From Chapter 6:

Exercises: Optimization Problems

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Extreme Problems!

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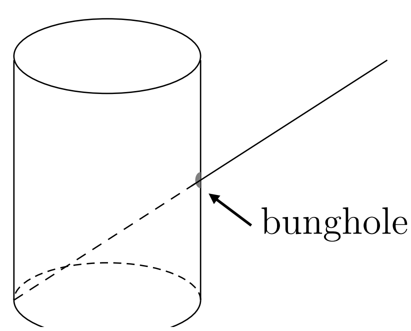
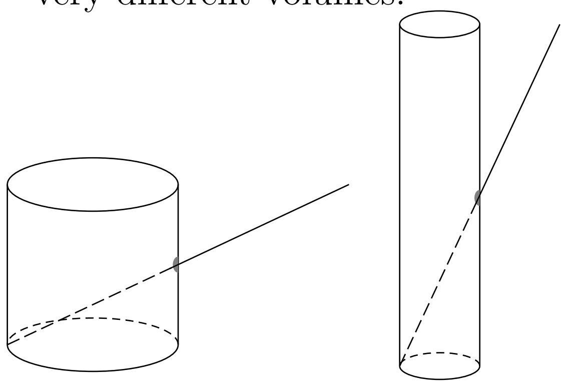
1. This is a famous problem. It was invented by the astronomer, physicist, and mathematician, Johannes Kepler—invented even before Newton and Leibniz discovered the Fundamental Theorem of Calculus. Kepler noticed that cylindrical wine barrels were measured by sticking a rod through a bung-hole in the middle of the side of the barrel and seeing how far it would go.

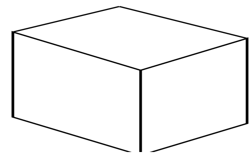


Kepler realized that this was not a good measure of the volume, that two casks with the same bung-hole measure might have very different volumes.



He then wondered what the ideal proportions were for a cask, that is, what is the largest cask with a bung-hole measure of, say, 1?

2. A piece of wrapping paper is $12'' \times 12''$. Of all the boxes that you can wrap with this paper, which has the largest volume? By "box," we mean the usual sort of box, you know, a rectangular parallelpiped.



When you wrap, every point of the surface of the box must be covered. The wrapping paper must not be cut in any way. By "you," we mean you, the reader. What's the best *you* can do? In the back, we'll tell you the best *we* can do, but we don't have a proof that it's the best *anyone* can do.

3. What is the largest rectangle one can construct in the first quadrant below the graph of the function, $f(x) = \frac{1}{x}$?