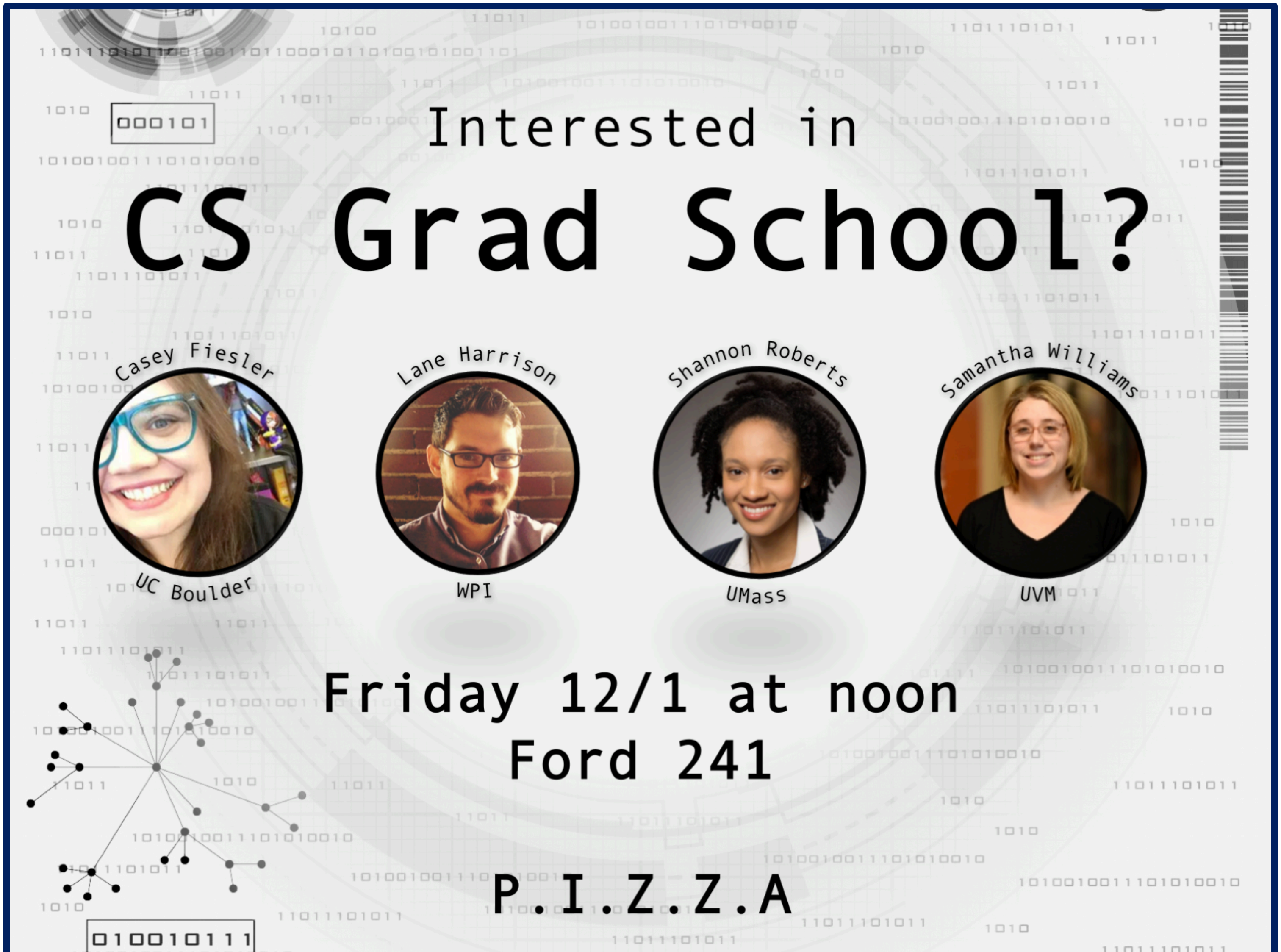
LECTURE 21:
SUPPORT VECTOR MACHINES PT. 2

November 29, 2017
SDS 293: Machine Learning

Announcements 1/3



Announcements 2/3

- FP2 feedback:
  - If you submitted **before** break, went out this morning
  - If you submitted **after** break, will be out by tomorrow
- FP3 has been posted, including a detailed rubric

Announcements 3/3

- FP poster template is now available on Moodle
- You're welcome to make changes / reformat; just please keep it 3'x4' portrait
- Printing:
  - **Option 1**: upload PDF to Moodle on or before ~December 7th
  - **Option 2:** arrange printing on your own (Paradise Copies, etc.)

Outline

- Maximal margin classifier
- Support vector classification
  - 2 classes, linear boundaries
  - 2 classes, nonlinear boundaries

quick recap

- Multiple classes
- Comparison to other methods
- Lab

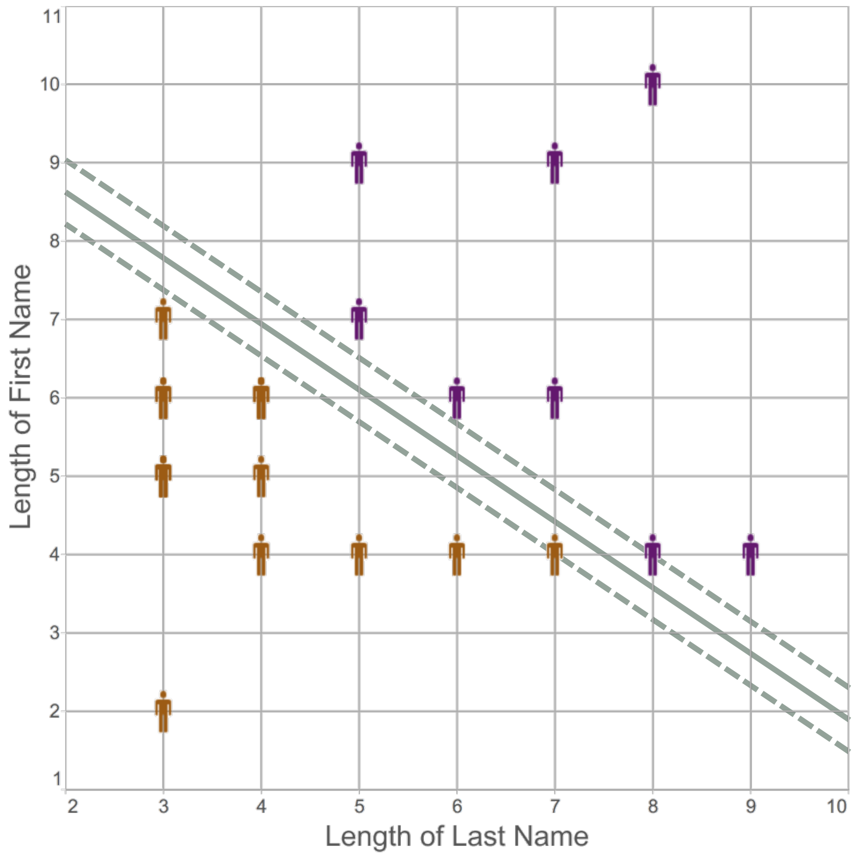
Recap: maximal margin classifier

Big idea:

find the dividing hyperplane with the **widest margin**

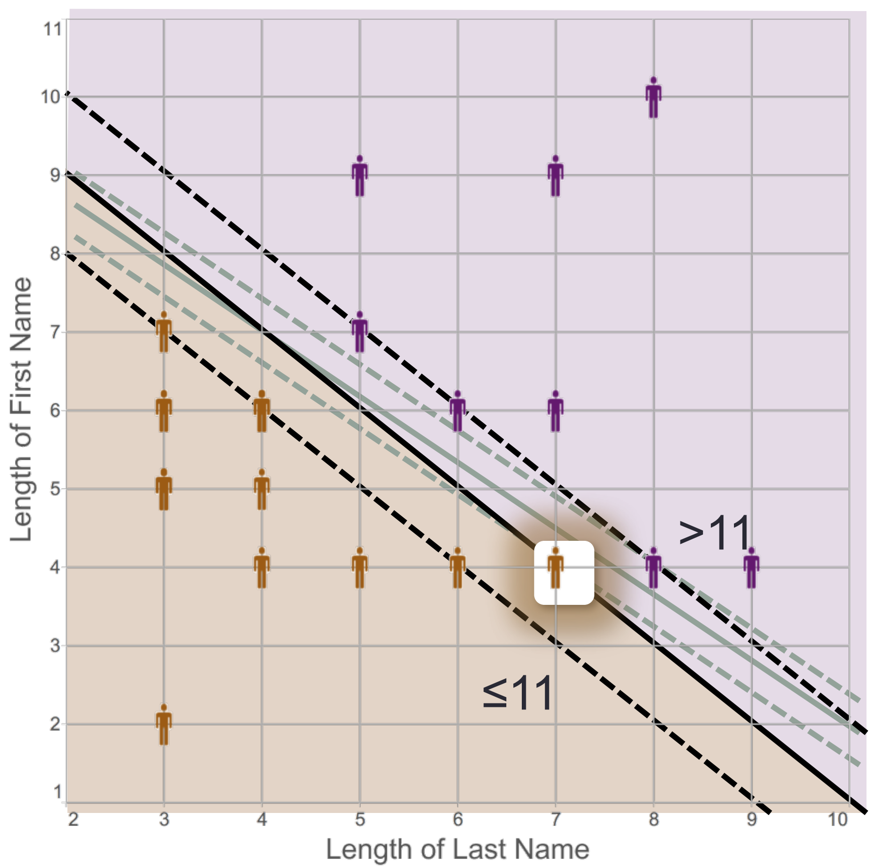
$$\max(M) \; such \; that$$

$$y_i\left(\beta_0 + \beta_1 x_{i1} + \ldots + \beta_p x_{ip}\right) \geq M$$



Problem: sometimes we can't be perfect…

Recap: support vector classifier



Big idea:

we might be willing to

sacrifice a few

in order to give the rest

a better margin

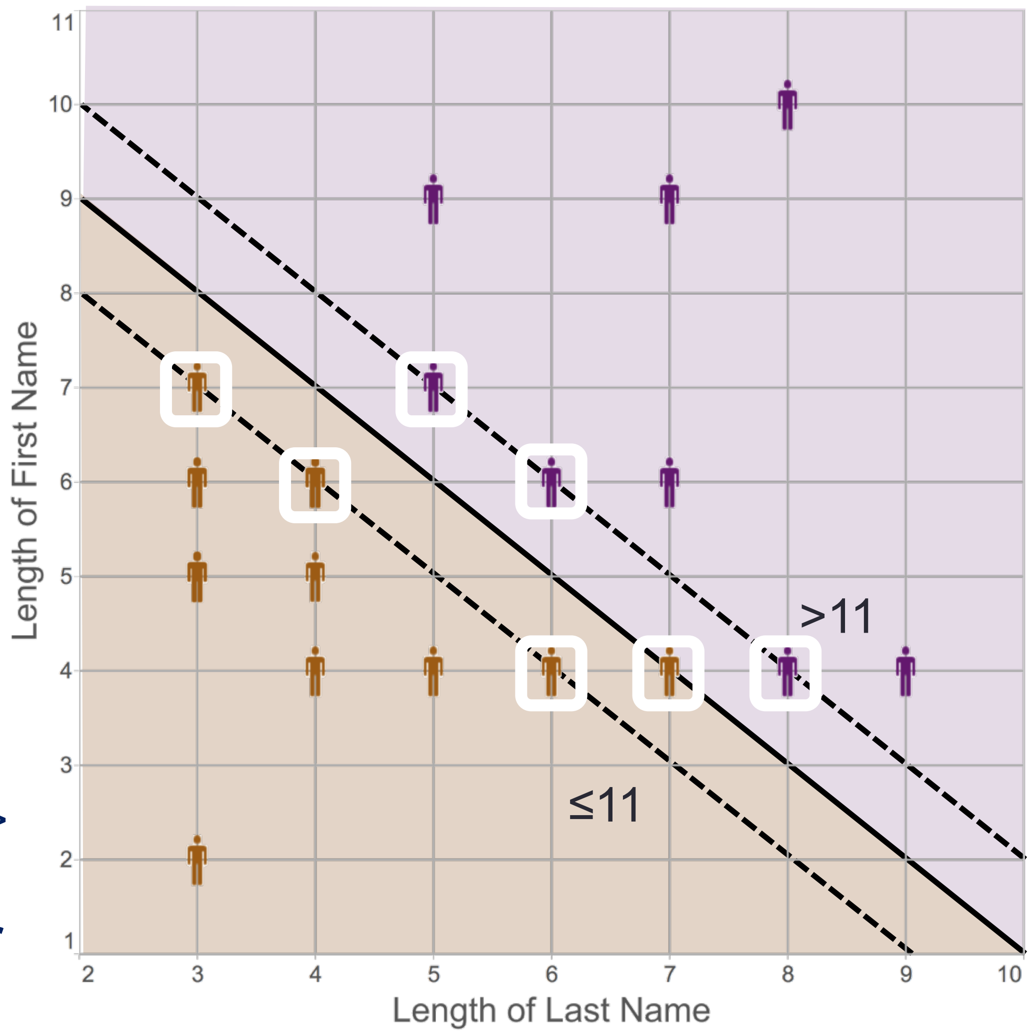
$$\max(M) \text{ such that}$$

$$\boxed{y_i\left(\beta_0 + \beta_1 x_{i1} + \ldots + \beta_p x_{ip}\right)} \geq M(1-\varepsilon_i)$$

$$\varepsilon_i \geq 0, \ \sum_{i=1}^{n} \varepsilon_i \leq C$$

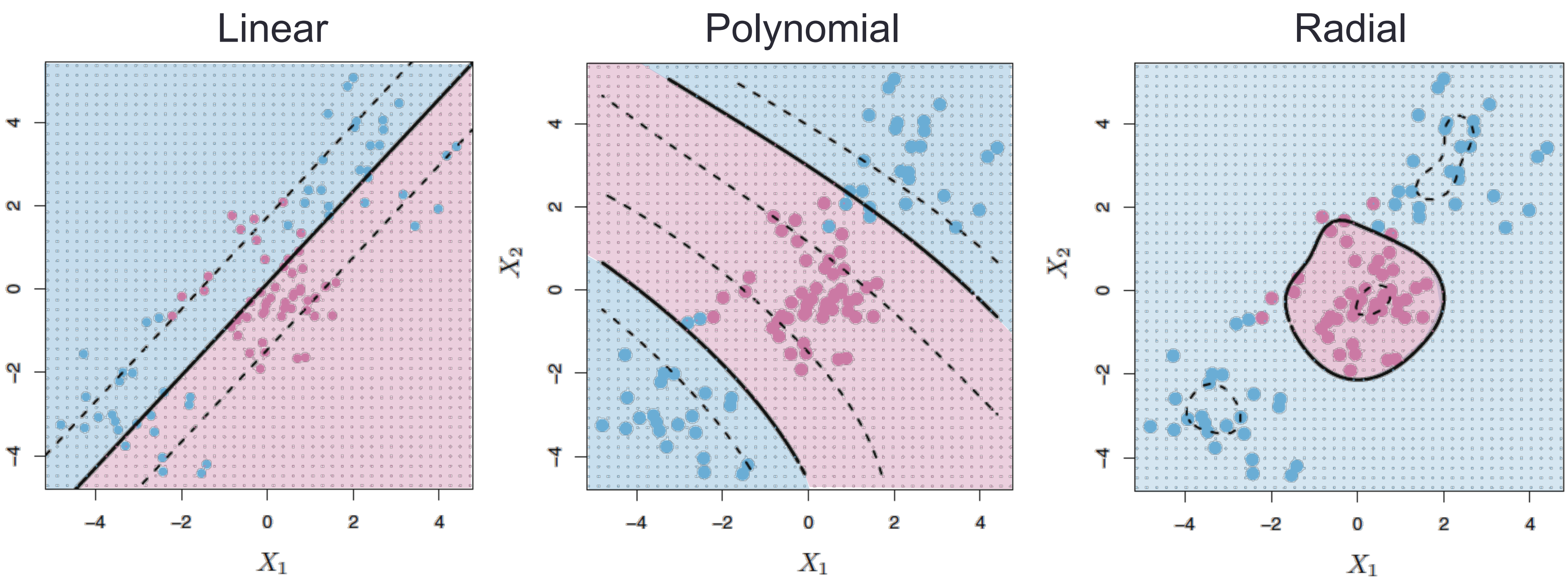
Bigger value = further from margin = **more confident**

Recap: support vectors



Decision rule is based only on the support vectors => SVC **is robust to strange behavior** far from the hyperplane!

Recap: kernel



$$K\left(x_i, x_{i'}\right) = \sum_{j=1}^{p} x_{ij} x_{i'j}$$

$$K\left(x_i, x_{i'}\right) = \left(1 + \sum_{j=1}^{p} x_{ij} x_{i'j}\right)^d$$

$$K\left(x_i, x_{i'}\right) = e^{-\gamma \sum_{j=1}^{p} \left(x_{ij} - x_{i'j}\right)^2}$$

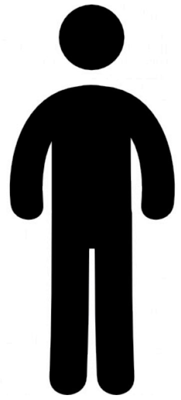
Big idea: using different ways of measuring "similarity" allows you to partition the feature space in different ways

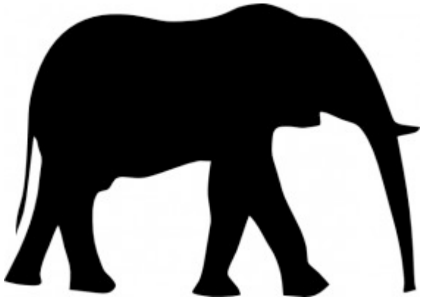
Discussion

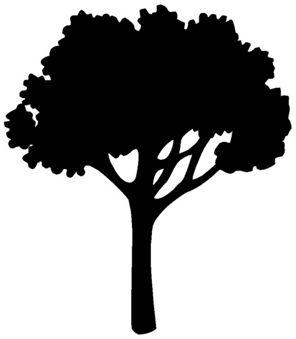
- **Question:** what's the problem?
- **Answer:** regardless of kernel shape, a SMV can only divide the data into **two** parts… but the data in the real world sometimes has multiple classes

So what can we do?

Goal: assign observation to 1 of 4 groups







One-versus-one classification

- **Big idea**: build an SVM for each pair of classes

- **To predict:** classify each observation using all of the *(k choose 2)* classifiers, keep track of how many times the observation is assigned to each: majority wins

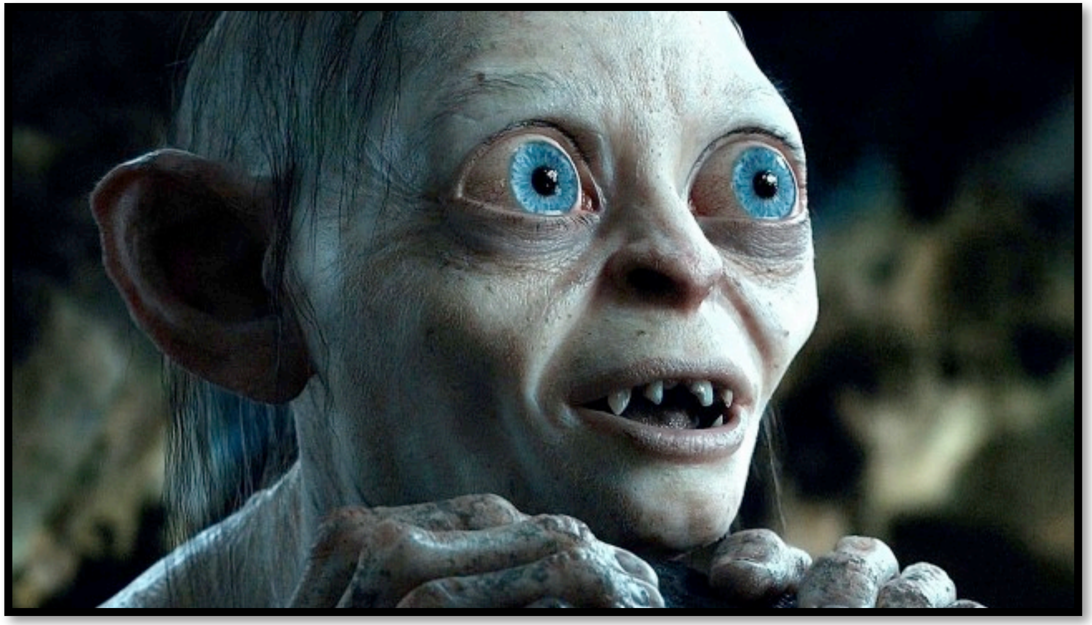
One-versus-all classification

- **Big idea**: build an SVM for each class

- **To predict:** classify each observation using each of the k classifiers, keep track of how confident we are of each prediction: most confident class wins

Quick history lesson

- Mid-90s: SVMs come on the scene
- **Reaction**: OoOoOoooOoooooohh…
  - Good performance
  - "Mysterious"– felt entirely different from classical approaches



Flashback: loss functions

- Many of the methods we've seen so far take the form:

$$\min_{\beta}\left\{\boxed{L(X,y,\beta)}+\boxed{\lambda P(\beta)}\right\}$$

"loss"
(consequence for poor prediction)

"penalty"
(consequence for being complicated)

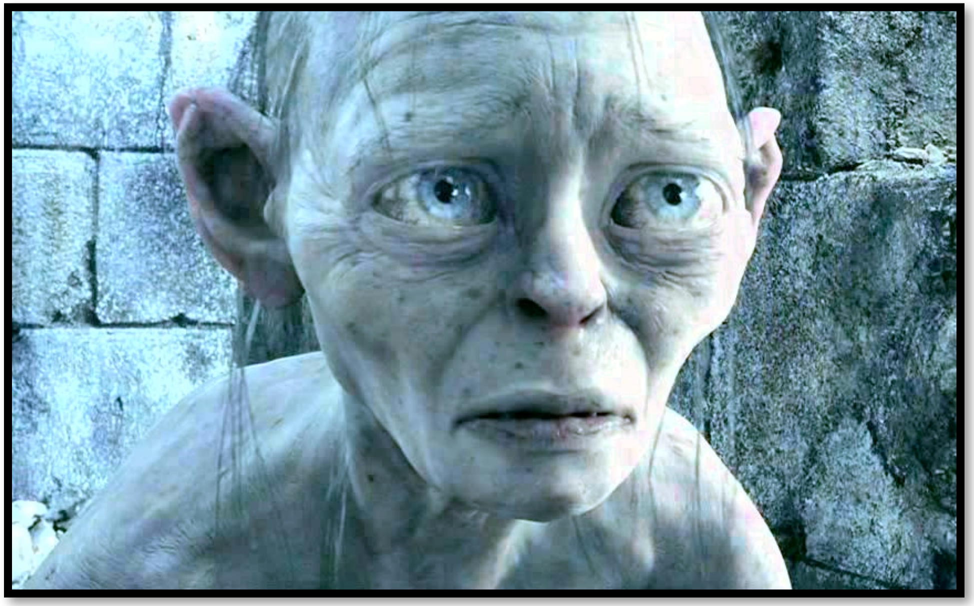
- With a little manipulation, we can rewrite our SVM as:

$$\min_{\beta}\left\{\boxed{\sum_{i=1}^{n}\max\left[0,1-y_i\left(\beta_0+\ldots+\beta_p x_{ip}\right)\right]}+\boxed{\lambda\sum_{j=1}^{p}\beta_j^2}\right\}$$

hinge loss

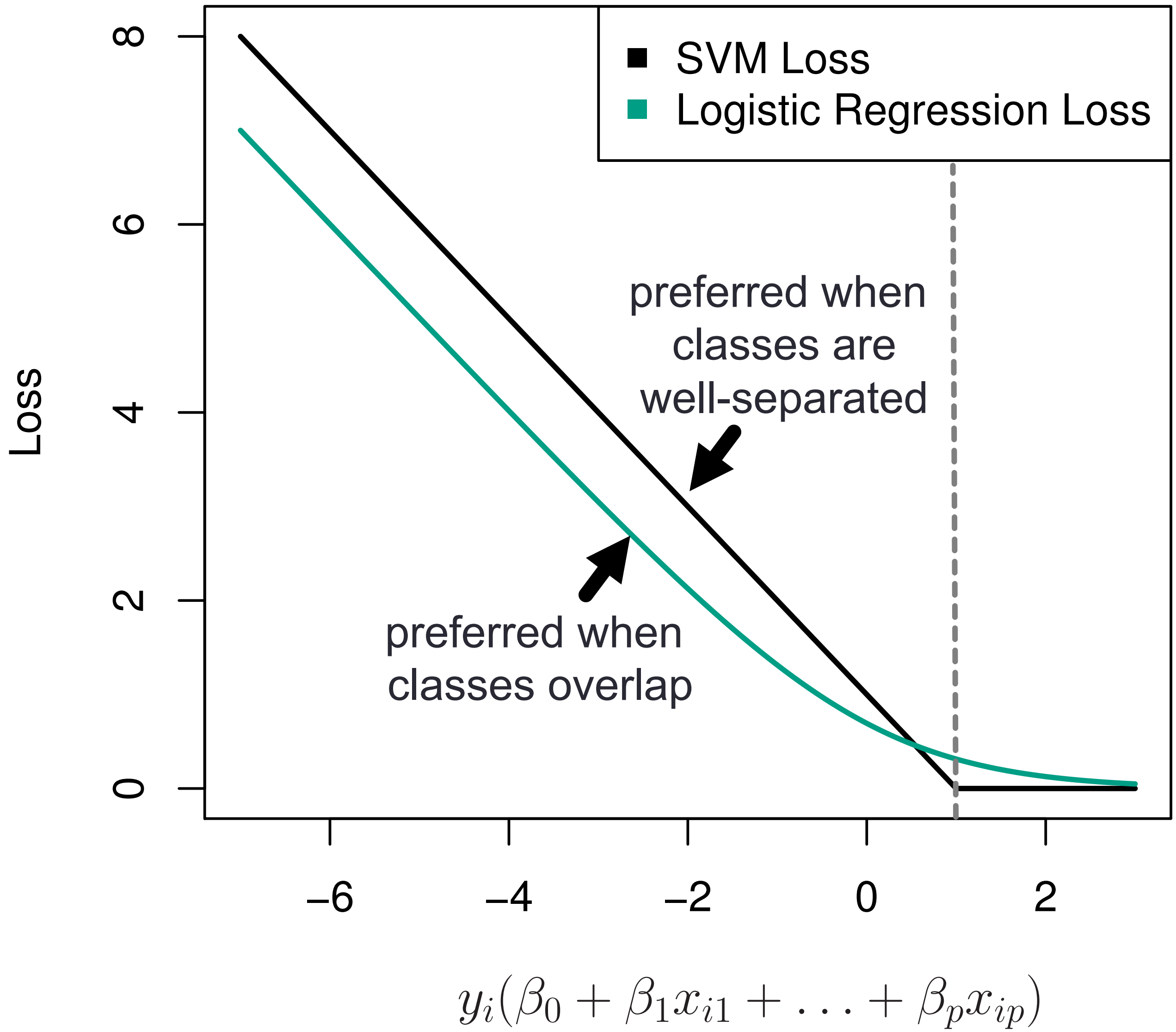
Alas…

- Despite appearances, SVMs are quite closely related to logistic regression and other classical statistical methods



- And what’s worse, most other classification methods can use non-linear kernels, too! (recall Ch. 7…)

SVMs vs. logistic regression loss functions



Lab: Multiclass SVMs

- To do today's lab in R: **`e1071, ROCR`**
- To do today's lab in python: <nothing new>
- Instructions and code:

[course website]/labs/lab15-r.html

[course website]/labs/lab15-py.html

- Full version can be found beginning on p. 366 of ISLR

Coming up

- **Next week:** unsupervised techniques