From Chapter 3:

3.7 *Application:* Paying Back Your Student Loan

Student loans, mortgage loans, and retirement accounts are all quite similar and surprisingly complex mathematically. Most customers are unprepared to check the bank's calculations. The mathematics is partly *continuous* (quantities changing all the time) and partly *discrete* (quantities changing at fixed intervals). Appropriately, the mathematics we use to describe student loans will be partly continuous and partly discrete.

Let's say you've borrowed \$10,000 at 7% interest. Without payments, the amount you owe, the principal L, increases at a rate proportional to itself,

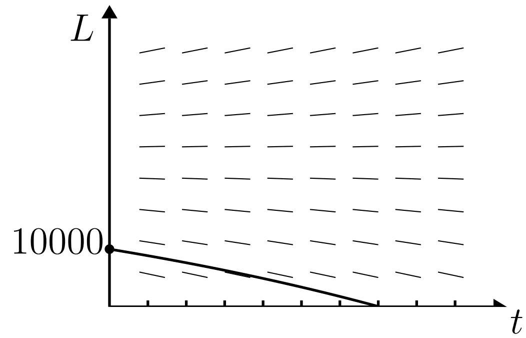
$$L' = .07L \text{ dollars per year.}$$

We've seen equations like this before in the example of the money market account in Chapter 1. We can use this equation, which models continuous change, even though interest is usually calculated monthly, which is a discrete change. The accuracy we lose by assuming continuous change is not much compared to the insight we'll gain and the calculations we can make.

Suppose you'll be making payments of \$150 a month. That's \$1800 per year, and since we're measuring t in years, we adjust our continuous model so that it becomes

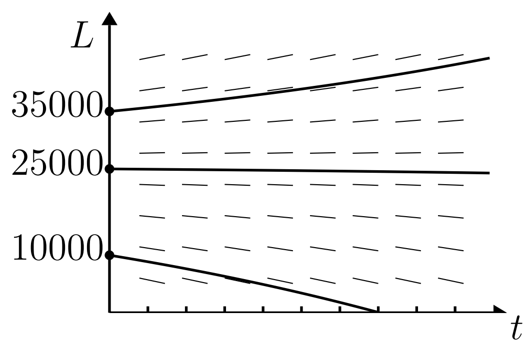
$$L' = .07L - 1800 \text{ dollars per year.}$$

If we enter the equation into an integrator, along with initial value $L(0) = \$10,000$, we'll see the solution to the resulting initial value problem, and we'll see our progress in repaying the loan.



In particular, we can see that the loan will be paid back in about 7 years.

What if we initially borrow amounts other than \$10,000? Here's a picture of solutions to $L' = .07L - 1800$ with three different initial values for $L(0)$.



The picture shows how different initial values (different loan amounts) fare. With monthly payments of \$150, small loans are paid back sooner and large loans later (or not at all). Of course, that's why large loans require large monthly payments.

Laboratory: **Student Loans**

In this laboratory, you're going to do a little financial planning. You're going to test a number of payback strategies. To keep things manageable, let's assume a constant 7% interest rate throughout.

Assignment #1:

1. As the preceding graphs show, if the initial loan is too large, it will never be repaid with monthly payments of \$150. Where's the break-point? That is, what is the smallest loan that will never be repaid at the rate of \$150 per month?
2. Student loan payments are generally set so that the loan is paid back after 10 years. What should the monthly payment be to pay back \$10000 in exactly 10 years?
3. Find an approximate value for $L(1)$, the amount of loan principle at the end of one year, for the system: $\begin{bmatrix} L' &=& .07L - 1800 \\ L(0) &=& 10000 \end{bmatrix}$ using Euler's method of sums with $\Delta t = .2$.
4. Find the exact (or nearly exact) solution to the system in the previous problem using an integrator. Compare your answers.

---

5. In reality, students make loan payments once a month and interest accrues once a month. Finding the value of L using Euler sums with discrete steps of $\Delta t = \frac{1}{12}$ is another way of solving (approximately) the initial value problem $\left[\begin{array}{rcl} L' & = & .07L - 1800 \\ L(0) & = & 10000 \end{array} \right]$. What is the value of L after one year with this method?

Our student loan examples above are based on federal rules prior to 1994. New rules put into effect by the Clinton administration that year eased the repayment of student loans. The idea behind the rule was that the payment rate should be a fixed percentage of the payer's income after graduation. Suppose, for example, you want to teach when you graduate. You might make as little as \$20,000 per year. Monthly payments of \$150 would be very difficult. \$100 might be more manageable. \$150 per month is \$1800 per year, or 9% of your salary, while \$100 per month amounts to 6% of your salary. But you can expect your salary to grow at a modest rate, say 5% a year, and so your payments will increase. To model this, we need a pair of equations, since there are two quantities changing, the loan amount L and your salary S.

⋮