Mathematical Treasures from Sid Sackson

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JIM HENLE

This is a column about the mathematical structures that give us pleasure. Usefulness is irrelevant. Significance, depth, even truth are optional. If something appears in this column, it's because it's intriguing, or lovely, or just fun. Moreover, it is so intended.

> Games mean many things to many people; to me they are an art form of great potential beauty.
>
> —Sid Sackson, *Beyond Tic Tac Toe*

Sid Sackson (1920–2002) was a professional game inventor. He was a seminal figure in twentieth-century game design, with pioneering board games, card games, party games, paper-and-pencil games, solitaire games, and word games. In this column I will argue:

1. That (many) games are mathematical structures.
2. That games and mathematics share many of their most important aesthetics.
3. That Sid Sackson created games that pose interesting mathematical problems.

But before I do, let me tell you a little about Sid. Sid Sackson was inventing games as a child. He designed games all his life, though he had a day job as an engineer into his fifties.

Acquire was probably his most successful game. It's a business game with hotels, stocks, and mergers, with geometric elements borrowed from war games. Acquire launched 3M's collection of bookshelf games.

Many of Sackson's games, Twixt and Sleuth, for example, were published by 3M. More than a few were published in Germany by Ravensburger, Piatnik, and other companies. One of these German games, Focus, won the *Spiel des Jahres* (game of the year) and *Essen Feather* awards.

Sackson lived in the Bronx. Game designer Bruce Whitehill said of him:

> Math was always his passion; he wanted to be a math teacher when he was in college, but was told his voice wasn't good for teaching, so he went into engineering. He loved to dance ... He hobnobbed with celebrities sometimes (and loved it), such as Tony Randall. Shari Lewis came to the house once. Sid shared a TV spotlight with Omar Sharif.

I was struck by another of Whitehill's recollections. After Sackson had passed away, he asked Bernice, Sid's wife of sixty-one years,

"Are there any funny stories?"

She replied,

"I wish there were."[1]

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[1] Everyone I've told this to agrees that this is a funny story.

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Mathematical Structures

For centuries philosophers have disputed the definition of mathematics. We don't have to touch that argument. We have only to agree what a mathematical structure is. I have a simple definition, one that is understandable to non-mathematicians and at the same time (I hope) is amenable to philosophers of mathematics:

> A mathematical structure is anything that can be described completely and unambiguously.

Whatever your definition of mathematics, the field definitely involves logical deduction. This is the case even in such fields as probability and quantum logic. Mathematical structures as defined above are both sufficient and necessary as subjects for logical deduction.

Most games are mathematical structures. The rules of a game, if properly written, describe the game completely and unambiguously. I don't mean to claim here, as formalist philosophers do, that mathematics in its entirety is simply a game.[2] Clearly, much mathematics came into being as a tool for understanding and capturing aspects of the physical world. But then there are games with the same purpose—war games, social games, economic games, etc.

This column, however, is about those mathematical structures that are intended to please, and most games are so intended.

Game Aesthetics

If we restrict our attention to games of strategy and tactics, the characteristics that make a game especially intriguing are exactly those that attract us to a mathematical theory. In mathematics, we appreciate a deep result with minimal hypotheses. Quite similarly, a game is especially attractive if its play is complex and difficult to master, yet the rules are simple and easy to follow.

Tic-tac-toe has simple rules, but the strategy is not challenging and is easily mastered. Whist is a complex game, but the rules are not simple; they are complicated and fussy.

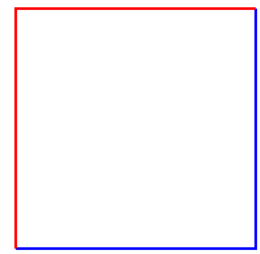
In contrast, the Japanese game of go is an exemplar of game beauty. The rules are few and elegant (much simpler than those of chess). Yet the strategy is enormously complex. As with chess, many go players devote their lives to improving their play.

The game of soccer—known internationally as "football"—is often called the "beautiful game." While not a mathematical game, I believe this description reflects the same aesthetics.

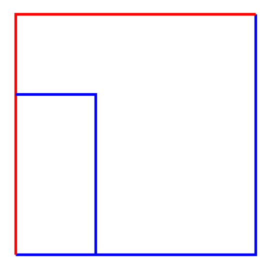
Sackson's "Cutting Corners"

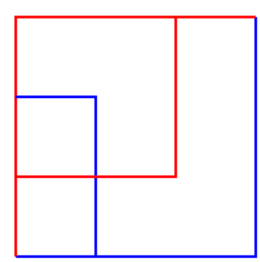
I first met the work of Sid Sackson through his book *A Gamut of Games*.[3] It contains 38 games, a few old, many by friends, and 22 by Sackson himself. They are all gems. Whenever I see this book at a used bookstore or at a yard sale, I buy it. Martin Gardner said that *Gamut of Games* was "the most important book on games to appear in decades."

Cutting Corners is a Sackson game from *Gamut*. It illustrates the simplicity and complexity of his work. It is played on a small square edged in two colors.



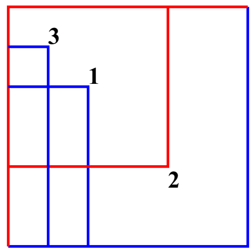
Blue is the color of Player I, the player who goes first. Red is the color of Player II.[4] A move consists in drawing an ell-shaped line that starts perpendicularly from an edge (but not at a vertex), makes a right-angled turn, and ends on another edge. Each move must either cross a line of the other color or end on an edge of the other color.

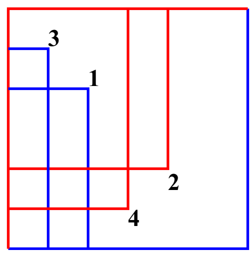




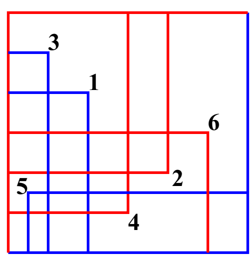
Additionally, the nth move, for every n, must cross exactly $n-1$ lines.

[2]But I might do that somewhere else. And maybe I did do that somewhere else. Ignore this footnote.
[3]Castle Books 1969, Pantheon 1982, and now available through Dover Publications.
[4]I will use these colors for all the games. They aren't always necessary, but they make the progress of the games easier to follow.

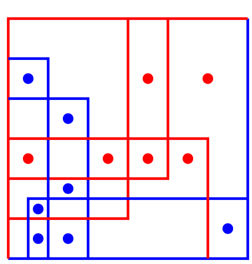




A game ends after a total of six moves.



To determine the winner, examine each region created by the lines. Mark it for Player I if it has more blue edges than red edges. Similarly, mark it for Player II if it has more red edges than blue.



The winner is the player who has more marked regions. In this case, Player I (blue) wins.

Complex! But we can trim the game down by reducing the number of moves.

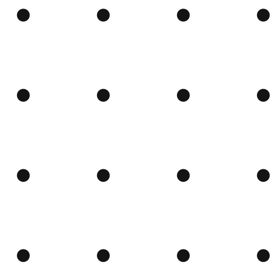
It's pretty easy to see that Player II has a winning strategy in the two-move game. In the three-move game, Player I, who gets to draw two lines, can always beat Player II, who draws only one. This is what you would expect, but it still takes checking. There is only one possible first move, but there are three possible second moves.

The game to tackle next, if you want a challenge, is the four-move game, in which each player makes two moves. My money is on Player II, but the game doesn't look easy to analyze. I counted 23 three-move patterns. You might have to explore all of them.

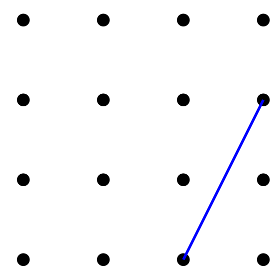
By the way, is it possible to have a tie in this game? I don't know the answer. Sackson doesn't mention the possibility. It seems like an intriguing mathematical question.

Sackson's "Hold That Line"

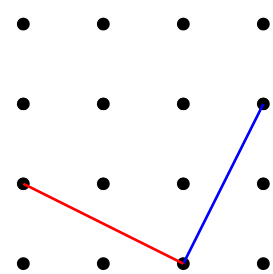
Hold That Line is played on a 4 × 4 array of dots:



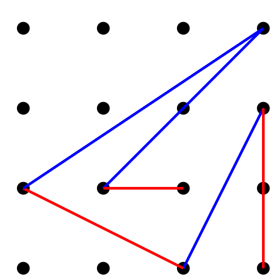
A play consists in drawing a straight line from one dot to another:



Each line after the first must begin at one of the two free ends of the broken line drawn so far and end on a free dot:

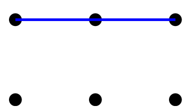


Lines may not cross. The last player to make a legal move is the *loser*.



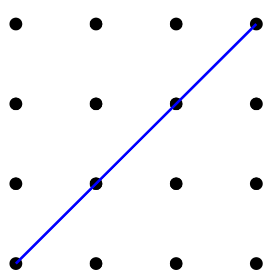
It's a tricky game and clearly a subject for logical analysis. To start, let's examine the rules as applied to smaller boards.

It's easy to see that in the game with a 2×2 array (four dots), Player II has a winning strategy. Every game on this board takes exactly three moves. It's more work, but it's not hard to show that Player I has a winning strategy in the 3×2 game. This initial move works:

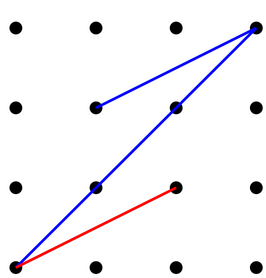


A similar move also works for Player I in the 4×2 game. It doesn't work, though, for the 5×2 game. A different strategy works, however. Can the reader find one?

Hold That Line is a misère game; that is, the last player to move loses. Why did Sackson choose to make the last player the loser? Nothing is written about this, but I'm sure it's because there is a simple strategy for the game in which the last player to move wins. Player I starts with the diagonal,



and then mimics Player II's moves symmetrically, for example:



If Player II has a legal move, then Player I will have one too, all the way to the end. Thus, Player I will make the last move.

Is the full (4×4, misère) Hold That Line too difficult to analyze? All I can say is that it has so far defeated me. If you see a way, let me know! I will report any discoveries on the column website:

www.math.smith.edu/~jhenle/pleasingmath/

"Springerle"

Sackson wrote a series of inspired paperbacks, each containing a collection of his games with a common theme. One book featured cooperative games; another featured solitaire games; another featured calculator games; yet another featured word games.

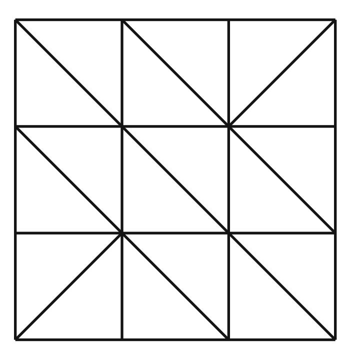
The two-person game Springer is from one of these books, *Beyond Tic Tac Toe.*[5] The name of the game is a reference to the German painter and printmaker Ferdinand Springer, some of whose work resembles a board of Springer at the end of a game.

Espace gris-bleu (ca. 1942), oil on canvas board, private collection, Paris. *(Courtesy of Mathias Springer.)*

Ferdinand Springer was the son of Ferdinand Springer junior, the son of Ferdinand Springer senior, the son of Julius Springer, the founder, in 1842, of Springer-Verlag, the publisher of the *Mathematical Intelligencer*. Serendipitously, there is a connection between Springer and games. The company's logo is a chess knight. This was chosen partly because *Springer* is the word in German for a chess knight, and partly because of the love Julius and Ferdinand senior had for chess.

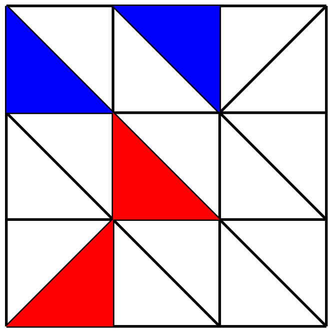
Springer has a large board and a somewhat elaborate set of rules for what constitutes winning. I have pared this down to a simpler game, which I call Springerle.

Springerle is played on this small board:

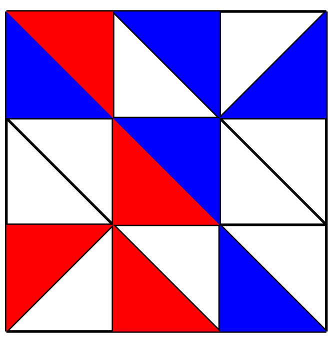


Players take turns coloring triangles with their color, subject to the restriction that no two of a player's triangles can share a side.

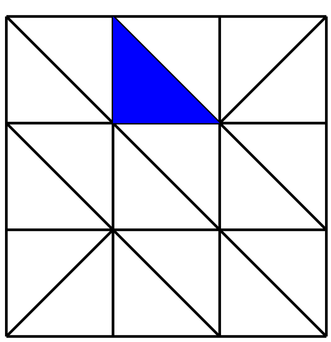
[5] Pantheon Books, 1975.



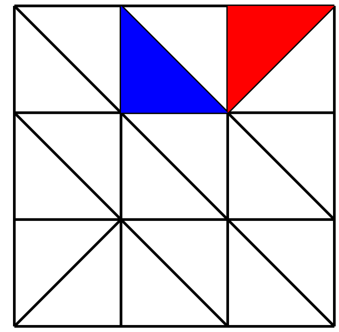
A player wins if his triangles form a network connected by vertices that contains part of each side of the square. Here is a win for Player I (blue).



One would think that the player going first has an advantage, but the restriction about neighboring triangles makes a difference. For example, if Player I starts out here,



then Player II can make sure Player I loses just by playing here,



because now it's impossible for Player I to reach the top edge.

The example above also shows that a draw is possible, that is, that neither player wins. I suspect, however, that Player I can force a win. I don't have a proof, though. Can someone help?

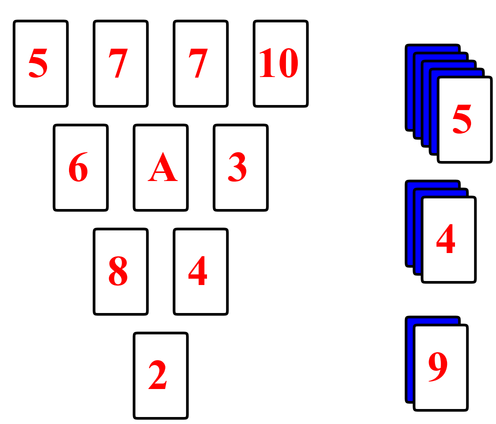
One could call Springerle a "crossing game," like hex.[6] There is a wonderful "strategy-stealing" proof that Player I has a winning strategy for hex. It works by showing that if Player II had a winning strategy, then Player I could "steal" it, that is, use it successfully to win the game. The above example shows that such a proof wouldn't work here.

The fun of Springerle lies in the ease with which a player can be blocked from touching an edge. That makes it difficult to design an appropriate board of a large size. This might explain Sid Sackson's more elaborate rules for his 9×9 board.

"Bowling Solitaire"

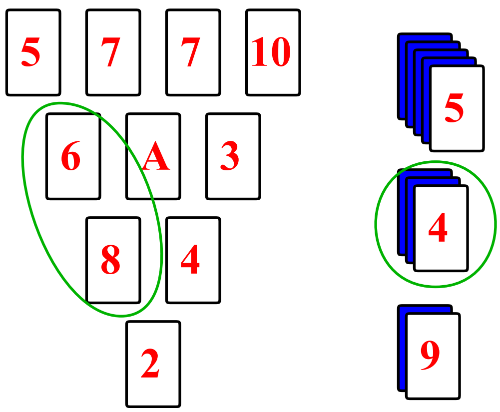
This is a solitaire game using twenty playing cards. Take two each of cards numbered 2 through 10 and two aces—the suits don't matter, it's only the denomination that counts—and think of the ace as representing the number 1. I will assume that the reader is familiar with the way bowling is scored.

Shuffle the cards and deal them out in the pattern of bowling pins and form piles of five cards, three cards, and two cards face down, each with the top card turned up.



[6]Invented independently by Piet Hein and John Nash.

The object, of course, is to knock down pins. You may knock down one, two, or three adjacent cards using a card from one of the piles if the sum of the pin cards has the same units digit as the card from the pile. For example, you can use the 4 card in the second pile to remove cards 6 and 8 ($6+8 = 14$).



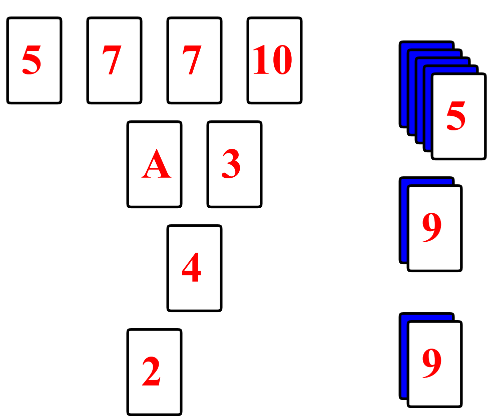
The 4 could instead be used to remove the ace and 3, or the 4, 3, and 7. But it can do only one of these. When pins are knocked down, those pin cards and the card from the pile are removed, and a new card is turned up on the pile.

Special rule: You aren't permitted to knock down any pins in the back row on the first card.

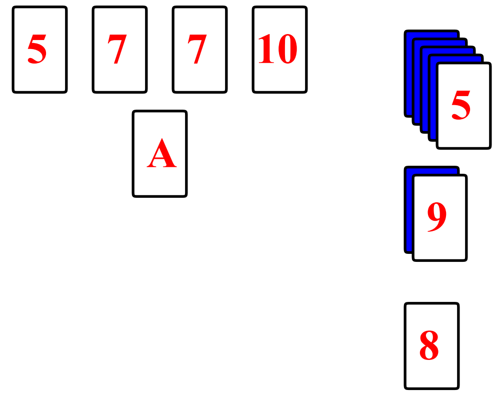
You continue to knock down pins until you are no longer able—or no longer wish to. That was your first ball. (This is bowling, remember?) At this point, you discard the top cards of the piles and turn over the new top cards.

Continue knocking down pins; this is your second ball. When you stop, discard the top cards of the piles and turn over the new top cards. Knock down pins once more. That's your third ball.

Let's think about the setup we have dealt here. We could knock out the 6 and 8 with the 4. Then we could knock out the 2, 4, and 3 with the 9. Let's do it. First the 6 and the 8.

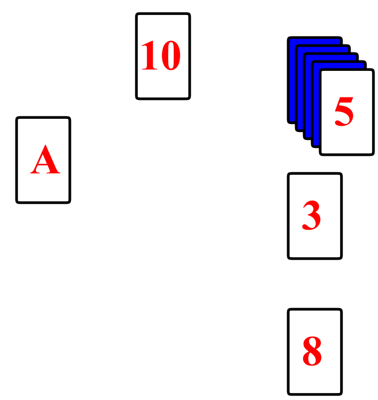


Ooh! Under the 4 was a 9. Now let's use one of the 9's.[7]



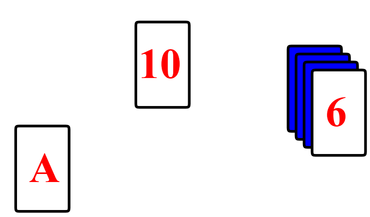
We could knock down the ace and two 7's with the 5. But then we would never get rid of the remaining 5, because all the 5's would be gone. We could instead just knock down the 5 with the 5 and then the 7 and the ace with the 8. But that would leave us with a 7 and a 10, which we could never knock down (there are no more 7's).

The last possibility is to use the other 9 to knock down the 5 and two 7's. That leaves us a chance of getting a strike or a spare. We *might* turn up a 10 and an ace. Here we go.



Phooey. That's the end of the first ball. We knocked down eight pins.

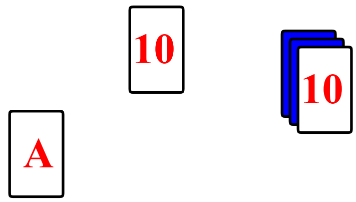
What do we get for the next ball?



Arggh.

And for the last ball?

[7]This is not a carefully constructed example. I shuffled the cards and dealt them. This is *real*.



Good! Okay. Knock down the 10 and turn the card. There are two cards underneath. One is a 2 and the other is an ace. We have a 50% chance of getting a score of ten for the frame.

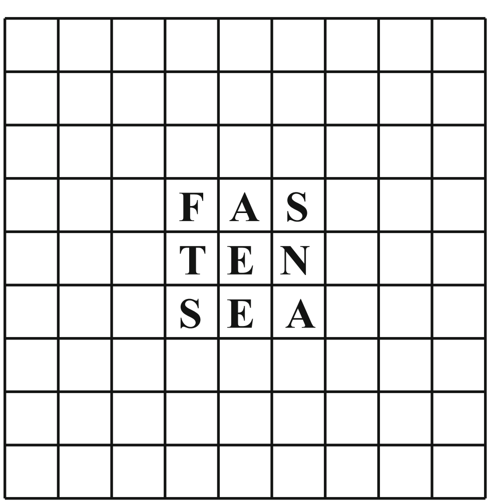
Yes!

"The Last Word"

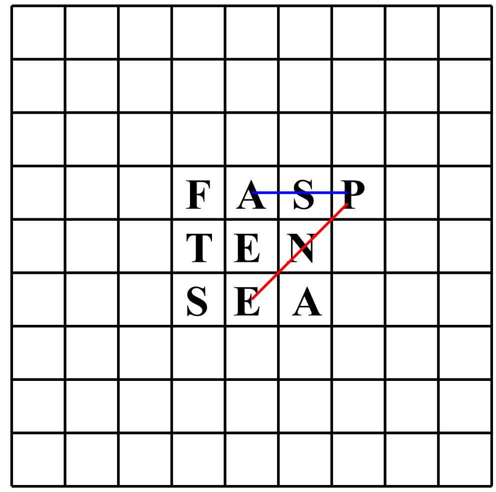
I'll close with one more Sackson game. It's a word game, but just legally mathematical by the definition I gave earlier—it can be described completely and unambiguously (once a dictionary has been chosen). I'm doing this to be nice. The game is a treat.

The fact that it's a word game means that it doesn't have the aesthetic of simplicity, since its set of rules contains an entire English-language dictionary. Nonetheless, it is an amazing game, elegant in the mathematical sense. It is a challenging, exciting word game that takes only a single sheet of paper and a pencil. Play it on a plane ride and the hours will pass quickly away.

The Last Word is played on a 9 × 9 grid. Start by filling in the middle nine squares with any letters you like. I usually pick up a book, find a page, and copy the first letters I see. We're on a plane, though; what do I see? "Fasten seat belts." Okay.



A move consists in placing a letter in a square next to a letter already placed. You get points by rearranging consecutive letters in a single direction to form a word. For example, by putting a P next to the S in the upper right corner,

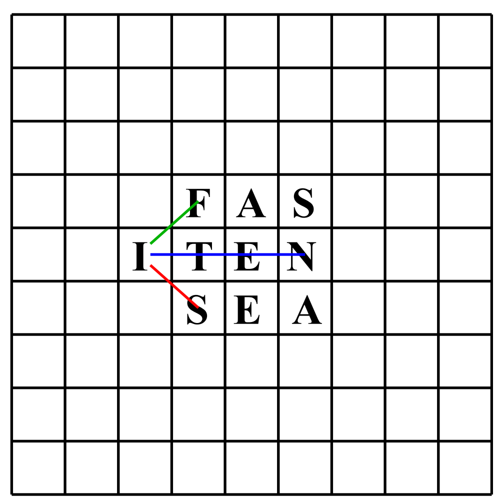


you can get the words SAP in one direction and PEN in another. You aren't allowed to jump over letters; that is, you couldn't use the S and the F without the A.

SAP and PEN are both three-letter words, but our score isn't the sum; it's the product. This move is worth nine points.

That's not the best first move, though. A T in the same place would get you FAST and TEN. That would be twelve points.

Still better would be an I next to the T:



That's worth sixteen points from IF, TINE, and IS.

What's especially nice about this game is that at later stages the scores get very high, hundreds of points, so that you can be losing by a large margin and catch up quickly. Do you see, by the way, that if Player I places the I as indicated, the second player could score 24 points?

The game ends when all four sides of the square have been reached.

As always, you can reach me at pleasingmath@gmail.com.