Excerpts from Chapter 1:

1.1 Derivatives

You're familiar with falling raindrops, rising temperatures and expanding populations. All of these involve change. In this section you'll learn how to use the language of calculus to describe rate of change.

Calculus uses one symbol to signify rate of change:

$$'$$

We pronounce the symbol "prime." Here's how we use it.

If a variable

$$x$$

represents a certain quantity, then

$$x'$$

represents the rate at which that quantity is changing.

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Exercises: Prime

The object here is to translate the mathematical content of English sentences into equations. *Warning:* Some sentences do not use the derivative ($'$). Some sentences use *several* derivatives. Also, some sentences have absolutely no mathematical content! We've done the first two for you.

1. a. *My net worth* (call it W) *right now* ($t = 0$) *is $55,000.* [Translation: $W(0) = 55,000$]

   b. *Thanks to some good investments, my net worth right now is growing at the rate of $6,000 per year.* [Translation: $W'(0) = \$6000$ per year.]

2. *I began keeping track of the population* (call it P) *of my hometown in 1996. Since then we've been adding 600 people per year to the population.* [Translation: $P'(t) = 600$ people per year for $t > 1996$.]

3. a. *The population of Malawi* (call it P)*was 5.04 million in 1974.*

   b. *The population of Malawi was increasing by 110,000 per year in 1974.*

4. *It was a dark and stormy night.*

5. *The area of the world's rain forests is decreasing constantly at the rate of 300 square miles each year.* (Let F stand for the number of square miles of rain forest in the world.)

6. The following appeared in an article by Richard Rothstein in *The New York Times* of May 10, 2000.

   a. *"As late as 1994* [$t = 0$], *unemployment* [call it U] *was still over 6 percent . . . "*

   b. *"Unemployment is now only 3.9 percent, its lowest level in 30 years."*

c. This wasn't in the article, but translate it anyway: *At the moment, unemployment is holding steady—neither rising nor falling.*

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1.2 Integrals

We've seen how the symbol $'$ stands for rate of change. In this section we'll introduce the second symbol at the core of the language of calculus. It stands for total change. That symbol is a wiggle:

$$\int$$

Until a later chapter we'll write the wiggle only with small letters (or numbers) attached: $\int_a^b$. We pronounce the symbol $\int_a^b$ "the **integral** from a to b." Here's how we use it in our language.

If d' is the rate of change of d, then

$$\int_a^b d'$$

represents the total changes in d in the time period from a to b.

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Exercises: Integral

Translate these sentences into mathematical notation using the integral sign where appropriate.

1. *Dorothy is petitioning to get a proposition on the ballot.* (Let $s(d)$ be the number of people who have signed as of the d^{th} day of petitioning and let $d = 0$ be the first day of the first week.)

   a. During the first week, she got 412 signatures.

   b. During the second week, she got 656 signatures.

2. a. *At noon* (call it $t = 0$)*the temperature* (call it T) *was 60^o.* (Translation: $T(0) = 60$.)

   b. *By 2:00* P.M.*, the temperature was 75^o and rising.*

   c. *Later, Horace said it was so hot he couldn't do the crossword puzzle.*

d. *At 4:00* P.M. *the temperature was 85º .*

e. *At 5:00* P.M. *it was still over 80º.*

f. *Between 5:00 and 6:00* P.M., *the temperature fell more than it had risen between 1:00 and 4:00* P.M..

3. The following appeared in *The Boston Globe* on May 10, 2000.

   a. *"Wal-Mart Stores Inc.'s first quarter earnings rose sharply from a year ago . . . "* (Let $E'(t)$ be the rate at which Wal-Mart's earnings are growing at time t (in years), and let $t = 0$ be Jan. 1, 2000.)

   b. *"The Bentonville, Ark.–based chain [Wal-Mart] reported that it earned $1.326 billion [in the first quarter of 2000]."*

   c. *" . . . compared with $916 million [in the first quarter of 1999]."*

4. The following was taken from U.S. Census data.

   a. *In 1980, the population of Florida was 9,746,000.* (Let $P(t)$ the be population in year t.)

   b. *In 1986, the population of Florida was 11,675,000.*

   c. *Between 1980 and 1986 there was a net change in the population of 1,928,000.*

5. The following appeared in *The New York Times* on May 8, 2000: *"Serious crimes reported to the police dropped in 1999 for an 8th consecutive year, down 7 percent from the year before . . . "* (Let $C(t)$ be the total number of serious crimes reported during year t.)

   a. 323,150 serious crimes were reported in New York in 1998.

   b. There were 299,523 serious crimes reported in 1999.

   c. There were 23,627 fewer serious crimes reported in 1999 than there were in 1998.

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1.3 Models

When we use the language of calculus to describe something, we're building a model. Mathematics is not reality, but it can describe reality. We call a mathematical description a **model**.

Example: *At the start of the year I put $357 into a money market account. They're only giving me 2% interest per year. I'll never have enough for a Lexus.*

This is reality. We have a real quantity, the money that's been placed in an account. If we give that quantity a name, we have a model. Since the quantity changes over time, it's most appropriate that we use calculus.

Let's call the amount of money in the account m. Since m changes it is a function of time, hich we'll call t. We'll interpret the phrase, "at the start" as $t = 0$. With all this, we can translate the first sentence above as:

$$m(0) = 357 \text{ dollars.}$$

The second sentence introduces a new idea, growth as a percentage. Since 2% of $357 is $7.14, the second sentence implies that when the account was opened (when $t = 0$) the balance started growing at the rate of \$7.14 per year. We can express that fact with the mathematical sentence:

$$m'(0) = .02 \cdot 357 = 7.14 \text{ dollars per year.}$$

But the second sentence says more than that. It says that the account grows at a rate of 2% throughout the year, not just at the start. In other words, *at every instant* t the account balance is growing at a rate of $.02m(t)$. So we can translate the second sentence into:

$$m'(t) = .02m(t) \qquad \text{for all times } t.$$

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Exercises: Models

Translate the English below into Calculus.

1. *My hometown is growing rapidly. Right now* ($t = 0$) *its population* (call it H) *is growing at a rate of 4 percent annually.*

2. a. *The population of Malawi* (call it M) *was 4.9 million in 1973 and was increasing at 2.8% per year.*

   b. *The population of Malawi was 5.04 million in 1974, and increasing at 2.3% per year.*

   c. *I don't know what the population of Malawi was in 1983, but it was increasing at the rate of 2.7% per year.*

   d. *I don't know what the population of Malawi is, and I* *don't really know how fast it's increasing, except that I know the rate of increase at every instant is directly proportional to the population.*

3. *The number of people with O-negative blood* (call it O^-) *in this country is proportional to the total population* (call it P).

4. *A secret is spreading. Each day, the number of people who hear the secret for the first time is a certain multiple of the number of people who have already heard the secret.* (Let $s(t)$ be the number of people who know the secret at time t.)

5. *The annual population growth rate of China is .77%.* (The CIA World Factbook 1999) (Let $P(t)$ be the population of China at time t.)

6. *The rate at which people catch colds is proportional to the number of people who have colds.* (Let $S(t)$ be the number of people who have a cold at time t.)

7. *Radioactive elements tend to decay. Barium 140 is a good example. If you leave a lump of this stuff alone, it will gradually change into something else. The rate at which it does this is always proportional to the amount of barium 140 left.* (Let A stand for the amount of barium 140.)