From Chapter 5:

2.5 *Application:* The End of the World

> *Through the animal and vegetable kingdom, nature has scattered the seeds of life abroad with the most profuse and liberal hand. She has been comparatively sparing in the room and nourishment necessary to rear them.*
>
> Thomas Malthus

Will the world run out of food?

Our oceans are overfished, our farms are exhausted, and our rivers are polluted. At the same time, world population is racing out of control. Huge population increases in the next century are considered unstoppable. What's going to happen? Where does calculus come in?

We're going to examine the ideas of one of the first economists to consider the problem, Thomas Malthus (1766–1834). Malthus predicted that human misery was our certain fate as a consequence of two fundamental laws of growth. The first was that resources, principally food, grow **"arithmetically"** over time. That means that resources (which we'll denote by r) grow every year by the same amount. Putting it another way, the growth rate is constant. In the language of calculus,

$$r'(t) = A$$

for all t, or simply $r' = A$. Here the A is constant.

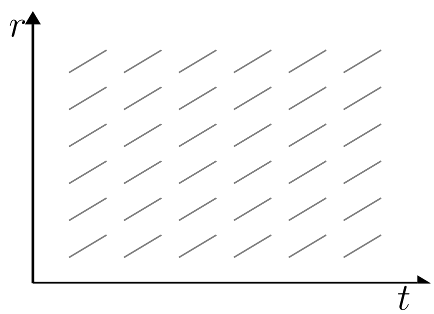
The second law Malthus cited was that over time population (which we'll denote by p) grows **"geometrically."** That means population growth rate is proportional to population. In the language of calculus,

$$p'(t) = Bp(t),$$

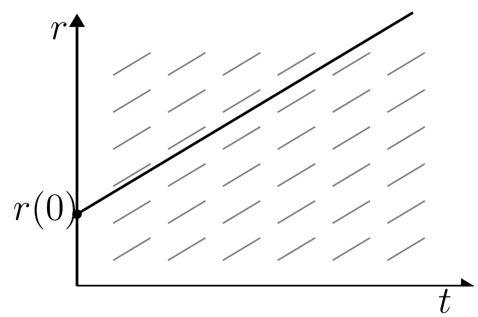
for all t, or $p' = Bp$. Here B is a constant, but the population growth rate is *not* constant. The growth rate p' changes as p changes.

Malthus concluded that population will ultimately exceed resources, and there will be mass starvation. The equations alone may not convince you, but suppose we turn them into pictures.

The equation describing resource growth, $r' = A$, with $A > 0$, has a constant slope field,

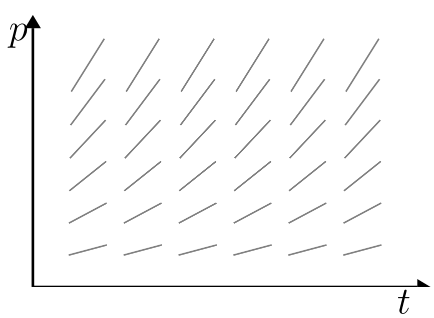


so every initial value $r(0)$ gives a straight line solution.

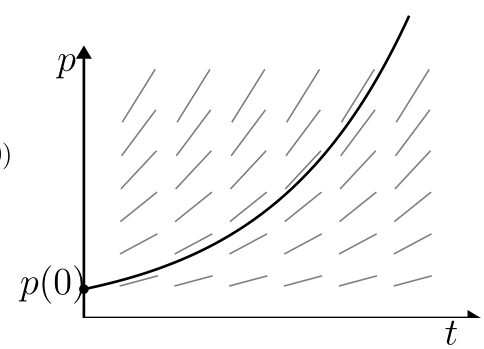


That's how resources grow, according to Malthus.

The equation describing population growth, $p' = Bp$ with $B > 0$, has a nonconstant slope field,



and initial values $p(0)$ produce curved solutions.



That's how population grows, according to Malthus.

$$\vdots$$