Appropriate any time after Chapter 4:

## Fuel

On June 4, 2001, *The New York Times* printed a piece on its Op-Ed page on the subject of the fuel supply. The author, Evar D. Nering, explored different strategies for dealing with the fuel shortages we were experiencing at that time. Specifically, he addressed the question of whether we were better off increasing the fuel supply or decreasing consumption.

To save time, we'll put his argument in the language of calculus.

Let F(t) be the supply of, say, oil in the world at time t. That would be the best estimate of all the oil we can feasibly extract and use. We'll take t = 0 to be the present (June 4, 2001). Nering notes that our current rate of fuel consumption, F'(0) is increasing at the rate of 5% per year and implies that this has more or less been the case for some time, so let's assume that

$$F''(t) = .05F'(t).$$
  
Laboratory: Fuel Supply

Comments and hints begin on page ??

Your task in this laboratory is to verify mathematically the assertions of Prof. Nering (he is Professor Emeritus of mathematics at Arizona State University). 1. Prof. Nering supposes that we have a 100-year supply of oil, if consumption remains at the present rate, that is,

$$\frac{F(0)}{F'(0)} = 100$$

But of course, consumption won't stay at that rate, it's increasing at the rate of 5% a year. In that case, Prof. Nering says, the oil will last about 36 years. Amazing? Let's see.

- a. Find an expression for F'(t) in terms of t by solving the differential equation, F''(t) = .05F'(t).
- b. Find an expression for how much oil is consumed in q years, i.e., compute  $\int_0^q F'(t) dt$  using your answer to a.
- c. For what value of q is your answer to b. equal to F(0)?
- 2. Prof. Nering next supposes instead that we have a 1,000-year supply of oil. He says that again, because consumption is increasing, that supply won't last 1,000 years, but only 79 years. Incredible! Check it out.
- 3. Prof. Nering then supposes that we have a 10,000-year supply of oil. This time he says it will last 125 years. Check it out!
- 4. Prof. Nering goes on to say that no matter what F(0) is, doubling it adds less than 14 years to the life of the fuel supply (assuming, as always that consumption continues to increase at 5% per year). True? Check it out.
- 5. On the other hand, Prof. Nering says that cutting in half the rate of growth of consumption (from 5% to 2.5%) almost doubles the life of the fuel supply. True? As Prof. Nering notes, he is not suggesting that the rate of consumption be cut—only that we cut the rate of growth of the rate of consumption. *Check it out!*

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