Computational Geometry Column 38

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Abstract

Recent results on curve reconstruction are described.

Reconstruction of a curve from sample points ("connect-the-dots") is an important problem studied now for twenty years. Early efforts, primarily by researchers in computer vision, pattern recognition, and computational morphology, relied on ad hoc heuristics (e.g., my own [OBW87]). The heuristics were placed on a firmer footing with α-shapes [EKS83] and β-skeletons [KR85] and other structures, whose underlying proximity graphs were later shown to support accurate reconstruction from uniformly sampled curves [FMG95, Att98, BB97]. User selection of the α or β parameter is still necessary.

A breakthrough was achieved by Amenta, Bern, and Eppstein [ABE98], who designed two algorithms that guarantee correct reconstruction of smooth closed curves even with (sufficiently dense) nonuniform samples, and which lift the burden of selecting a parameter. One of their algorithms computes what they call the *crust*, a subgraph of the complete graph on the sample points that coincides with the correct polygonal curve under the right conditions. One of the novelties of their approach is to define the sample density to increase on exactly those portions of the curve Γ where more points are needed for reconstruction. They demand that for every point $x \in \Gamma$, there is a sample point p such that $|xp| < \epsilon\mu(x)$, where $\mu(x)$ is the distance from x to the medial axis/skeleton. This distance is small wherever two sections of the curve are close (in the vicinity of a sharp turn, or a narrow neck), for such sections are separated there by a branch of the medial axis. They can guarantee reconstruction for all $\epsilon \lesssim \frac{1}{4}$. Fig. 1(a) illustrates a reconstruction using their algorithm, and (b) shows the crust when the sample density is below their threshold.[1]

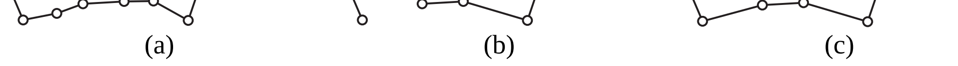

Figure 1: (a) [ABE98] algorithm on a densely sampled curve. (b) The crust [ABE98], with 12 fewer sample points. (c) TSP algorithm on sparse set. Computations performed at `http://review.mpi-sb.mpg.de:81/Curve-Reconstruction/` [AMNS00].

Their work was followed by a flurry of improvements and extensions: a computational improvement [Gol99], a simpler "nearest-neighbor crust" that raises the density threshold to $\epsilon = \frac{1}{3}$ [DK99], and an extention to curves with endpoints [DMR99]. Another line of investigation was opened by Giesen [Gie99], who proved that the TSP tour reconstructs the curve for uniformly sampled nonsmooth curves (and that no larger class of connected curves can be correctly reconstructed within the Delaunay subgraph.) This was quickly extended

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[1] It should be noted that some applications call for multiple components, or nodes of degree > 2, as in (b) of the figure.

by Althaus and Mehlhorn to nonuniform samples [AM00], who in addition established that the TSP instance can be solved in polynomial time.

Althaus et al. [AMNS00] have now implemented all the major curve reconstruction algorithms using LEDA [MN99], and made them available for interactive comparison on the Web. Their experiments show that the TSP algorithm is both the most time-intensive computation (13 times the fastest competitor, [DK99]), but also the most robust for sparsely sampled curves,[2] as indicated by Fig. 1(c).

The next frontier in provable reconstruction is reconstruction of 2D surfaces embedded in 3D. See [ABK98, AB99, AC99, DL99] for a start.

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[2] Or at least for certain classes of curves, with uniform random sampling.

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