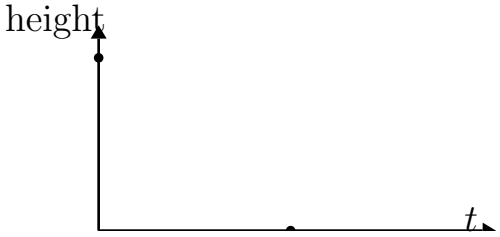
Excerpts from Chapter 2:

2.1 Picturing Derivatives

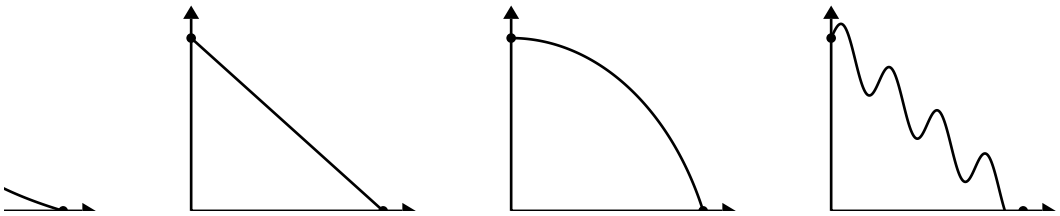
In Chapter One, we translated words into equations. We translated from English to Calculus. In this chapter, we translate English and equations into the language of pictures.

Example: *I dropped the book when I was leaning over the railing. It fell slowly at first, then faster and faster, until it hit the ground.*

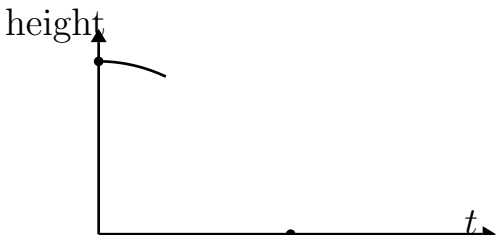
These words paint a picture. The picture is the graph of height as a function of time. Let's say that time is 0 at the start of the fall. We'll use the variable t for time. At $t = 0$, the book is at a certain height (we don't know what that is) and later it's at height 0. That gives us two points for the graph we want to draw.



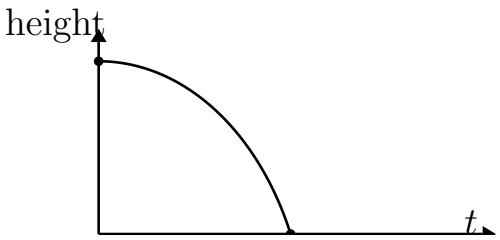
One can imagine many possible curves connecting these two points.



But the description in English actually tells us the general shape of the graph depicting the falling book. "It fell slowly at first" means that at the start, the height didn't change very much as time increased.



Then the words "faster and faster, until it hit the ground" tells us that the height changed more and more rapidly. Thus, the graph becomes steeper and steeper.



Of course, we only have the general shape of the graph, not the exact curve. The English itself was not exact, but it did contain information, and we turned that information into a picture.

Odd-numbered solutions begin on page ??

Exercises: Pictures

Translate these passages into curves on a plane with appropriately labeled axes. In all cases, the horizontal axis is the time axis. The vertical axis will vary with the problems, but will always be some measure of quantity (not rate).

1. *I held the balloon 5 feet under water and let it go. It rose slowly at first, but by the time it reached the surface of the pool, it was rising very rapidly.*

2. a. *I held the stone at the surface of the water at the deep end of the pool and let it go. It fell slowly at first, but picked up speed as it fell.*

   b. *After a short while, it reached a maximum rate of fall, and then fell at a steady rate to the bottom of the pool.*

3. *At first, there were very few cases of the flu. People were falling ill, but so slowly that the news didn't catch the attention of the papers. After two weeks, though, the infection reached epidemic proportions. But then things slowed down a bit. By the end of a month, nearly everyone had come down with the bug, and there were just a few new cases each day.*

4. a. *I held the clock pendulum to the left and let it go. It moved slowly to the right, then picked up speed.*

b. *Eventually, as the pendulum swung right, it began to slow down. At the end of its swing, it was motionless.*

c. *Then it started back to left, slowly at first then faster and faster.*

d. *And this pattern continued, the pendulum moving back and forth, back and forth.*

e. *When I was tired of this, I placed the pendulum straight down and let go. Well of course, it just stayed there.*

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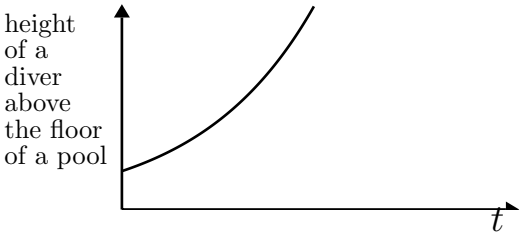
Use the language of rates to describe in English the situations depicted in these curves:

5.



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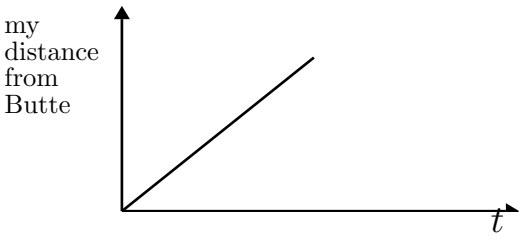

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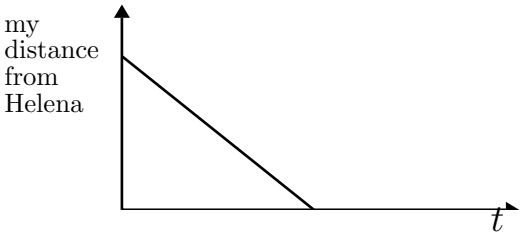

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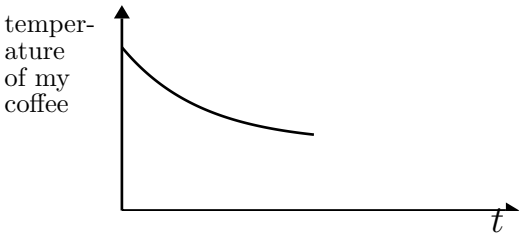

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