# Lab 2 - Linear Regression in Python

February 24, 2016

This lab on Linear Regression is a python adaptation of p. 109-119 of "Introduction to Statistical Learning with Applications in R" by Gareth James, Daniela Witten, Trevor Hastie and Robert Tibshirani. Written by R. Jordan Crouser at Smith College for SDS293: Machine Learning (Spring 2016).

## 1 3.6.1 Importing Libraries

```
In [2]: # Tells matplotlib to display images inline instead of a new window
        %matplotlib inline

        import numpy as np
        import pandas as pd
        import statsmodels.api as sm
```

We'll start by importing the data from Boston.csv into a pandas dataframe:

```
In [105]: df = pd.read_csv('Boston.csv', index_col=0)
          df.head()
```

```
Out[105]:        crim  zn  indus  chas    nox     rm   age     dis  rad  tax  ptratio  \
          1  0.00632  18   2.31     0  0.538  6.575  65.2  4.0900    1  296     15.3
          2  0.02731   0   7.07     0  0.469  6.421  78.9  4.9671    2  242     17.8
          3  0.02729   0   7.07     0  0.469  7.185  61.1  4.9671    2  242     17.8
          4  0.03237   0   2.18     0  0.458  6.998  45.8  6.0622    3  222     18.7
          5  0.06905   0   2.18     0  0.458  7.147  54.2  6.0622    3  222     18.7

              black  lstat  medv
          1  396.90   4.98  24.0
          2  396.90   9.14  21.6
          3  392.83   4.03  34.7
          4  394.63   2.94  33.4
          5  396.90   5.33  36.2
```

## 2 3.6.2 Simple Linear Regression

Now let's fit a simple linear model (OLS - for "ordinary least squares" method) with `medv` as the response and `lstat` as the predictor:

```
In [106]: lm = sm.OLS.from_formula('medv ~ lstat', df)
          result = lm.fit()
```

To get detailed information about the model, we can print the results of a call to the `.summary()` method:

```
In [107]: print result.summary()
```

```
OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.544
Model:                            OLS   Adj. R-squared:                  0.543
Method:                 Least Squares   F-statistic:                     601.6
Date:                Wed, 03 Feb 2016   Prob (F-statistic):           5.08e-88
Time:                        21:36:56   Log-Likelihood:                -1641.5
No. Observations:                 506   AIC:                             3287.
Df Residuals:                     504   BIC:                             3295.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
Intercept     34.5538      0.563     61.415      0.000        33.448    35.659
lstat         -0.9500      0.039    -24.528      0.000        -1.026    -0.874
==============================================================================
Omnibus:                      137.043   Durbin-Watson:                   0.892
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              291.373
Skew:                           1.453   Prob(JB):                     5.36e-64
Kurtosis:                       5.319   Cond. No.                         29.7
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Want individual attributes? You can access them independently like this:

```
In [108]: result.rsquared, result.fvalue, result.params.Intercept, result.params.lstat
```

```
Out[108]: (0.5441462975864797,
           601.6178711098953,
           34.553840879383074,
           -0.9500493537579906)
```

For a complete list of attributes and methods of a `RegressionResults` object, see: http://statsmodels.sourceforge.net/devel/generated/statsmodels.regression.linear_model.RegressionResults.html?highlight=r

Now let's try making some predictions using this model. First, we'll set up a dataframe containing the `lstat` values for which we want to predict a response:

```
In [109]: new = pd.DataFrame([[1, 5], [1, 10], [1, 15]], columns=['Intercept', 'lstat'])
```

Now we just call the `.predict()` method:

```
In [110]: result.predict(new)
```

```
Out[110]: array([ 29.80359411,  25.05334734,  20.30310057])
```

Technically those are the right prediction values, but maybe it would be good to have the confidence intervals along with them. Let's write a little helper function to get that and package it all up:

```
In [111]: def predict(res, new):

              # Get the predicted values
              fit = pd.DataFrame(res.predict(new), columns=['fit'])
```

```
    # Get the confidence interval for the model (and rename the columns to something a bit mo
    ci = res.conf_int().rename(columns={0: 'lower', 1: 'upper'})

    # Now a little bit of matrix multiplication to get the confidence intervals for the predi
    ci = ci.T.dot(new.T).T

    # And finally wrap up the confidence intervals with the predicted values
    return pd.concat([fit, ci], axis=1)
```

In [112]:
```
predict(result, new)
```

Out[112]:

| | fit | lower | upper |
|---|---|---|---|
| 0 | 29.803594 | 28.317716 | 31.289472 |
| 1 | 25.053347 | 23.186975 | 26.919720 |
| 2 | 20.303101 | 18.056234 | 22.549967 |

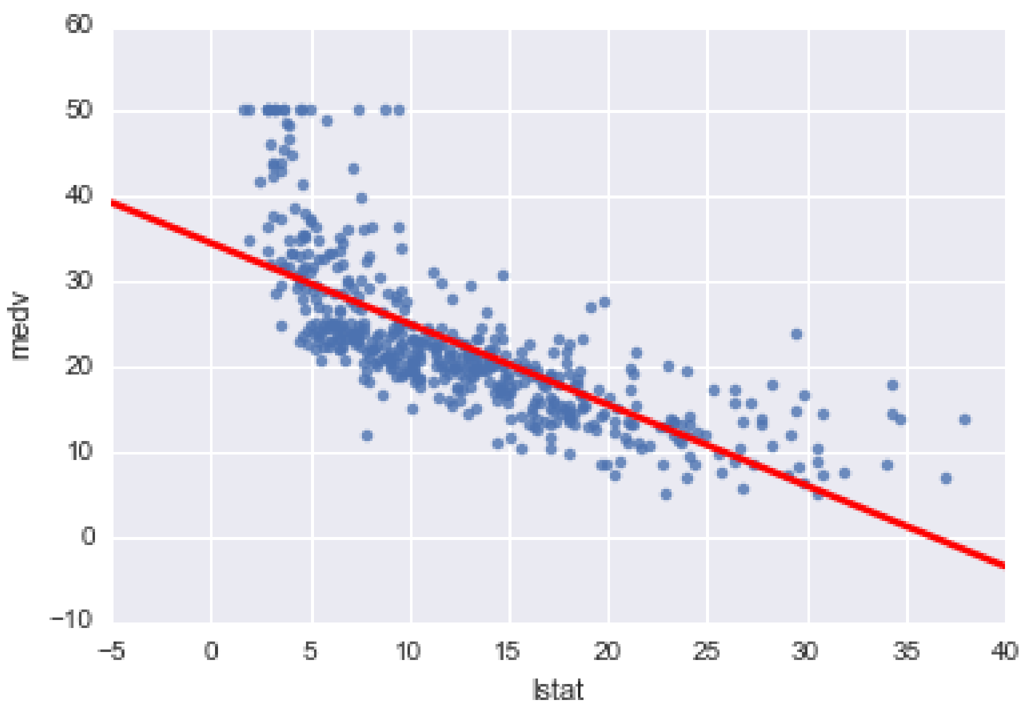
Seaborn is a Python visualization library based on matplotlib that provides a high-level interface for drawing attractive statistical graphics.

In [113]:
```
import seaborn as sns
```

We will now plot `medv` and `lstat` along with the least squares regression line using the `regplot()` function. We can define the color of the fit line using `line_kws` ("line keywords"):

In [114]:
```
sns.regplot('lstat', 'medv', df, line_kws = {"color":"r"}, ci=None)
```

Out[114]: <matplotlib.axes._subplots.AxesSubplot at 0x10aa0d3d0>




We can also plot the residuals against the fitted values:

```
In [115]: fitted_values = pd.Series(result.fittedvalues, name="Fitted Values")
          residuals = pd.Series(result.resid, name="Residuals")
          sns.regplot(fitted_values, residuals, fit_reg=False)

Out[115]: <matplotlib.axes._subplots.AxesSubplot at 0x10b5888d0>
```


Perhaps we want normalized residuals instead?

```
In [116]: s_residuals = pd.Series(result.resid_pearson, name="S. Residuals")
          sns.regplot(fitted_values, s_residuals,  fit_reg=False)

Out[116]: <matplotlib.axes._subplots.AxesSubplot at 0x10a843b10>
```


We can also look for points with high leverage:

In [130]:
```
from statsmodels.stats.outliers_influence import OLSInfluence
leverage = pd.Series(OLSInfluence(result).influence, name = "Leverage")
sns.regplot(leverage, s_residuals,  fit_reg=False)
```

Out[130]: <matplotlib.axes._subplots.AxesSubplot at 0x10bc653d0>


# 3 3.6.3 Multiple Linear Regression

In order to fit a multiple linear regression model using least squares, we again use the `from_formula()` function. The syntax `from_formula(y ∼ x1 + x2 + x3)` is used to fit a model with three predictors, $x1$, $x2$, and $x3$. The `summary()` function now outputs the regression coefficients for all the predictors.

```
In [131]: model = sm.OLS.from_formula('medv ~ lstat + age', df)
          result = model.fit()
          print result.summary()
```

```
OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.551
Model:                            OLS   Adj. R-squared:                  0.549
Method:                 Least Squares   F-statistic:                     309.0
Date:                Wed, 03 Feb 2016   Prob (F-statistic):           2.98e-88
Time:                        21:38:14   Log-Likelihood:                -1637.5
No. Observations:                 506   AIC:                             3281.
Df Residuals:                     503   BIC:                             3294.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
Intercept     33.2228      0.731     45.458      0.000        31.787    34.659
lstat         -1.0321      0.048    -21.416      0.000        -1.127    -0.937
age            0.0345      0.012      2.826      0.005         0.011     0.059
```

```
==============================================================================
Omnibus:                      124.288   Durbin-Watson:                   0.945
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              244.026
Skew:                           1.362   Prob(JB):                     1.02e-53
Kurtosis:                       5.038   Cond. No.                         201.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

The Boston data set contains 13 variables, and so it would be cumbersome to have to type all of these in order to perform a regression using all of the predictors. Instead, we can use the following short-hand:

```
In [132]: # All columns (except medv, which is our response)
          model = sm.OLS.from_formula('medv ~ ' + '+'.join(df.columns.difference(['medv'])), df)
          result = model.fit()
          print result.summary()
```

```
OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.741
Model:                            OLS   Adj. R-squared:                  0.734
Method:                 Least Squares   F-statistic:                     108.1
Date:                Wed, 03 Feb 2016   Prob (F-statistic):         6.72e-135
Time:                        21:38:15   Log-Likelihood:                -1498.8
No. Observations:                 506   AIC:                             3026.
Df Residuals:                     492   BIC:                             3085.
Df Model:                          13
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
Intercept     36.4595      5.103      7.144      0.000        26.432    46.487
age            0.0007      0.013      0.052      0.958        -0.025     0.027
black          0.0093      0.003      3.467      0.001         0.004     0.015
chas           2.6867      0.862      3.118      0.002         0.994     4.380
crim          -0.1080      0.033     -3.287      0.001        -0.173    -0.043
dis           -1.4756      0.199     -7.398      0.000        -1.867    -1.084
indus          0.0206      0.061      0.334      0.738        -0.100     0.141
lstat         -0.5248      0.051    -10.347      0.000        -0.624    -0.425
nox          -17.7666      3.820     -4.651      0.000       -25.272   -10.262
ptratio       -0.9527      0.131     -7.283      0.000        -1.210    -0.696
rad            0.3060      0.066      4.613      0.000         0.176     0.436
rm             3.8099      0.418      9.116      0.000         2.989     4.631
tax           -0.0123      0.004     -3.280      0.001        -0.020    -0.005
zn             0.0464      0.014      3.382      0.001         0.019     0.073
==============================================================================
Omnibus:                      178.041   Durbin-Watson:                   1.078
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              783.126
Skew:                           1.521   Prob(JB):                    8.84e-171
Kurtosis:                       8.281   Cond. No.                     1.51e+04
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
[2] The condition number is large, 1.51e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Note that we used the syntax `.join(df.columns.difference(['medv']))` to exclude the response variable above. We can use the same napproach to perform a regression using just a subset of the predictors? For example, in the above regression output, `age` and `indus` have a high p-value. So we may wish to run a regression excluding these predictors:

```
In [133]: # All columns (except medv)
          model = sm.OLS.from_formula('medv ~ ' + '+'.join(df.columns.difference(['medv', 'age', 'indus
          result = model.fit()
          print result.summary()
```

```
OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.741
Model:                            OLS   Adj. R-squared:                  0.735
Method:                 Least Squares   F-statistic:                     128.2
Date:                Wed, 03 Feb 2016   Prob (F-statistic):         5.54e-137
Time:                        21:38:18   Log-Likelihood:                -1498.9
No. Observations:                 506   AIC:                             3022.
Df Residuals:                     494   BIC:                             3072.
Df Model:                          11
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
Intercept     36.3411      5.067      7.171      0.000        26.385    46.298
black          0.0093      0.003      3.475      0.001         0.004     0.015
chas           2.7187      0.854      3.183      0.002         1.040     4.397
crim          -0.1084      0.033     -3.307      0.001        -0.173    -0.044
dis           -1.4927      0.186     -8.037      0.000        -1.858    -1.128
lstat         -0.5226      0.047    -11.019      0.000        -0.616    -0.429
nox          -17.3760      3.535     -4.915      0.000       -24.322   -10.430
ptratio       -0.9465      0.129     -7.334      0.000        -1.200    -0.693
rad            0.2996      0.063      4.726      0.000         0.175     0.424
rm             3.8016      0.406      9.356      0.000         3.003     4.600
tax           -0.0118      0.003     -3.493      0.001        -0.018    -0.005
zn             0.0458      0.014      3.390      0.001         0.019     0.072
==============================================================================
Omnibus:                      178.430   Durbin-Watson:                   1.078
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              787.785
Skew:                           1.523   Prob(JB):                    8.60e-172
Kurtosis:                       8.300   Cond. No.                     1.47e+04
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.47e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

# 4 3.6.4 Interaction Terms

It is easy to include interaction terms in a linear model using the `.from_formula()` function. The syntax `lstat : black` tells Python to include an interaction term between `lstat` and `black`. The syntax `lstat * age` simultaneously includes lstat, age, and the interaction term lstat × age as predictors; it is a shorthand for lstat + age + lstat : age.

```
In [134]: print sm.OLS.from_formula('medv ~ lstat*age', df).fit().summary()
```

```
OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.556
Model:                            OLS   Adj. R-squared:                  0.553
Method:                 Least Squares   F-statistic:                     209.3
Date:                Wed, 03 Feb 2016   Prob (F-statistic):           4.86e-88
Time:                        21:38:20   Log-Likelihood:                -1635.0
No. Observations:                 506   AIC:                             3278.
Df Residuals:                     502   BIC:                             3295.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
Intercept     36.0885      1.470     24.553      0.000        33.201    38.976
lstat         -1.3921      0.167     -8.313      0.000        -1.721    -1.063
age           -0.0007      0.020     -0.036      0.971        -0.040     0.038
lstat:age      0.0042      0.002      2.244      0.025         0.001     0.008
==============================================================================
Omnibus:                      135.601   Durbin-Watson:                   0.965
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              296.955
Skew:                           1.417   Prob(JB):                     3.29e-65
Kurtosis:                       5.461   Cond. No.                     6.88e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 6.88e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

# 5 3.6.5 Non-linear Transformations of the Predictors

The .from_formula() function can also accommodate non-linear transformations of the predictors. For instance, given a predictor X, we can create a predictor X^2 using np.square(X). We now perform a regression of medv onto lstat and lstat^2.

```
In [135]: lm.fit2 = sm.OLS.from_formula('medv ~ lstat + np.square(lstat)', df).fit()
          print lm.fit2.summary()
```

```
OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.641
Model:                            OLS   Adj. R-squared:                  0.639
Method:                 Least Squares   F-statistic:                     448.5
Date:                Wed, 03 Feb 2016   Prob (F-statistic):          1.56e-112
Time:                        21:38:23   Log-Likelihood:                -1581.3
No. Observations:                 506   AIC:                             3169.
Df Residuals:                     503   BIC:                             3181.
Df Model:                           2
```

```
Covariance Type:            nonrobust
==================================================================================
                       coef    std err          t      P>|t|      [95.0% Conf. Int.]
----------------------------------------------------------------------------------
Intercept           42.8620      0.872     49.149      0.000        41.149    44.575
lstat               -2.3328      0.124    -18.843      0.000        -2.576    -2.090
np.square(lstat)     0.0435      0.004     11.628      0.000         0.036     0.051
==============================================================================
Omnibus:                      107.006   Durbin-Watson:                   0.921
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              228.388
Skew:                           1.128   Prob(JB):                     2.55e-50
Kurtosis:                       5.397   Cond. No.                     1.13e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.13e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

The near-zero p-value associated with the quadratic term suggests that it leads to an improved model. We use the `anova_lm()` function to further quantify the extent to which the quadratic fit is superior to the linear fit.

```
In [136]: lm.fit = sm.OLS.from_formula('medv ~ lstat', df).fit()
          print sm.stats.anova_lm(lm.fit, lm.fit2)
```

```
df_resid           ssr  df_diff     ss_diff           F        Pr(>F)
0       504  19472.381418        0         NaN         NaN           NaN
1       503  15347.243158        1  4125.13826  135.199822  7.630116e-28
```

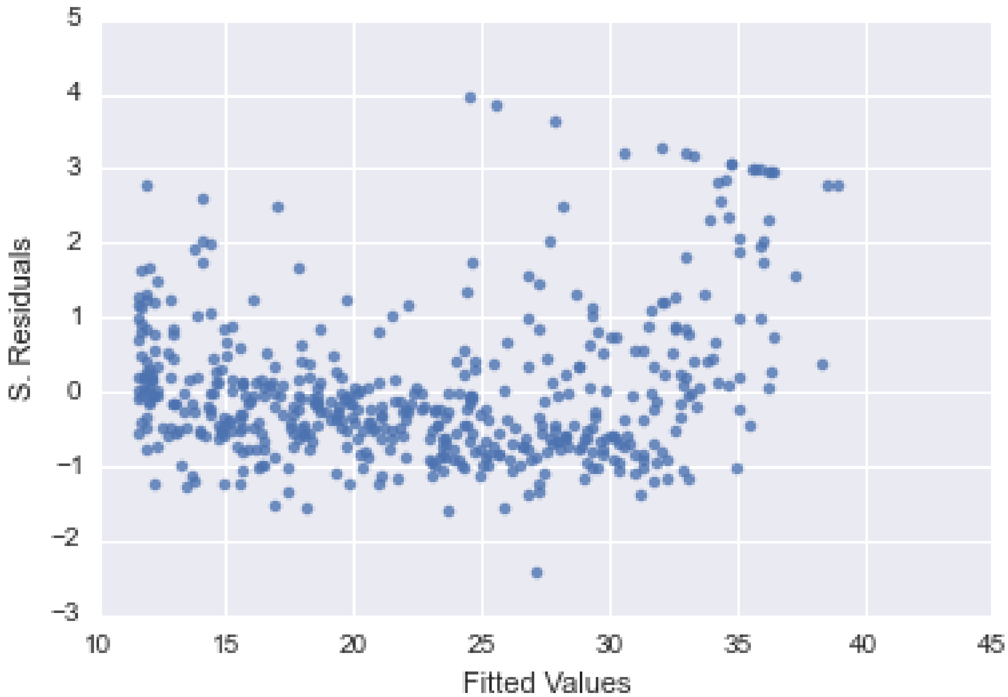
Here Model 0 represents the linear submodel containing only one predictor, `lstat`, while Model 1 corresponds to the larger quadraticmodel that has two predictors, `lstat` and `lstat2`. The `anova_lm()` function performs a hypothesis test comparing the two models. The null hypothesis is that the two models fit the data equally well, and the alternative hypothesis is that the full model is superior.

The F-statistic is 135 and the associated p-value is virtually zero. This provides very clear evidence that the model containing the predictors `lstat` and `lstat2` is far superior to the model that only contains the predictor `lstat`. This is not surprising, since earlier we saw evidence for non-linearity in the relationship between `medv` and `lstat`.

If we type:

```
In [137]: fitted_values = pd.Series(lm.fit2.fittedvalues, name="Fitted Values")
          residuals = pd.Series(lm.fit2.resid, name="S. Residuals")
          sns.regplot(fitted_values, s_residuals,  fit_reg=False)

Out[137]: <matplotlib.axes._subplots.AxesSubplot at 0x10bf09d50>
```




then we see that when the `lstat2` term is included in the model, there is little discernible pattern in the residuals.

In order to create a cubic fit, we can include a predictor of the form `np.power(x, 3))`. However, this approach can start to cumbersome for higher order polynomials. A better approach involves using list comprehension inside a `.join()`. For example, the following command produces a fifth-order polynomial fit:

```
In [138]: sm.OLS.from_formula('medv ~ ' + '+'.join(['np.power(lstat,' + str(i) + ')' for i in range(1,6

Out[138]: <class 'statsmodels.iolib.summary.Summary'>
          """
                                      OLS Regression Results
          ==============================================================================
          Dep. Variable:                   medv   R-squared:                       0.682
          Model:                            OLS   Adj. R-squared:                  0.679
          Method:                 Least Squares   F-statistic:                     214.2
          Date:                Wed, 03 Feb 2016   Prob (F-statistic):          8.73e-122
          Time:                        21:38:31   Log-Likelihood:                -1550.6
          No. Observations:                 506   AIC:                             3113.
          Df Residuals:                     500   BIC:                             3139.
          Df Model:                           5
          Covariance Type:            nonrobust
          ======================================================================================
                                 coef    std err          t      P>|t|      [95.0% Conf. Int.]
          --------------------------------------------------------------------------------------
          Intercept           67.6997      3.604     18.783      0.000        60.618    74.781
          np.power(lstat, 1) -11.9911      1.526     -7.859      0.000       -14.989    -8.994
          np.power(lstat, 2)   1.2728      0.223      5.703      0.000         0.834     1.711
          np.power(lstat, 3)  -0.0683      0.014     -4.747      0.000        -0.097    -0.040
          np.power(lstat, 4)   0.0017      0.000      4.143      0.000         0.001     0.003
```

```
np.power(lstat, 5) -1.632e-05   4.42e-06     -3.692      0.000      -2.5e-05 -7.63e-06
==============================================================================
Omnibus:                      144.085   Durbin-Watson:                   0.987
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              494.545
Skew:                           1.292   Prob(JB):                    4.08e-108
Kurtosis:                       7.096   Cond. No.                     1.37e+08
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.37e+08. This might indicate that there are
strong multicollinearity or other numerical problems.
"""
```

Of course, we are in no way restricted to using polynomial transformations of the predictors. Here we try a log transformation.

In [139]:
```
sm.OLS.from_formula('medv ~ np.log(rm)', df).fit().summary()
```

Out[139]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.436
Model:                            OLS   Adj. R-squared:                  0.435
Method:                 Least Squares   F-statistic:                     389.3
Date:                Wed, 03 Feb 2016   Prob (F-statistic):           1.22e-64
Time:                        21:38:35   Log-Likelihood:                -1695.4
No. Observations:                 506   AIC:                             3395.
Df Residuals:                     504   BIC:                             3403.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
Intercept    -76.4878      5.028    -15.213      0.000       -86.366   -66.610
np.log(rm)    54.0546      2.739     19.732      0.000        48.672    59.437
==============================================================================
Omnibus:                      117.102   Durbin-Watson:                   0.681
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              584.336
Skew:                           0.916   Prob(JB):                    1.30e-127
Kurtosis:                       7.936   Cond. No.                         38.9
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
"""
```

# 6 3.6.6 Qualitative Predictors

We will now examine the `Carseats` data, which is part of the `ISLR` library. We will attempt to predict `Sales` (child car seat sales) in 400 locations based on a number of predictors.

In [3]:
```
df2 = pd.read_csv('Carseats.csv')
df2.head()
```

```
Out[3]:    Sales  CompPrice  Income  Advertising  Population  Price ShelveLoc  Age  \
        0   9.50        138      73           11         276    120       Bad   42
        1  11.22        111      48           16         260     83      Good   65
        2  10.06        113      35           10         269     80    Medium   59
        3   7.40        117     100            4         466     97    Medium   55
        4   4.15        141      64            3         340    128       Bad   38

           Education Urban   US
        0         17   Yes  Yes
        1         10   Yes  Yes
        2         12   Yes  Yes
        3         14   Yes  Yes
        4         13   Yes   No
```

The `Carseats` data includes qualitative predictors such as `Shelveloc`, an indicator of the quality of the shelving location—that is, the space within a store in which the car seat is displayed—at each location. The predictor `Shelveloc` takes on three possible values, `Bad`, `Medium`, and `Good`.

Given a qualitative variable such as `Shelveloc`, Python generates dummy variables automatically. Below we fit a multiple regression model that includes some interaction terms.

```
In [5]: sm.OLS.from_formula('Sales ~ Income:Advertising+Price:Age + ' + "+".join(df2.columns.difference
```

```
Out[5]: <class 'statsmodels.iolib.summary.Summary'>
        """
                                    OLS Regression Results
        ==============================================================================
        Dep. Variable:                  Sales   R-squared:                       0.876
        Model:                            OLS   Adj. R-squared:                  0.872
        Method:                 Least Squares   F-statistic:                     210.0
        Date:                Sat, 06 Feb 2016   Prob (F-statistic):          6.14e-166
        Time:                        22:41:25   Log-Likelihood:                -564.67
        No. Observations:                 400   AIC:                             1157.
        Df Residuals:                     386   BIC:                             1213.
        Df Model:                          13
        Covariance Type:            nonrobust
        =======================================================================================
                                  coef    std err          t      P>|t|      [95.0% Conf. Int.]
        ---------------------------------------------------------------------------------------
        Intercept               6.5756      1.009      6.519      0.000         4.592     8.559
        ShelveLoc[T.Good]       4.8487      0.153     31.724      0.000         4.548     5.149
        ShelveLoc[T.Medium]     1.9533      0.126     15.531      0.000         1.706     2.201
        US[T.Yes]              -0.1576      0.149     -1.058      0.291        -0.450     0.135
        Urban[T.Yes]            0.1402      0.112      1.247      0.213        -0.081     0.361
        Income:Advertising      0.0008      0.000      2.698      0.007         0.000     0.001
        Price:Age               0.0001      0.000      0.801      0.424        -0.000     0.000
        Advertising             0.0702      0.023      3.107      0.002         0.026     0.115
        Age                    -0.0579      0.016     -3.633      0.000        -0.089    -0.027
        CompPrice               0.0929      0.004     22.567      0.000         0.085     0.101
        Education              -0.0209      0.020     -1.063      0.288        -0.059     0.018
        Income                  0.0109      0.003      4.183      0.000         0.006     0.016
        Population              0.0002      0.000      0.433      0.665        -0.001     0.001
        Price                  -0.1008      0.007    -13.549      0.000        -0.115    -0.086
        ==============================================================================
        Omnibus:                        1.281   Durbin-Watson:                   2.047
        Prob(Omnibus):                  0.527   Jarque-Bera (JB):                1.147
```

```
Skew:                           0.129   Prob(JB):                        0.564
Kurtosis:                       3.050   Cond. No.                     1.31e+05
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.31e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
"""
```

To learn how to set other coding schemes (or contrasts), see: http://statsmodels.sourceforge.net/devel/examples/notebooks/generated/contrasts.html

```
In [ ]:
```