Excerpts from Chapter 4:

As with the Product Rule, we advise extreme caution when differentiating quotients,

$$\cancel{\left(\frac{f}{g}\right)' = \frac{f'}{g'}}$$

With the Quotient Rule, we can extend Proposition 4.2.

Proposition 4.1 (**An Improved Power Rule**) For every integer n, positive or negative,

$$(x^n)' = nx^{n-1}.$$

Example: Find $(x^{-4})'$.

Solution: $(x^{-4})' = -4x^{-5}$.

Proof of Proposition 4.1: We must show that the Power Rule is true for $n = 0$ and for all negative integers. Notice first that the rule is true if $n = 0$, since $(x^0)' = (1)' = 0 = 0x^{-1}$.

Now consider a negative integer, $-n$, where n is positive. Then, using the Quotient Rule for the second line in the following calculation, we get

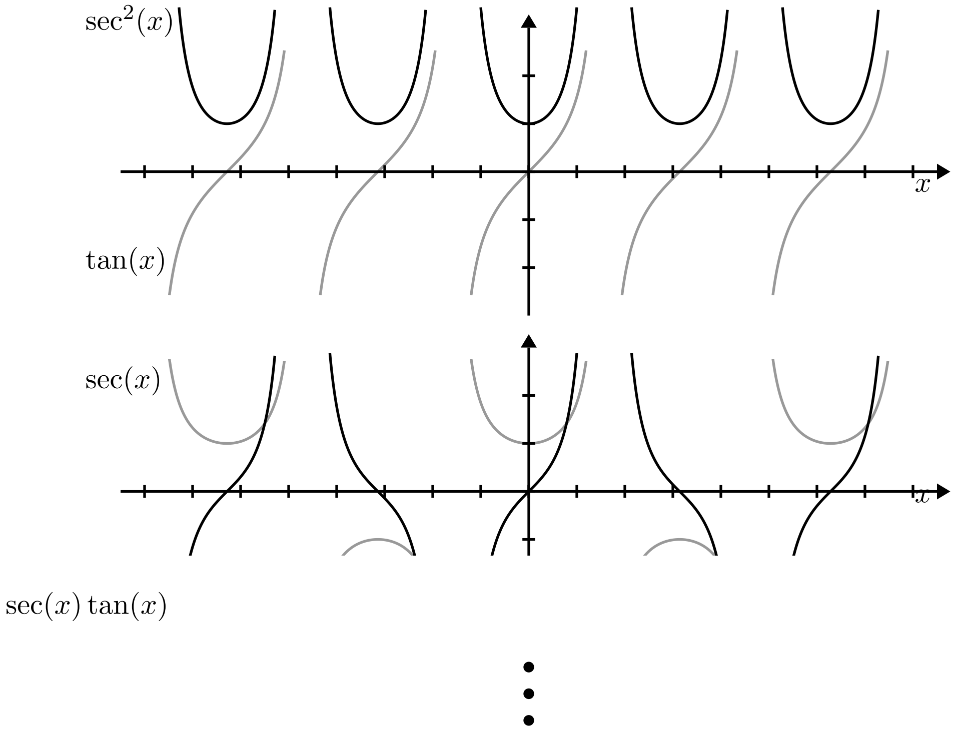
$$\begin{aligned}(x^{-n})' &= \left(\tfrac{1}{x^n}\right)' \\ &= \tfrac{(1)'(x^n)-(1)(x^n)'}{(x^n)^2} \\ &= \tfrac{0-nx^{n-1}}{x^{2n}} \\ &= -nx^{(n-1)-2n} \\ &= -nx^{-n-1}.\end{aligned}$$

So the Power Rule holds for $-n$. ■

With the Quotient Rule, we can compute the derivatives of the remaining trigonometric functions.

Proposition 4.2 $(\tan(x))' = \sec^2(x)$ $\quad (\sec(x))' = \sec(x)\tan(x)$
$(\cot(x))' = -\csc^2(x)$ $\quad (\csc(x))' = -\csc(x)\cot(x)$

Here's what these graphs look like:



Differentiating can be a confusing business at first. What you are doing is parsing functions, taking them apart, bit by bit. To help you get started parsing functions, we have a 6-step procedure for you to follow:

Step A. State the type of function (of the six listed in the box above) and identify the parts.
Step B. Differentiate by applying the formula or rule appropriate for the type.
Step C. Take the differentiation process one step further, identifying the type of each part that is not yet differentiated, (if there are such parts)
Step D. Take the differentiation process one step further, applying formula(s) and/or rule(s) appropriate for the type(s).
Step E. Complete the differentiation, if not already complete.
Step F. (optional) Diagram the function.

⋮

Example: Find $\left(\sqrt{\frac{\sin(z)-17}{\tan(5z)}}\right)'$.

Solution: **Step A:** This is a composition. The outside function is $\sqrt{\quad}$ and the inside function is $\frac{\sin(z)-17}{\tan(5z)}$.

Step B: $\left(\sqrt{\frac{\sin(z)-17}{\tan(5z)}}\right)' = \left(\left(\frac{\sin(z)-17}{\tan(5z)}\right)^{.5}\right)'$
$= .5\left(\frac{\sin(z)-17}{\tan(5z)}\right)^{-.5}\left(\frac{\sin(z)-17}{\tan(5z)}\right)'$.

Step C: The part $\frac{\sin(z)-17}{\tan(5z)}$ is a quotient, $\sin(z)-17$ divided by $\tan(5z)$.

Step D:

$$\begin{aligned} & .5\left(\frac{\sin(z)-17}{\tan(5z)}\right)^{-.5}\left(\frac{\sin(z)-17}{\tan(5z)}\right)' \\ = \; & .5\left(\frac{\sin(z)-17}{\tan(5z)}\right)^{-.5}\frac{(\sin(z)-17)'\tan(5z)-(\sin(z)-17)(\tan(5z))'}{\tan^2(5z)}. \end{aligned}$$

The differentiation is still not yet complete.

Step E:

$$
\begin{aligned}
& .5\left(\frac{\sin(z)-17}{\tan(5z)}\right)^{-.5} \frac{(\sin(z)-17)'\tan(5z)-(\sin(z)-17)(\tan(5z))'}{\tan^2(5z)} \\
= \quad & .5\left(\frac{\sin(z)-17}{\tan(5z)}\right)^{-.5} \frac{\left((\sin(z))'-(17)'\right)\tan(5z)-(\sin(z)-17)\sec^2(5z)(5z)'}{\tan^2(5z)} \\
= \quad & .5\left(\frac{\sin(z)-17}{\tan(5z)}\right)^{-.5} \frac{(\cos(z)-0)\tan(5z)-(\sin(z)-17)\sec^2(5z)(5)}{\tan^2(5z)}
\end{aligned}
$$

Step F:

$$
\sqrt{\begin{matrix}\sin(z) \ - \ 17 \\ \\ \tan \quad 5z\end{matrix}}
$$