

A Survey of Map Labeling with Sliding Labels*

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Abstract

In this paper, I survey the status of research on map labeling with sliding labels. Some open problems are also listed.

1 Introduction

Map labeling is a common problem in our daily life. It is an old art in cartography and finds new applications in recent years in GIS, graphics and graph drawing. For early heuristic methods readers are referred to [DF92]. For a complete list of references, readers are referred to the Map Labeling Page maintained by Alex Wolff [Wo]. In many applications, for the ease of presenting automatic solutions using computers, people use discrete model, i.e., each of the n given sites can only have a constant number of candidate labels (among which one will be chosen). For example, in the paper by Formann and Wagner, any point site can only be labeled with 4 candidate (axis-parallel) squares each of which has a vertex anchored at the site and the objective is to maximize the square size. Even this simplest version is shown to be NP-complete and moreover; it is NP-hard to approximate the problem within factor 2 [FW91]. The first continuous model was proposed in 1982 by Hirsch to use repelling forces between labels to find acceptable labeling [Hir82], but no theoretical results was given.

In the past several years this more generalized (and natural) model has been proposed for further research in map labeling (i.e., to either maximize the number of sites labeled or maximize the size of the labels). The basic idea is to allow each site to have an infinite number of possible candidate labels (see [DMMMZ97, IL97, KSW99, SW01, ZP99]). This model is more natural than the previous discrete models (like the one in [FW91]) and has been coined as the *sliding model* in [KSW99]. On the other hand, designing efficient algorithms for map labeling under the sliding model is a new challenge to map labeling researchers.

In this paper, we first briefly review the current status of map labeling research with sliding labels and then we list some open problems. We mainly focus on maximizing the size of the labels. (On maximizing the number of sites labeled, readers are referred to [EJS01]. We also skip bicriteria approximation algorithms.) For all the problems discussed, we assume that n input sites are given, no two labels can overlap, a site must be on the boundary of its label and all the labels are of the same size. We use $O_{problem}^*$ to denote the optimal solution (value) of a problem.

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2 The Survey

2.1 1APsquare

In this case, the label for each site is an axis-parallel square. van Kreveld *et al.* proved that the problem is NP-hard, in fact, NP-hard to approximate within a factor of 1.33 (the proof was originally for maximizing the number of sites labeled with uniform axis-parallel squares and their proof also works for this size-maximization problem) [KSW99]. A factor-4 solution is easy to obtain: It is easy to see that if we use the discrete model of Formann and Wagner we can always achieve at least $1/2$ of $O_{1APsquare}^*$. This latter problem is NP-hard and Formann and Wagner obtained $1/2$ of the optimal solution under the discrete model [FW91]. So we can achieve at least $1/4$ of $O_{1APsquare}^*$ in $O(n \log n)$ time. The best known approximation factor is 2 [QZ02]. While this solution is complex, there is a simple factor-3 approximation [ZJ04].

2.2 2APsquare

In this problem, one needs to label each site with a pair of axis-parallel squares. It was proved recently that this problem is NP-hard and in fact, NP-hard to approximate within a factor of 1.33 [Sp00]. The first approximation algorithm is due to Zhu and Poon, who presented a factor-4 approximation in $O(n \log n)$ time [ZP99]. This factor was improved to 3 by Zhu and Qin using a graph theoretical method with the same time complexity [ZQ02]. Later, Qin *et al.* improved the factor further to 2 [QWXZ00], using the idea of Formann and Wagner together with a technique of [PZC98]. It also runs in $O(n \log n)$ time.

2.3 3APsquare

This is a new problem in which one needs to label each site with a triple of axis-parallel squares. (Note that when we use 4 axis-parallel squares to label each site then the problem is trivial.) Duncan *et al.* showed recently that the problem can be solved in optimal $\Theta(n \log n)$ time under the discrete model and in $O(n^2 \log n)$ under the sliding model [DQVZ03].

2.4 1Circle

Map labeling with circles was first studied as a theoretical problem, but recently we were contacted by some GIS company for real applications. In this case the label for each site is a circle. The first approximation bound is by Doddi *et al.*, $8(2 + \sqrt{3})$, which is roughly 29.86 [DMMMZ97]. Strijk and Wolff improved the factor to 19.35 and proved that the problem is NP-hard (in fact, NP-hard to approximate within a factor better than 1 — though they could not actually compute this factor) [SW01]. Doddi *et al.* presented a new technique and a concept, which was later referred to as *maximal feasible region*, which improves the factor significantly to about 3.60 [DMM00]. Recently, Jiang *et al.* improved the factor further to $3 + \epsilon$ using a new combinatorial lemma of labeling circles with points [JQQZC03]. With some further tuning and a revised definition of *maximal feasible regions*, a $2.931 + \epsilon$ bound was obtained recently [JZBQ04].

2.5 2Circle

Map labeling with 2-circles is a very natural generalization of 2APsquare. The first approximation algorithm, proposed by Zhu and Poon, has a factor of 2 [ZP99]. Qin *et al.* improved the

factor to 1.96 by restricting the labels to be within the Voronoi region of each site; moreover, they proved that it is NP-hard to approximate the optimal solution within a factor better than 1 (the constant stated in the conference paper, 1.37, was not right) [QWXZ00]. (A lower bound of 1.0349 has been obtained for both 1Circle and 2Circle recently [JZBQ04].) A 1.5-factor was achieved by Wolff *et al.* a few years ago [WTX00]. The best bound is $1.495 + \epsilon$ [JZBQ04].

2.6 1Fsquare

In the problem 1Fsquare one can label a site with a square along any direction as long as the site is on the boundary of the square label. It is not hard to prove that problem is NP-hard. The first approximation algorithm was proposed by Doddi *et al.* [DMMMZ97], which has an approximation factor of $8\sqrt{2}/\sin(\pi/10)$ (roughly 36.6). In [ZQ02], as a by-product of their results Zhu and Qin showed that 1Fsquare can be approximated with a factor of $4/\sin(\pi/10)$ (roughly 12.95). With the recent technique of Jiang *et al.* [JZBQ04], the bound can be improved to 4.14.

3 Open problems

The following table summarize the status of the current research in map labeling under the sliding model. In the table ρ represents the best known approximation factor.

	1APsquare	2APsquare	3APsquare	1Circle	2Circle	1Fsquare
NP-hard?	Yes	Yes	No	Yes	Yes	Yes
ρ	2	2	—	2.931	1.495	4.14

Table 1. A summary of results on map labeling with sliding labels.

We now present some open problems.

(1) For the two problems using circular labels (1Circle and 2Circle, or MLUC and MLUCP in the map labeling community), only very weak inapproximability results (roughly 1.0349) are known [JZBQ04]. Are they inapproximable within some explicit constant factors (say, 1.3)?

(2) For the five approximation factors, is it possible to further improve them?

(3) For the 3APsquare problem, is it possible to improve the $O(n^2 \log n)$ time bound? $\Omega(n \log n)$ is the only known lower bound.

(4) For the algorithm in [DMM00], the running time is $O(n \log n + n \log O_{1Circle}^*)$ which is pseudo-polynomial. Is it possible to make it fully polynomial? Similarly, the algorithms in [JQQZC03, JZBQ04] have a running time $O(n(O(\frac{1}{\epsilon}))^{O(\frac{1}{\epsilon^2})})$. Is it possible to make it practical?

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