Fall 2019 FINAL EXAM
Course Number and Section: CSC 240 Section 01
Course Title: Introduction to Computer Graphics
Instructor: Nicholas R. Howe
Exam Format: Self-scheduled, limited notes
Number of Blue Books per student: 1

STUDENT NAME: ____________________________
ID NUMBER: _______________________________
CLASS YEAR: ______________________________
EXAM DATE: _______________________________
EXAM OUT: ________________________________
EXAM DUE: _________________________________
EXAM IN: _________________________________

ACADEMIC HONOR CODE
Students and faculty at Smith are part of an academic community defined by its commitment to scholarship, which depends on scrupulous and attentive acknowledgement of all sources of information and honest and respectful use of college resources.

Smith College expects all students to be honest and committed to the principles of academic and intellectual integrity in their preparation and submission of course work and examinations. All submitted work of any kind must be the original work of the student who must cite all the sources used in its preparation.

Student Signature: _______________________________

Note: All blue books, used or unused, must be returned with the exam.

EXAM INSTRUCTIONS

ALL ANSWERS SHOULD BE WRITTEN IN YOUR EXAM BOOKLET.

YOU MAY USE A CALCULATOR AND TWO 8.5"x11" DOUBLE-SIDED SHEETS OF NOTES ON THIS EXAM.

YOU MAY NOT USE THE TEXTBOOK, A COMPUTER, OR ANY OTHER INFORMATION SOURCE BESIDES YOUR PAGES OF NOTES.

THIS EXAM CONSISTS OF THREE (3) PAGES WITH SEVEN (7) QUESTIONS
Line drawing (18 pts)

A red line is to be drawn between the center of pixel (0,0) to the center of pixel (4,10) in screen coordinates using one of the algorithms shown. For each algorithm, determine (i) how many pixels will be colored completely red, (ii) how many pixels will be colored some shade of pink between white and red, and (iii) whether pixel (1,4) will be colored at all.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th># red</th>
<th># pink</th>
<th>(1,4) colored?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.) Basic midpoint algorithm</td>
<td></td>
<td></td>
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<tr>
<td>b.) Incremental midpoint algorithm</td>
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<td>c.) Anti-aliased line drawing algorithm</td>
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</tbody>
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Matrices & Transformations (16 pts)

Consider the following sets of matrices in answering the questions that follow. (Note: matrix labels are chosen alphabetically and do not indicate the matrix's role.) Assume that the scene contains an excavator and a tree, and the excavator is a hierarchical model with the engine as parent and digger arm as a subpart.

A = Converts projected coordinates to screen pixel coordinates
B = Converts camera view coordinates to world coordinates
C = Converts excavator’s coordinates to the digger arm coordinates
D = Converts excavator’s coordinates to world coordinates
E = Converts world coordinates to the tree’s model coordinates
F = Converts camera reference frame coordinates to world coordinates

\[
G = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
H = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0.5 & 0
\end{bmatrix}
\]

\[
I = \begin{bmatrix}
0.6 & 0.8 & 0 & 0 \\
-0.8 & 0.6 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

a.) Using the matrices shown and/or their inverses as necessary, compose the matrix that would be necessary to compute the pixel coordinates for the tip of the digger arm under a perspective projection.
b.) Using the matrices shown and/or their inverses as necessary, compose the matrix that would be necessary to compute the pixel coordinates for the bottom of the excavator engine under orthographic projection.
c.) Using the matrices shown and/or their inverses as necessary, compose a matrix that would convert the digger arm coordinates to world coordinates and add one to its x coordinate.
d.) Using the matrices shown and/or their inverses as necessary, compose a matrix that would rotate the tree coordinates and then transform them to excavator coordinates.
Clipping (12 pts)

Suppose that a camera is defined using the Three.js commands shown below. Answer the questions that follow.

```javascript
camera = new THREE.PerspectiveCamera(90, 1, 10, 100);
camera.position.set(0,0,10);
```

a.) What are the coordinates of the four corners of the view plane?
b.) What are the coordinates of the four corners of the far side of the view frustrum?
c.) Determine whether the following entities will be fully, partially, or not visible in the scene (assuming they are not blocked by other objects):
   i.) Point with world coordinates (0,0,-95)
   ii.) Point with world coordinates (8,8,-8)
   iii.) Line from world coordinates (-100,0,-20) to (100,0,-20)
   iv.) Line from world coordinates (0,0,5) to (0,5,-5)

Lighting (16 pts)

In the scene below, identify (a) any point(s) on the surfaces of the gray objects that will receive maximum illumination from the light source shown at the center, and (b) any point(s) that will receive illumination \( f = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \).

Texture Mapping (10 pts)

Suppose that the image shown below is used as a texture map for the shape at right. UV coordinates for several of the vertices are shown. Determine the color of the point at the center of each of the labeled faces A through J. (Note: the coordinates of the center point can be computed by averaging all three corners.)
Ray Tracing (16 pts)

Suppose that a ray from the camera located at the origin strikes a vertical plane (equation $z = -8$) at the point $(0,6,-8)$ and reflects.

   a.) Draw a side view of this situation.  
   b.) Give the equation of the reflected ray.  
   c.) Give the equation of the ray from the point of contact to a light source at $(16,6,4)$.  
   d.) At what point would the ray to the light source hit a sphere of radius 14 centered at $(0,-6,2)$?

Particle Systems (12 pts)

Particle positions can follow any sort of progression with time. Although physical simulations are common, another possibility is to follow a Bézier curve. Suppose that for each particle we define three control points and animate its position over a two-second lifespan. Assuming 30 frames per second, where would each of the following particles be located? Show your work.

   a.) Particle at frame 0, control points $P_0 = (1,2,3)$; $P_1 = (4,5,6)$; $P_2 = (7,8,9)$.  
   b.) Particle at frame 30, control points $P_0 = (100,0,0)$; $P_1 = (0,100,0)$; $P_2 = (0,0,100)$.  
   c.) Particle at frame 40, control points $P_0 = (9,0,18)$; $P_1 = (9,27,0)$; $P_2 = (81,27,18)$.  