

A translucent sphere is to be rendered with long-range subsurface scattering. The sphere has radius 11 and its center is at (0,0,-20). There is a light source located behind the sphere at (6,1,-40). Although the scattering is usually simulated using standard rendering with a z-buffer, for this problem you should use raycasting to answer the questions below.



a.) Give an equation for the ray that goes from the light source to the camera focal point located at the origin.

$$R(t) = (0,30,-30) + \left(0, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) t = (0,30,-30) + (0,-0.707,0.707)t$$

b.) Find the coordinates of the point(s) where the ray you computed in part (a) intersects the surface of the sphere.

$$R_0' = R_0 - S = (0,30,-30) - (0,0,-14) = (0,30,-16)$$

$$a = R_d \cdot R_d = \left(0, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \left(0, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = 0 + \frac{1}{2} + \frac{1}{2} = 1$$

$$b = 2\vec{R}_0' \cdot \vec{R}_d = 2(0,30,-16) \cdot \left(0, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = 2\left(0 - 15\sqrt{2} - 8\sqrt{2}\right) = -46\sqrt{2}$$

$$c = \vec{R}_0' \cdot \vec{R}_0' - r^2 = (0,30,-16) \cdot (0,30,-16) - 100 = 30^2 + (-16)^2 - 100 = 1056$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{46\sqrt{2} \pm \sqrt{4232 - 4 \cdot 1 \cdot 1056}}{2} = \frac{46\sqrt{2} \pm \sqrt{8}}{2} = \frac{46\sqrt{2} \pm 2\sqrt{2}}{2}$$

$$= 23\sqrt{2} \pm \sqrt{2}$$

$$R(t) = (0,30,-30) + \left(23\sqrt{2} \pm \sqrt{2}\right) \cdot \left(0, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$= (0,30,-30) + (0,-22,22)$ or $(0,30,-30) + (0,-24,24)$

= (0,8,-8) or (0,6,-6)

c.) If the intensity of subsurface scattered light is given by $I_{SS} = e^{-d/10}$, where d is the thickness of the material, what is the intensity that will be computed for the point in part (b)?

$$d = \|(0,6,-6) - (0,8,-8)\| = \sqrt{(0-0)^2 + (6-8)^2 + (-6-(-8))^2} = 2\sqrt{2} = 2.83$$

$$I_{SS} = e^{-d/10} = e^{-0.283} = 0.754$$

d.) Suppose that the display is 220 pixels wide and 220 pixels tall, with the viewport at $z = -1$ and extending to ± 1.1 on both x and y axes. The focal point is at the origin and the camera points in the –z direction. Give the standard form of the ray equation that goes through pixel (130,0).

$$x = \frac{130}{220} \cdot 2.2 - 1.1 = 0.2$$

$$y = \frac{220 - 0}{220} \cdot 2.2 - 1.1 = 1.1$$

$$z = -1$$

$$R_d = \frac{(0.2,1.1,-1)}{\|(0.2,1.1,-1)\|} = \frac{(0.2,1.1,-1)}{1.5} = \left(\frac{4}{30}, \frac{22}{30}, \frac{-20}{30}\right) = (0.133,0.733,-0.667)$$

$$R(t) = (0,0,0) + \left(\frac{4}{30}, \frac{22}{30}, \frac{-20}{30}\right)t = (0,0,0) + \left(\frac{2}{15}, \frac{11}{15}, -\frac{2}{3}\right)$$

e.) Find the 3D coordinates of the point where the ray from (0,0,0) through (1,2,-2) intersects the triangle ABC, where A=(0,0,-4), B = (4,0,-2), and C = (0,4,-4).

$$\vec{R} = (0,0,0) + \left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right)t$$

$$x_0 + tx_d = a_x + u(b_x - a_x) + v(c_x - a_x)$$

$$y_0 + ty_d = a_y + u\left(b_y - a_y\right) + v(c_y - a_y)$$

$$z_0 + tz_d = a_z + u(b_z - a_z) + v(c_z - a_z)$$

$$0 + t = 0 + u(4-0) + v(0-0)$$

$$0 + 2t = 0 + u(0-0) + v(4-0)$$

$$0 - 2t = -4 + u\left(-2-(-4)\right) + v(-4-(-4))$$

$$t = 4u$$

$$2t = 4v$$

$$-2t = -4 + 2u$$

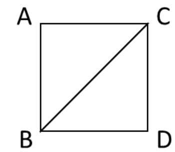
Solving, $-8u = -4 + 2u$, *so* $u = 0.4$. *Thus* $t = 1.6$ *and* $v = 0.8$. *Our point is:*

$$\vec{p} = \vec{A} + u\left(\vec{B} - \vec{A}\right) + v(\vec{C} - \vec{A})$$

$$\vec{p} = (0,0,-4) + 0.4(4,0,2) + 0.8(0,4,0) = (1.6,3.2,0.8) = \left(\frac{8}{5}, \frac{16}{5}, -\frac{16}{5}\right)$$

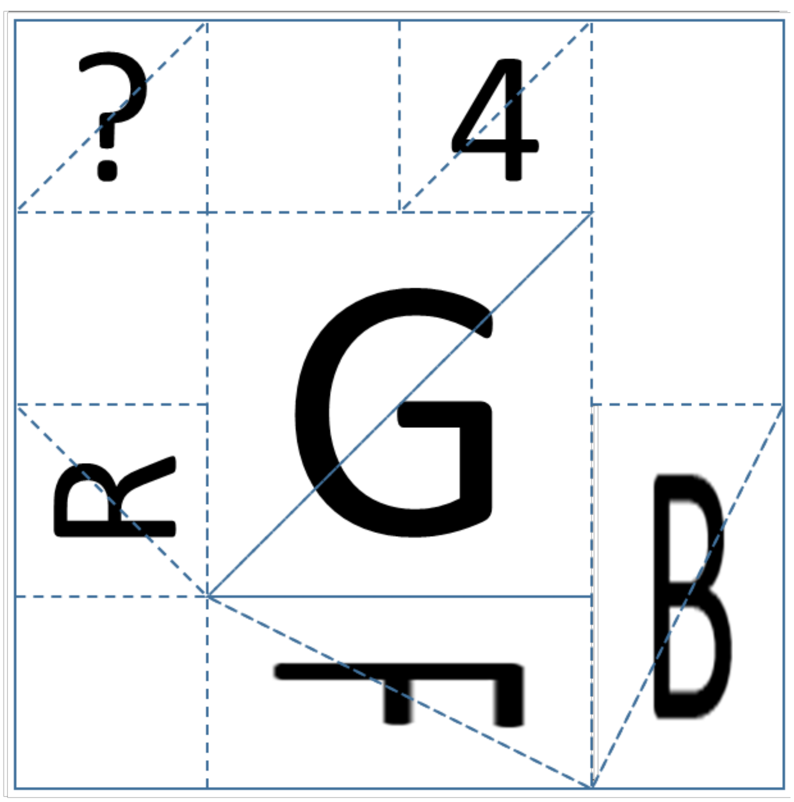
(Note that $u + v > 1$, so the point does not lie within the polygon.)

Texture Mapping (12 pts)



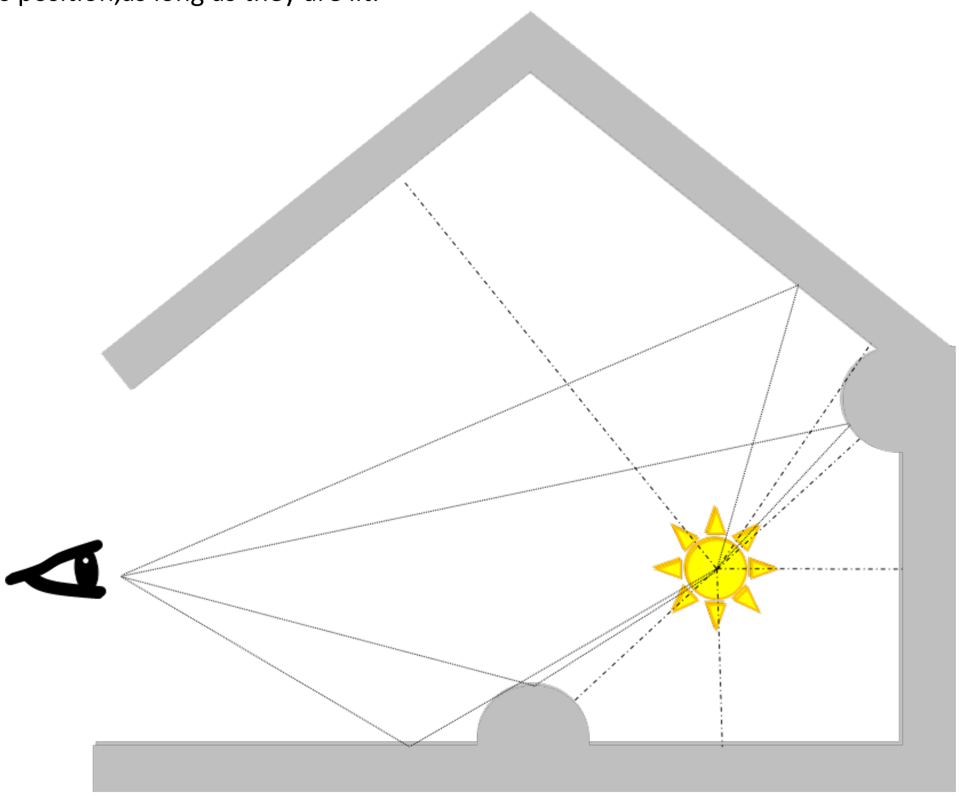
Letters are shown on the faces of a cube by mapping a texture onto it using the UV coordinates generated with the code shown below. Each face of the cube is made of two triangles, as shown in the image at right, with vertices always listed in the order ABC, then BDC. The six faces of the cube are labeled with ?, 4, R, F, B and G as indicated in the code comments. Infer the appearance of the texture source image, and draw what it must look like. Pay attention to both position and orientation. (You may assume that any unused portions of the texture source image are blank.)

```
geometry.faceVertexUvs[0] = [
    [new THREE.Vector2(0.00,1.00),new THREE.Vector2(0.00,0.75),new THREE.Vector2(0.25,1.00)],  // upper left ?
    [new THREE.Vector2(0.00,0.75),new THREE.Vector2(0.25,0.75),new THREE.Vector2(0.25,1.00)],  // lower right ?
    [new THREE.Vector2(0.50,1.00),new THREE.Vector2(0.50,0.75),new THREE.Vector2(0.75,1.00)],  // upper left 4
    [new THREE.Vector2(0.50,0.75),new THREE.Vector2(0.75,0.75),new THREE.Vector2(0.75,1.00)],  // lower right 4
    [new THREE.Vector2(0.00,0.25),new THREE.Vector2(0.25,0.25),new THREE.Vector2(0.00,0.50)],  // upper left R
    [new THREE.Vector2(0.25,0.25),new THREE.Vector2(0.25,0.50),new THREE.Vector2(0.00,0.50)],  // lower right R
    [new THREE.Vector2(0.75,0.25),new THREE.Vector2(0.25,0.25),new THREE.Vector2(0.75,0.00)],  // upper left F
    [new THREE.Vector2(0.25,0.25),new THREE.Vector2(0.25,0.00),new THREE.Vector2(0.75,0.00)],  // lower right F
    [new THREE.Vector2(0.75,0.50),new THREE.Vector2(0.75,0.00),new THREE.Vector2(1.00,0.50)],  // upper left B
    [new THREE.Vector2(0.75,0.00),new THREE.Vector2(1.00,0.00),new THREE.Vector2(1.00,0.50)],  // lower right B
    [new THREE.Vector2(0.25,0.75),new THREE.Vector2(0.25,0.25),new THREE.Vector2(0.75,0.75)],  // upper left G
    [new THREE.Vector2(0.25,0.25),new THREE.Vector2(0.75,0.25),new THREE.Vector2(0.75,0.75)]   // lower right G
];
```



Shading (10 pts)

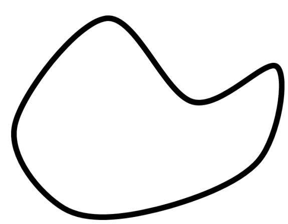
Clearly identify all the surface points in the scene below whose shading intensity will be a local maximum (i.e., brighter than the neighboring surface points on either side). Assume perspective projection and a Phong reflectance model with large alpha. Include points that may not be visible from the camera's position,as long as they are lit.



Diffuse maxima appear where angle of incidence is maximized (dash-dot lines). Specular maxima appear where reflections occur (dotted lines).

Splines (8 pts)

Please answer the questions below concerning splines



a.) A smooth closed loop curve will be represented using a cubic Bézier spline with five individual segments. How many unique control points will be required to specify the entire curve?
Each segment requires 4 points. However, the curve is closed and continuous so the endpoints must coincide. There are five shared endpoints. Thus the total number of unique control points is 15.

b.) A smooth closed loop curve will be represented using a cubic Bézier spline with five individual segments. How many unique control points must the curve pass through?

Normally each curve segment will only pass through the end control points. There would normally be two of these per segment, but each segment shares a control point with its neighbors. There are 5 unique control points that the curve must pass through.

c.) A smooth closed loop curve will be represented using a cubic polynomial spline with five individual segments. How many unique control points will be required to specify the entire curve?
Each segment has four control points, but these are all shared with other curves in the spline. There will be just five unique control points in all.

d.) A smooth closed loop curve will be represented using a cubic polynomial spline with five individual segments. How many unique control points must the curve pass through?
The curve muss pass through all five unique control points.

Matrices & Transformations (16 pts)

Using the transformation matrices provided below, compose a transformation that will have the effect described. If the effect is not possible using the matrices given, say "not possible."

F: Convert the model coordinates of a flowerpot into world coordinates with root position p_f
G: Convert the model coordinates of a garden gnome into world coordinates with root position p_g
H: Convert world coordinates to camera coordinates
O: Orthographic projection
P: Perspective projection with a focal length of 4
Q: Convert projected coordinates to screen coordinates
R: Perform 3D rotation by 180 degrees around the z axis
S: In 3D, scale the z coordinate by a factor of 2
T: Translate in 3D coordinates from p_f to p_g

a.) Place the garden gnome in the scene at position p_f *$T^{-1}G$*
b.) Place the flowerpot in the scene upside-down at position p_f *FR*
c.) Convert world coordinates to screen coordinates under perspective projection *QPH*
d.) Convert screen coordinates to world coordinates under orthographic projection
Not possible (you cannot recover the z coordinates)
e.) Convert flowerpot model coordinates to camera coordinates (assuming the flowerpot is at p_f)
HF
f.) Convert camera coordinates to flowerpot model coordinates (assuming the flowerpot is at p_f)
$F^{-1}H^{-1}$
g.) Convert camera coordinates to projected coordinates under perspective projection with a focal length of 2.
PS
h.) Convert camera coordinates to projected coordinates under perspective projection with a focal length of 6.
Not possible using the matrices given.

Three.js (8 pts)

Answer the questions below about Three.js. You may use the online Three.js documentation to research your answers.

a.) Class used to represent a homogeneous 3D transformation
THREE.Matrix4

b.) Class used to represent a UV coordinate
THREE.Vector2

c.) Class used to perform raycasting
THREE.Raycaster

d.) Class used to create a virtual object for purposes of hierarchical modeling
THREE.Object3D

e.) Class used for representing the shape of 3D text in a scene
THREE.TextGeometry

f.) Class used for adding a particle cloud to a scene
THREE.Points (also accepted THREE.PointCloud)

g.) Expression to create a camera object for a canvas 120 pixels wide by 80 pixels tall, using perspective projection with a 90 degree field of view, clipping objects nearer than 2 units and farther than 80
new THREE.PerspectiveCamera(90,1.5,2,80)

h.) Expression to create a pure green light source of default intensity, extending 15 units in every direction, without decay
new THREE.PointLight(0x00ff00,1,15,0)

Rendering (10 pts)

The diagram below shows a cross-section of objects in world coordinate space. The camera is placed at the origin and points in the negative z direction. Please answer the questions that follow.

a.) Under the painter's algorithm, in what order would the objects be rendered?
N, K, H, A, C, M, G, J, B, F, E, O, D, I, L

b.) Using z-buffering and orthographic projection, and assuming that the objects are rendered in alphabetical order, which objects would never be drawn in the scene at all?
G, H, K, M, N

c.) Using z-buffering and orthographic projection, and assuming that the objects are rendered in alphabetical order, which objects would be partially or completely overpainted?
A, B, C, D, E

d.) Using z-buffering and orthographic projection, and assuming that the objects are rendered in alphabetical order, what would be the contents of the z-buffer after objects A-G have rendered?
Top to bottom: -∞, -9, -9, -∞, -14, -14, -∞, -8, -4, -4, -7, -7

e.) Assuming perspective projection, with field of view 90 degrees, near clipping plane at z=-0.5 and far clipping plane at z=-13.5, which objects would be visible in the final rendering?
I, O, B, M, J, F, D. L is outside the frustum, and C and H are beyond the far clipping plane.