CS 240: Computer Graphics
Solution to Midterm: Fall 2020

Part 1: Pixels and Coordinates

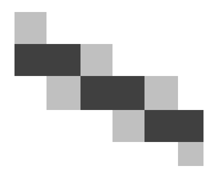
A line is to be drawn on a screen from (30,20) to (90,50) using antialiased graphics.

Note: this question proved hard to grade. In some cases different answers were accepted based upon assumptions that were not spceified in the problem. Answers that received full credit were clear, correct, and openly stated the assumptions that were made.

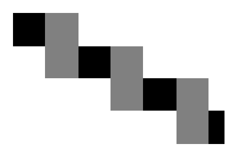
a.) Does the length of the drawn line change under the pixel center origin vs. corner origin conventions? Why or why not?
The length of the line should not change. All coordinates are shifted by 0.5, and should be rounded in the same manner. (Specific rounding details could change this answer.)

b.) Does the position of the drawn line change under the pixel center origin vs. corner origin conventions? Why or why not?
The position will shift downwards and to the right by 0.5 pixel under pixel center origin. This will be visible as a shift of the darkest intensity pixels.

c.) If you examine a small window of pixels near the center of the line, will it be possible to distinguish between lines drawn under the pixel center origin vs. corner origin conventions? Why or why not?
Yes. Examined closely enough, you will see a difference in the fill pattern because the line cuts through the pixel grid differently.



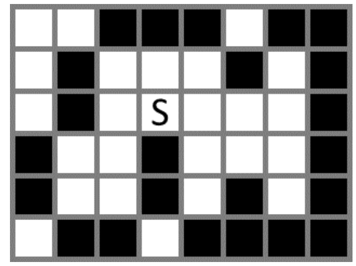
d.) If you examine a small window of pixels near an end of the line, will it be possible to distinguish between lines drawn under the pixel center origin vs. corner origin conventions? Why or why not?
Yes. Examined closely enough, you will see a difference in the fill pattern because the line cuts through the pixel grid differently.

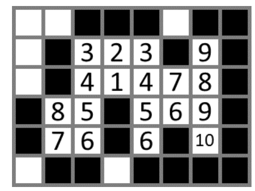


e.) Which of your answers above would change if the line were drawn using simple graphics (no antialiasing) instead?
Answer c will change. Answer b might change depending on the rounding conventions chosen.

Part 2: Fill Algorithms

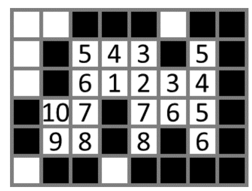
To avoid a call stack overflow with the recursive fill algorithm, the important statistic to track is the recursion depth, or the number of invocations of a recursive function that are active (have not yet returned) at any given moment. An error will occur when the recursion depth reaches a predefined limit set by the system. For the image at right, and the specified recursion order below, give the maximum recursion depth that would occur. You may show your work for partial credit.

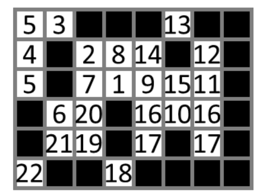




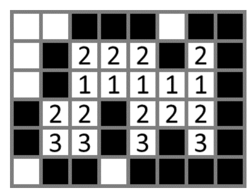
a.) Recursion order {North, South, East, West}. *11*
(Note that the 10^{th} square makes calls that return immediately.)

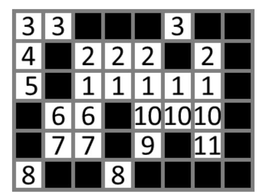
a.) Recursion order {East, South, West, North} *11*





b.) Recursion order {Northwest, Northeast, Southeast, Southwest, North, East, South, West}. *23*



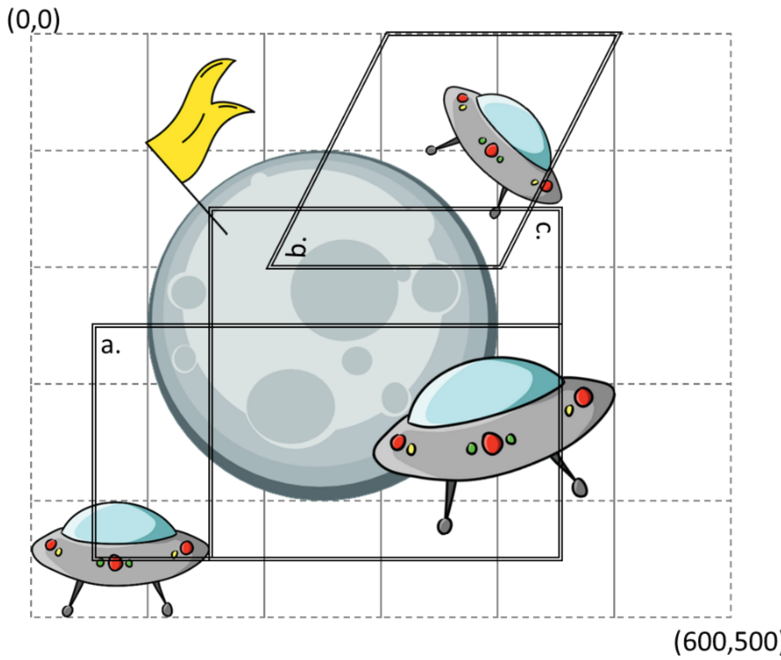


c.) Using sweep fill with 4-connection, and recursion order {North, South}, what is the maximum recursion depth? *4*

d.) Using sweep fill with 8-connection, and recursion order {Northwest, North, Northeast, Southwest, South, Southeast}, what is the maximum recursion depth? *12*

Part 3: Transformations.

Suppose that we have the scene shown below, with world coordinates as specified at the corners. Our viewport is 100x100 pixels.



a.) Give the rendering transform matrix (in homogeneous coordinates) that would display the portion of the scene enclosed by the box marked a above. Assume that the scene will be is oriented with the a label at the top left.

The selected area must be scaled and translated so that it maps onto the viewport.

Corner points are $P = \begin{bmatrix} 50 & 50 & 450 & 450 \\ 250 & 450 & 250 & 450 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. *Translating the top left corner to (0,0) requires* $T = \begin{bmatrix} 1 & 0 & -50 \\ 0 & 1 & -250 \\ 0 & 0 & 1 \end{bmatrix}$. *Now* $TP = \begin{bmatrix} 0 & 0 & 400 & 400 \\ 0 & 200 & 0 & 200 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. *Scaling to 100x100 requires* $S = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. $STP = \begin{bmatrix} 0 & 0 & 100 & 100 \\ 0 & 100 & 0 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ *as desired.*

The final transform is $ST = \begin{bmatrix} 0.25 & 0 & -12.5 \\ 0 & 0.5 & -125 \\ 0 & 0 & 1 \end{bmatrix}$.

b.) Give the rendering transform matrix (in homogeneous coordinates) that would display the portion of the scene enclosed by the box marked b above. Assume that the scene will be oriented with the b label at the top left.

The selected area must be scaled, rotated, sheared, and translated to the viewport.

Corner points are $P = \begin{bmatrix} 200 & 400 & 300 & 500 \\ 200 & 200 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. *Translating the top left corner to (0,0) requires* $T = \begin{bmatrix} 1 & 0 & -200 \\ 0 & 1 & -200 \\ 0 & 0 & 1 \end{bmatrix}$. *Now* $TP = \begin{bmatrix} 0 & 200 & 100 & 300 \\ 0 & 0 & -200 & -200 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. *Shearing to restore a square requires* $H = \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. *Now* $HTP = \begin{bmatrix} 0 & 200 & 0 & 200 \\ 0 & 0 & -200 & -200 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

Rotation by 90 degrees is $R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. *Now* $RHTP = \begin{bmatrix} 0 & 0 & 200 & 200 \\ 0 & 200 & 0 & 200 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

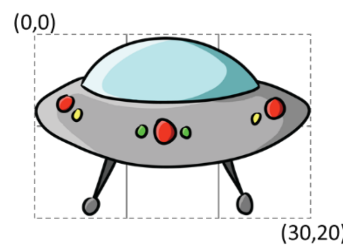
Scaling to 100x100 requires $S = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. $SRHTP = \begin{bmatrix} 0 & 0 & 100 & 100 \\ 0 & 100 & 0 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

The final transform is $SRHT = \begin{bmatrix} 0 & -0.5 & 100 \\ 0.5 & 0.25 & -150 \\ 0 & 0 & 1 \end{bmatrix}$.

c.) On the image above, draw a box showing the portion of the viewport that would be shown if the rendering transform was $\begin{bmatrix} 0 & 0.33 & -50 \\ -0.33 & 0 & 150 \\ 0 & 0 & 1 \end{bmatrix}$. Label the top left corner <u>c</u>.

Corner points are $P = \begin{bmatrix} 450 & 150 & 450 & 150 \\ 150 & 150 & 450 & 450 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. *Answer shown on image above.*

d.) Assume that we have the object template shown at right, with scale given by the coordinates in the corners. Give the modeling transform that would place this object into the scene at the position of the UFO at the lower left.



Corner points in model coordinates are $P_M = \begin{bmatrix} 0 & 0 & 30 & 30 \\ 0 & 20 & 0 & 20 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ *and target world coordinates are* $P_W = \begin{bmatrix} 0 & 0 & 150 & 150 \\ 400 & 500 & 400 & 500 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. *Scale is* $S = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Now $SP_M = \begin{bmatrix} 0 & 0 & 150 & 150 \\ 0 & 100 & 0 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. *Translation is* $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 400 \\ 0 & 0 & 1 \end{bmatrix}$.

Full transform is $TS = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 400 \\ 0 & 0 & 1 \end{bmatrix}$

e.) Suppose that the object is placed into the world with a modeling transform of $M_M = \begin{bmatrix} 2 & 0 & 200 \\ 0 & -2 & 300 \\ 0 & 0 & 1 \end{bmatrix}$, and the world is displayed with a rendering transform of

$M_R = \begin{bmatrix} 0.2 & 0 & -100 \\ 0 & 0.2 & 100 \\ 0 & 0 & 1 \end{bmatrix}$. What single matrix would transform points in object coordinates to viewport coordinates?

The combined transform is $M_R * M_M = \begin{bmatrix} 0.4 & 0 & -60 \\ 0 & -0.4 & 160 \\ 0 & 0 & 1 \end{bmatrix}$

Part Four: Curves

Answer the questions below concerning curves.

a.) Given a third-order Bézier curve with $p_0 = (0,7)$, $p_1 = (1,2)$, $p_2 = (4,6)$, and $p_3 = (5,-1)$, what point would lie on the curve at $t = 0.4$?

$$p_{01} = 0.6 * p_0 + 0.4 * p_1 = (0.4,5)$$

$$p_{12} = 0.6 * p_1 + 0.4 * p_2 = (2.2,3.6)$$

$$p_{23} = 0.6 * p_2 + 0.4 * p_3 = (4.4,3.2)$$

$$p_{012} = 0.6 * p_{01} + 0.4 * p_{12} = (1.12,4.44)$$

$$p_{123} = 0.6 * p_{12} + 0.4 * p_{23} = (3.08,3.44)$$

$$p = p_{0123} = 0.6 * p_{012} + 0.4 * p_{123} = (2.296,4.04)$$

b.) Given a first-order Bézier curve with $p_0 = (3,3)$ and $p_1 = (5,8)$, what is the t value for the point (3.7,4.75)?
3.7 is 35% of the way between 3 and 5, and 4.75 is 35% of the way from 3 to 8. Therefore $t = 0.35$.

c.) Given a second-order Bézier curve with $p_0 = (-2,0)$, $p_1 = (3,6)$, and $p_2 = (4,1)$, what is the t value at the point (3.44,2.56)?

The equation for a second-order curve is $p = (1-t)^2 p_0 + 2t(1-t)p_1 + t^2 p_2$.
Substituting the x coordinates we have $3.44 = (1-t)^2(-2) + 2t(1-t)3 + t^2 4$.
Simplifying gives $3.44 = -2 + 4t - 2t^2 + 6t - 6t^2 + 4t^2$, *or* $4t^2 - 10t + 5.44 = 0$.
Using the quadratic formula we get t=0.8 or 1.7.

Substituting the y coordinates we have $2.56 = (1-t)^2(0) + 2t(1-t)6 + t^2 1$, *and simplifying gives* $11t^2 - 12t + 2.56 = 0$. *Using the quadratic formula we get t = 0.29 or 0.8*

Both coordinates are consistent with t=0.8, therefore this is the answer.

d.) Suppose we want a second-order Bezier curve with $p_0 = (0,4)$ and $p_2 = (4,0)$ to be symmetric around the line $y = x$ and pass through the point (4,4). Where should we place p_1?

Since the curve is symmetric around y=x, we know that at (4,4) we must have t=0.5. Plugging into the curve equation, we get $4 = (0.5)^2 0 + 2(0.5)(0.5)x_1 + (0.5)^2 4$ *and* $4 = (0.5)^2 4 + 2(0.5)(0.5)y_1 + (0.5)^2 0$. *Solving we get* $p_1 = (x_1, y_1) = (6,6)$.

Part 5: Line Clipping

Consider the diagram at right while answering the questions below. For each question you are asked to give the endpoints of a line or lines that would satisfy the given description, or state that no such line exists using the available endpoints shown. (Assume that the first endpoint given is p_0 and the second is p_1.) You should reference the case breakdown in the revised handout on Cohen-Sutherland line clipping.

a.) Any line or lines that immediately trigger Case 1. *GH or HG*

b.) Any line or lines that immediately trigger Case 2, starting from F. *FA, FJ, FK.*

c.) Any line or lines that do not trigger Case 2 immediately, but eventually do trigger Case 2 during evaluation of a recursive call. *JL/LJ and CE/EC*

d.) Any line or lines that will immediately trigger Case 3e, followed by Case 3h. *HA*

e.) Any line or lines that will immediately trigger Case 3f, followed by Case 3h. *None*

f.) Any line or lines that will immediately trigger Case 3a, followed by Case 3f. *BK, BL, BM, BN, CK, CL, CM, CN; possibly DK, DL*

g.) Any line or lines that will immediately trigger Case 3g, followed by Case 3d. *None*

h.) Any line or lines that will trigger five or more cases in total. *AN/NA*