Application: Soaking the Rich

In *Rich and Poor* we introduced the Lorenz curve, which describes the distribution of wealth, and the Gini coefficient, which measures how unequal that distribution is.

In this article, we examine the effect of a flat tax and a tax rebate on the Gini coefficient. We'll see whether they increase or decrease the gap between the rich and the poor.

We'll need from *Rich and Poor* the formula for the Gini coefficient:

$$G = 2\left(\frac{1}{2} - \int_0^1 L(x)\,dx\right).$$

We'll also need the wealth function from *The Wealth of Families*,

$$W = \frac{N}{F} \cdot L',$$

or,

$$L = \int \frac{F}{N} W\,dx.$$

$W(x)$ is the wealth of a family x from the bottom. For example, if a certain family has more money than 67% of all families and less than 33% of all families, then the family has assets of $W(.67)$.

First we analyze the flat tax: Suppose we collected from every family 1% of their assets. Would the Gini coefficient change?

The tax plan changes the Lorenz function. Let's call L_{new} the new Lorenz function. We'll also have a new wealth function, W_{new}, and the total national wealth changes too, to N_{new}. We see first of all, that

$$W_{\text{new}}(x) = .99W(x).$$

That's the tax. Also,

$$N_{\text{new}} = .99N,$$

since everyone is now 1% poorer. To calculate the new Lorenz function, we compute:

$$\begin{aligned} L_{\text{new}} &= \int \tfrac{F}{N_{\text{new}}} W_{\text{new}}\, dx \\ &= \int \tfrac{F}{(.99)N} .99W\, dx \\ &= \int \tfrac{F}{N} W\, dx \\ &= L. \end{aligned}$$

In other words, the Lorenz function doesn't change. Thus, the Gini coefficient doesn't change. A simple flat tax doesn't affect the gap between rich and poor.

Now the tax rebate. We promised that we would analyze this, but we're going to let you do it.

Laboratory: The Tax Rebate

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Suppose we give every family the same amount, r dollars. In effect, this is what Congress implemented such a rebate in 2001. Let L, N, W, and G be respectively the Lorenz function, the nation's wealth, the wealth function, and the Gini coefficient before the rebate, and let L_{new}, N_{new}, W_{new}, and G_{new} be the corresponding functions and constants after the rebate.

Show that

1. $W_{\text{new}}(x) = W(x) + r.$
2. $N_{\text{new}} = N + rF.$
3. $L_{\text{new}}(x) = \frac{NL(x)+xrF}{N+rF}.$
4. $G_{\text{new}} = \frac{N}{N+rF} G.$