Optimal Vehicle Control of Four-Wheel Steering

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Abstract

Automotive vehicles call for a range of steering activity: one extreme is highway driving with negligible turning. Another is steering during U-turn maneuvers, which calls for agile turning and a small turning radius to increase vehicle stability. System modeling and simulation are becoming widely used in autonomous vehicle engineering to reduce development time and improve the design and miniaturization of complex systems. This capstone project focuses on steering control system modeling, kalman filter design, and simulation for optimal vehicle tracking. A four-wheel steering (4WS) system control strategy is established and the concept of four-wheel steering is discussed in detail. A vehicle model is developed using appropriate steering system dynamics. A kalman filter is designed to estimate the position and velocity of a vehicle in Simulink. The filter performance is validated by simulating a U-turn maneuver and randomly generated measurement noise. Implementing the control strategy of a steer-by-wire four-wheel steering conversion mechatronic control system is found to reduce the turning radius of a vehicle and to improve maneuvering in tight spaces. Finally, the simulation results show that the kalman filter design improves the position measurements and provides accurate velocity estimates for a vehicle.
Declaration

The work in this capstone design project final report is based on research carried out at Smith College, Picker Engineering Program, Massachusetts.

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Chapter 1

Introduction

People recently have been paying more attention to not only the security and comfort in their vehicle, but also to having optimal steering performance. Specifically, there is an issue regarding steering performance in vehicles performing a U-turn maneuver with a too small turning radius, which may be the case for large trucks and cars towing trailers. Amidst the fast development in control theory and technology, research on automotive mechatronic control systems for enhancing the steering performance of a vehicle is becoming an increasingly popular and important area of work in the automotive field.

Four-wheel steering (4WS) is an advanced control technique which can improve steering characteristics. Compared with traditional two-wheel steering (2WS), four-wheel steering systems steer the front wheels and rear wheels individually when cornering, according to vehicle motion states: speed, yaw velocity and lateral acceleration. Four-wheel steering can enhance handling stability, improve the active safety for a vehicle, and allow a vehicle to turn in a significantly smaller turning radius.

This capstone design project includes the development for a mechatronic control system design to improve handling performance of the automotive vehicle under special steering circumstances and to reduce turning radius. This project develops an understanding in the basics of vehicle dynamics, four-wheel steering mechatronic
control systems, steering kinematics, and kalman filtering. Work to create a function for a vehicle trajectory while minimizing turning radius (a trajectory based on the car’s center of mass) is developed. A trajectory curve using vehicle kinematics, steering geometry, desired steering angle, and turning radius is parametrized. A design and test procedure is executed to validate a kalman filter’s performance by simulating a U-Turn maneuver, (shown in Figure 1.1) and to determine the state of a vehicle. This will provide more information on how to control and optimize performance parameters of a vehicle in a specific situation.
Chapter 2

Background and Literature Review

2.1 The Steering System

Generally, wheeled vehicles are divided into two different types: single track vehicles and track-trailers. This project deals with single track vehicles, which are passenger cars or car-like vehicles/robots that are viewed as a single steering tractor. From the viewpoint of steering, single track vehicles are further classified into two types of vehicles. The first is a front-steering vehicle (2WS) in which only the two front tires are steered and second is a full-steering vehicle (4WS) in which the front and rear tires are steered independently. The primary task of vehicle lateral motion control is path/lane following, or more plainly, keeping a vehicle on a road or in a lane. Vehicle lateral dynamics are relatively easy to control because they almost solely depend on controlling a steering subsystem.

2.1.1 Steer-By-Wire

This project provides an overview into the initiatives and techniques for vehicle lateral motion (steering) control with an emphasis on lateral motion monitoring and steering controller design. An optimal vehicle control strategy includes increasing the requirements of safe and comfortable driving with steer-by-wire integration into
the steering control. These steer-by-wire systems are computer-controlled subsystems that are connected through in-vehicle computer networks. A steer-by-wire system (as shown in Figure 2.1) replaces the traditional mechanical linkage between the steering wheel and the road wheel actuator (the rack and pinion steering system) with an electronic connection. Since it removes direct kinematical relationship between the steering and road wheels, it enables control algorithms to enhance driver steering command.

![Diagram of Steer-By-Wire System](image)

Figure 2.1: Diagram of Steer-By-Wire System

One of the direct reasons why the steer-by-wire system is chosen to be integrated into the design is because steer-by-wire systems help the driver steer the vehicle more easily. On average, the steering torque required at the hand-wheel during normal driving ranges from 0 Nm to 2 Nm, while emergency maneuvers can demand up to 15 Nm of torque. The actuator that would be installed into the vehicle would provide a maximum steering torque of 17.1 Nm with a maximum steer rate of 700 degrees per second [1].

The second purpose of integrating a steer-by-wire system is to filter out the disturbance torque generated by road roughness and parameter changes caused by tire pressure/temperature and loading variations. The aim of a steer-by-wire
system integrated in the controller is to track the commanded steering angles with minimal error. With an alignment moment added to the feedback and feed-forward control, the actuator effort is effectively eliminated in most of the steering disturbances that would arise when turning at a specific speed.

The response in steer-by-wire systems is quicker and more accurate than that of a conventional steering system. This improves vehicle stability and provides a basis to advance the technology for steering system fault detection. In general, steer-by-wire systems are expected to provide a better operation platform of lateral motion controllers. It is estimated to take ten or twenty years to widely employ steer-by-wire systems into the real world to provide a better operational platform of lateral motion controllers.

\section*{2.1.2 Four-Wheel Steering}

Full steering vehicles, or two-wheel steering (2WS) vehicles significantly outperform front steering, or four-wheel steering (4WS) vehicles in handling and stability. When a vehicle enters a curved path, the rear wheels first steer in the opposite direction of the front wheels in order to generate sufficient yaw motion to follow the desired yaw rate. Then, the rear wheels synchronize with the front wheel to keep the desired yaw rate value and to control the lateral motion for path tracking.

The lateral motion in the y-axis of an automotive vehicle is considered when analyzing steering systems. Lateral motion of the automotive vehicle implies how the vehicle responds to steering input. A human driver (HD) controls the lateral dynamics of a vehicle by indirectly affecting the forces generated by the wheels of the vehicle. These forces are influenced by many systems, including the steering system of an automotive vehicle.

The response of the automotive vehicle to steering input is predominantly influenced by a steer-by-wire (SBW) all-wheel-steered (AWS) conversion mechatronic control system. Conventionally, vehicle steering systems are used to control the lateral motion of the vehicle. Research and development (RD) on this subject is broken
down along the following lines; RD work on active front-wheel steering (FWS), active rear-wheel steering (RWS) and all-wheel steering (AWS) systems. Specifically, this project focuses on the SBW four-wheel-steered (4WS) conversion mechatronic controller that influences the wheels direction in different modes, as shown in Figure 2.2.

![Figure 2.2: Active SBW 4WS Conversion Mechatronic Control System](image)

Four-wheel steering (4WS) systems control both front and rear steering angles as a function of driver input and vehicle dynamics. The front-wheel steering (FWS) controller alters the direction of the front wheels as a function of the drivers input with or without a mechanical link. Active FWS provides an electronically controlled superposition at an angle to the steering wheel angle. This additional degree of freedom enables a continuous and driving-situation dependent adaptation of the steering characteristics. Active FWS optimizes features such as steering comfort, effort, and steering dynamics. However, the rear-wheel steering (RWS) controller does not influence the front-steering angle (this task is left to the driver) but rather affects the vehicle dynamics by adjusting the steering angle of the rear wheels. For vehicles operating under normal operation circumstances, controlling lateral dynamics using a SBW 4WS conversion mechatronic control system is
desirable; here the front and rear steering angles are the two control inputs.

2.1.3 SBW 4WS Conversion Mechatronic Control System

With the SBW 4WS conversion mechatronic control system, the rear wheels are turned in the same or opposite senses of direction as the front wheels, depending on vehicle velocity or the angle at which the steering hand-wheel (HW) is turned. All of this occurs as soon as the SBW 4WS mechatronic control system is used, allowing a tighter sight and turning line for a vehicle during cornering. At a predetermined vehicle velocity, when the steering HW is turned to the desired angle, the rear wheels may turn in the same sense of direction as the front wheels. At that steering angle or vehicle velocity, the rear wheels may either move to a straight-line position or turn in the opposite sense of direction.

The aim of a steer-by-wire (SBW) four-wheel steering (4WS) conversion mechatronic control system is better stability during overtaking maneuvers, reduction of vehicle oscillation, reduced sensibility to lateral wind, neutral behavior during cornering, and improvement of active safety [9]. The SBW 4WS conversion mechatronic control system is computer-controlled and automatically adjusts the angles of the rear wheels according to steering wheel position, vehicle velocity, and other variables.

The design proposed is for a modernized SBW 4WS conversion mechatronic control system that is controlled by a set of drive shafts (or mechanically, depending on the type of vehicle platform). Additional components are added to the underside of these platforms to monitor and control the RWS mechanical components. An additional steering gearbox that is similar to the FWS unit is implemented to control the predetermined rear-wheel angles. A safety device or fail-safe unit locks the alignment of the rear wheels in the conventional straight-ahead mode if a problem develops.

A SBW 4WS conversion mechatronic control system has many advantages. Mechatronically controlled, the rear wheels change the way a vehicle turns based on driving

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parameters. When the vehicle is moving slowly, the rear wheels turn in the opposite sense of direction from the front wheels to improve maneuvering performance. At high values of vehicle velocity, the rear wheels turn in the same sense of direction as the front wheels to reduce yaw and improve stability [1].

Additionally, double axle SBW 4WS Conversion Kinematics offer greater maneuverability than single axle SBW 2WS conversion by moving the turn center closer to the center of the vehicle. A SBW 4WS vehicle accomplishes half the turn radius of a SBW 2WS vehicle for the same alteration in wheel heading. A double track, full vehicle physical model of the double axle SBW 4WS conversion is shown in Figure 2.3 [8].

![Figure 2.3: Double Track Vehicle Model for Double Axle SBW 4WS Conversion](image)

The objective of a vehicle model is to convey the user-prescribed dynamics upon a vehicle traveling at high velocity values under normal operation. A relatively simple physical model of an automotive vehicle is used. The physical model
chosen, a two-degrees-of-freedom (2-DOF) one, commonly referred to as the single-track half-vehicle (bicycle) physical model, encompasses the dominant dynamics of the vehicle under prescribed conditions [5]. The vehicle states are yaw rate and lateral velocity.

Figure 2.4: Vehicle States Associated with Single-Track Half-Vehicle Bicycle Model

Figure 2.4 describes the automotive vehicle states. Vehicle velocity is the time rate of change of position of a vehicle; it is a vector quantity that has a sense of direction as well as magnitude [5]. Vehicle speed is the time rate of change of position of a vehicle without regard to a sense of direction, in other words, the magnitude of the vehicle-velocity vector [7]. For a wheeled vehicle (WV) the forces and torques enforced on the steering mechanisms follow from those created at the wheel-ground interface as shown in Figure 2.5 [6].
Automotive obligations require a variety of steering activity: one is highway driving with minor turning for hundreds of kilometers; another is urban handling that requires agile turning [1]. In order to figure out how to minimize the turning radius of a vehicle, it is important to understand the responsibilities of steering and propulsion (traction for a variety of steering activities). The knowledge gained from steering forces and torques (moments of force), enables an establishment on the terminology and the forces and torques mandatory for an automotive vehicle during a turn.

### 2.2 Steering Kinematics and Turning Radius

Analyzing kinematics of different steering configurations (also explained further in the project’s concept generation and selection in Chapter 3) allows the characteristics of diverse steering modes to be practical and efficient with regards to various performance standards. However, all kinematic studies are idealized analyses in light of the fact that the wheel to ground interaction isn’t taken into account [1]. Straight driving benefits as a footing for evaluating steady-state turning. Additionally, to reduce the lateral forces on the wheels during a turn, all of the wheels should be in
a rolling condition [1]. For wheels to continue in a pure rolling event during turning, the wheels must follow a curved path with different radii starting from a common center. The relationship between the steering angle of the inside and outside front wheels may be attained from physics and geometry as shown in Figure 2.6 [1].

\[ s = \sqrt{r^2 - \left(\frac{l}{2}\right)^2} - \frac{b}{2}, \]

\[ \delta_f = \tan\left(\frac{l}{s}\right). \]

\[ \delta_r = \tan\left(\frac{l}{b + s}\right). \]

Figure 2.6: Equations to Obtain Relation Between Steering Angle of Inside and Outside of Front Wheels

Because the outer wheels travel a longer path distance than the inner wheels, the velocities must be distributed to match the path lengths. The kinematics analysis of the four-wheel steering conversion mechatronic control system helps in determining wheel angular velocities given a vehicle’s dimensions, the desired radius, and the desired turn rate. This can be shown in Figure 2.7 and Figure 2.8.
2.3 Previous Solutions

A FWS system without a mechanical link between steering hand wheel (HW) and the steering angle (i.e. an active FWS SBW conversion mechatronic control system) is reviewed [2]. With regard to consumer truck and smart-utility vehicles (SUV) platforms, scientists and engineers are designing and developing a SBW 4WS conversion mechatronic control system that will be offered on select vehicles, as well as full-size pickup platforms. The considerable function certainly is that this improvement allows larger vehicle platforms a shorter turning radius for tight maneuvering and better road handling manners, especially under loaded and towing circumstances [3].

Delphi Corporation created an innovative system which is scheduled to reach consumers in 2016 and is termed Quadrasteer [4]. It is a SBW 4WS conversion mechatronic control system using RWS on the two rear wheels. This system is designed to assist large trucks and SUVs while maneuvering in small spaces as well as to increase stability by highway values of vehicle velocity.

2.4 Environmental Context of Possible Steering Solutions

Additional research work in this capstone design project includes the environmental context within current steering systems. There is a possibility for electric power steering rather than hydraulic power steering. Electric power steering is more efficient compared to hydraulic power steering because the electric power steering motor only needs to provide assistance when the steering wheel is turned on. The
added benefit of this solution is the elimination of environmental hazard posed by
leakage and disposal of hydraulic power steering fluid.

The main characteristics of a vehicle moving on a road is related to driver com-
mand response and to the environmental factors affecting the direction of motion
of a vehicle. The two basic problems regarding vehicle handling include controlling
it along a desired path and stabilizing the direction of motion against disturbances.
The topic of hybrid ground vehicles is also researched and a greater understanding
is attained regarding how hybrid ground vehicles motivate electric and steer-by-wire
steering system technology due to the restrictions on power source availability. Al-
though these two steering systems are efficient, flexible, and environmental friendly,
the steer-by-wire system provides the opportunity for semi-autonomous and au-
tonomous vehicle operation, as well as favoring a drive-by-wire architecture.

Additionally, the research of alternative technologies in the field of automotive
propulsion is an important topic of modern vehicle solutions. Due to this trend,
many problems such as energy efficiency and environmental impact represent a pri-
mary objective in automotive design combined with fundamental aspects concerning
driver safety and consequently vehicle stability as a whole.
Chapter 3

Methodology

3.1 Concept Generation

A problem within the capstone design project that is important to generate concepts/options for is a design for a system/method that reduces the turning radius of a vehicle. Two different concept generation techniques are used include a patent/literature search (using books and published papers) and a TRIZ matrix. Included in the Appendix is a thorough documentation of the concept generation work completed in the design project.

Concepts generated from the patent/literature search include:

1. Turning Control Device and Method that Reduces Turning Radius (by braking the rear wheel on the inside of a turn in accordance with the steering operation when a vehicle is turning).

2. Passively Articulated Axle Steering

3. Vehicle Speed Sensing Type Four-Wheel Steering (4WS)

4. Rack and Pinion Mechanism for Steering the Wheels

5. Motion of Steering Wheel Fed to Sensor (which is sent to a controller to give motion to the tie rods for turning the wheels of a vehicle).

6. Bell Crank Lever and Toggle Disc System

For the patent/literature search process, an external patent search and literature
research search is conducted and serves as a useful information-gathering process for the concept generation in this capstone design project. This process is used in the project to find existing concepts relating to both the overall problem and sub-problems identified in the design problem clarification step. Additionally, this project looks at how implementing an existing solution is easier, cheaper, and faster than developing a new solution. The idea of optimizing a pre-existing solution or applying it as-is to a sub-problem and pairing it with an original concept for another sub-problem yields a novel and an improved overall design for the project.

The first step in the patent/literature search included gathering information relating to the design problem. Next, the search focuses on the scope of the design project by exploring more direct details using design requirements. Given the immense quantity of design information available online in data sources, a methodical search is conducted, specifically in patent archives. Through this approach, innovative conceptual designs are developed from existing solutions. Keywords are used in the search relating to the design requirements for the control system. These keywords include: steering, turning, turning radius, turning circle, maneuvering, mechatronic control, four-wheel steering, turning control, steering control, and vehicle lateral dynamic control. Considering at previous solutions and integrating multiple aspects of each into one solution worked well for the development of useful design concepts/options.

Concepts generated from the TRIZ matrix technique include:
1. Skid SBW 4WS Conversion Kinematics
2. Axle Articulated SBW 2WS Conversion Kinematics
3. Independent explicit SBW 4WS Conversion Kinematics

The TRIZ matrix concept generation process is based on the idea that many problems engineers face are already solved in a different industry, for an unrelated situation, or using different technology. The first step in the TRIZ matrix process is finding major contradictions that make the design problem (minimizing the turn-
3.2. Concept Selection

A specific topic is identified and articulated for which to apply the concept selection process. The important topic selected is steering configurations for a steer-by-wire four-wheel/two-wheel steering vehicle system. For the selection matrix, five concepts are chosen and include:

1. **Independent Explicit Steer-By-Wire Four-Wheel Steering**
   The Independent Explicit SBW 4WS control system explicitly articulates each of the wheels to a desired heading. To accomplish this, the system changes the heading of the wheels to yield a change in heading of the vehicle. This concept is different from the previous design for the independent explicit SBW 4WS system because of the integration of crab steering. Crab steering is when all of the wheels of a vehicle turn by the same amount in the same direction. The all-terrain vehicle then moves in a sideways fashion. Lastly, individual drive FM/PM/EM motors are used inside each wheel with the necessary gearing.

2. **Coordinated Ackerman Steer-By-Wire Two-Wheel Steering**
   The Coordinated Ackerman SBW 2WS system has the most common steering configuration concept. It involves a single axle SBW 2WS conversion mechatronic control system where the front two wheels are pivoted. This means that
the coordinated Ackerman 2WS is mechatronically controlling the coordinates of the angles for the front two wheels. All wheels in a pure rolling event during turning follow curved paths with different radii originating from a common center. The outer wheels travel a longer path distance than the inner wheels so that the velocity components are spread out to match path lengths.

3. **Frame Articulated Steer-By-Wire Four-Wheel Steering**

This system is ideal for all-terrain vehicles. The heading of the vehicle alters by folding the hinged chassis units. This system allows the vehicle significantly more maneuverability than a vehicle with Ackerman steering. During a turn, the maximum value of thrust is provided and maintained by the traction elements. The turning conditions (i.e. running vehicle velocity), the steering angle, and steering time, affect steering torque required to steer the vehicle.

4. **Skid Steer-By-Wire Four-Wheel Steering**

This concept exhibits agility from point turning to line driving using only the motions, components, and swept volume needed for straight driving. Skid SBW 4WS is achieved by creating a differential thrust between the left and right sides of the vehicle, which causes an alteration in heading. Skid SBW 4WS creates differential angular velocities between the inner and outer wheels. The motion of the wheels is limited to rotation about one axis. Centralized drive passes the propelling and tractive torques directly to each wheel. The system allows a preliminary determination of wheel angular velocities given the vehicle dimensions, desired radius, and desired turn rate.

5. **Axle Articulated Steer-By-Wire Two-Wheel Steering**

This mechatronic control system is achieved by adding a free pivot to one of the axles of the vehicle. The steering configuration concept is derived from ones in wagons and carts. The wheels run on separate tracks when going around curves. This requires an increase in the drive propulsion as each wheel is running over fresh terrain. The design to be implemented on a vehicle is the mechatronic control of angular velocity for the front wheels to maintain a desired angle of the front axle. The steered front axle has the ability to
point the perception sensors with the front axle. This system has a joint that includes two free rotations, one about the vertical axis and the other allowing a rolling motion of the front axle to enable all four wheels to contact the ground over rough terrain. There is no steering actuator in this system.

The relevant criteria for concept selection are listed below. Each of the relevant criteria for concept selection originates from an analysis of the section on steering configurations in the book by B.T. Fijalkowski: *Automotive Mechatronics, Operational and Practical Issues*.

- Maneuverability
- Mechanical Complexity
- Control Complexity
- Propulsion Power During Steering Maneuvers

The concept selection documentation for the Concept Screening, Rating Scales, Weighted Criteria, Concept Scoring, and Sensitivity Analysis is found in Appendix A.2.

### 3.3 State Estimation with Kalman Filter

#### 3.3.1 Kalman Filter Overview

Kalman filtering, also known as linear quadratic estimation (LQE), is an algorithm using a series of measurements observed over time, containing statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more precise than those based on a single measurement alone. The kalman filter has numerous applications in technology, including guidance, navigation, and control of vehicles [11]. Kalman filters are a main topic in the field of robotic motion planning and control, and are sometimes included in trajectory optimization. The algorithm for the kalman filter works in a two-step process.
3.3. State Estimation with Kalman Filter

- Step 1: This is the prediction step where the Kalman filter produces estimates of the current state variables, along with their uncertainties.

- Step 2: Once the outcome of the next measurement (necessarily corrupted with some amount of error, including random noise) is observed, these estimates are updated using a weighted average, with more weight being given to estimates with higher certainty.

The Kalman filter algorithm is recursive. It can run in real time, using only the present input measurements and the previously calculated state and its uncertainty matrix; no additional past information is required. The Kalman filter doesn't require any assumption that the errors are Gaussian. However, the filter yields the exact conditional probability estimate in the special case that all errors are Gaussian-distributed.

3.3.2 Estimating State of System

The Kalman filter utilizes the system's dynamics model (such as the physical laws of motion), known control inputs to that system, and multiple sequential measurements (e.g. from sensors) to form an estimate of the system's varying quantities or the state that is better than the estimate obtained by using any one measurement alone. All measurements and calculations based on models are estimated to some degree. Noisy sensor data, approximations in the equations that describe how a system changes, and external factors that are not accounted for introduce some uncertainty about the inferred values for a system's state.

The Kalman filter averages a prediction of a system's state with a new measurement using a weighted average. The purpose of weights is that values with better (smaller) estimated uncertainty are trusted more. The weights are calculated from the covariance, a measure of the estimated uncertainty of the prediction of the system's state. The result of the weighted average is a new state estimate that lies between the predicted and measured state, and have an optimal estimated uncertainty than either alone. This process is repeated every time-step, with the new
3.3. State Estimation with Kalman Filter

estimate and its covariance informing the prediction. This means that the kalman filter works recursively and requires only the last best guess, rather than the entire history, of a systems state to calculate a new state.

Because the certainty of the measurements is often difficult to measure precisely, it is common to explore the filters behavior in terms of gain. The kalman gain is a function of the relative certainty of the measurements and current state estimate, and is tuned to achieve particular performance. With a high gain, the filter places more weight on the measurements and thus follows them more closely. With low gain, the filter follows the model predictions more closely, smoothing out noise but decreasing the responsiveness. At extremes, a gain of one causes the filter to ignore the state estimate entirely, while a gain of zero causes the measurements to be ignored. When performing the actual calculations for the filter, the state estimate and covariances are coded into matrices to handle the multiple dimensions involved in a single set of calculations. This allows for a representation of linear relationships between different state variables (such as position, velocity, and acceleration) in any of the transition models or covariances.

This capstone design project estimates states of linear systems using a time-varying kalman filter in Simulink. The first method is utilizing a Kalman Filter Block in the Control System Toolbox library to estimate position and velocity of a vehicle based off of noisy position measurements. An example of these noisy position measurements is GPS sensor measurements. Additionally, the plant model in the kalman filter is given time-varying noise properties [10].

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Figure 3.1 shows the diagram for estimating the position and velocity of a vehicle in the east and north directions. It is determined that the vehicle maneuvers without constraints in two-dimensional space. This section of the paper explains the design for a versatile and multi-functional tracking and guidance system used for a vehicle. In Figure 3.1, $x_e(t)$ and $x_n(t)$ represent east and north positions of the vehicle from the origin, $\theta(t)$ represents vehicle orientations from the east, $u_\psi(t)$ represents the steering angle, and $t$ represents the continuous-time variable.
3.3. State Estimation with Kalman Filter

The Simulink model shown in Figure 3.2 (and Appendix B.1) consists of a vehicle model and a kalman filter design.

3.3.3 Vehicle Model

The tracked vehicle is defined by a simple point-mass model:

$$\begin{align*}
\frac{d}{dt} \begin{bmatrix} x_c(t) \\ x_n(t) \\ s(t) \\ \theta(t) \end{bmatrix} &= \begin{bmatrix} s(t) \cos(\theta(t)) \\ s(t) \sin(\theta(t)) \\ (P \frac{ur(t)}{s(t)} - A C_d s(t)^2)/m \\ s(t) \tan(\psi(t))/L \end{bmatrix}
\end{align*}$$

The car model is put into the Simulink model’s subsystem. The design of the Simulink model incorporates two different PI controllers. The purpose of the first PI controller is to track the desired orientation for a car. The second PI controller is used to track the desired speed of a car (as shown in Figure B.4 in Appendix B). The main benefit of the PI controllers in the Simulink Model is that the user can specify multiple different operating modes for the car and test the performance of the kalman filter design.

The vehicle states are:

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The vehicle parameters are:

\[
\begin{align*}
    x_e(t) & \quad \text{East position [m]} \\
    x_n(t) & \quad \text{North position [m]} \\
    s(t) & \quad \text{Speed [m/s]} \\
    \theta(t) & \quad \text{Orientation from east [deg]}
\end{align*}
\]

The control inputs are:

\[
\begin{align*}
    u_t(t) & \quad \text{Throttle position in the range of -1 and 1 [Unitless]} \\
    u_\psi(t) & \quad \text{Steering angle [deg]}
\end{align*}
\]

In this capstone design project, the longitudinal dynamics for the vehicle model neglect the resistance due to tire rolling. Since the important aspect of the project is focused on lateral dynamics, the lateral dynamics of the model accept that a desired steering angle is achieved immediately and neglects the yaw moment of inertia.

### 3.3.4 Kalman Filter Design

For the kalman filter design used in the capstone design project, the kalman filter is utilized as an algorithm to estimate unknown variables that are of interest to the project depending on a linear model. The linear model chosen explains the change of the estimated variables over time in response to the model’s initial conditions, known inputs, and unknown inputs.

The estimated parameters are shown below:

\[
\begin{align*}
    P & = 100000 \quad \text{Peak engine power [W]} \\
    A & = 1 \quad \text{Frontal area [m\textsuperscript{2}]} \\
    C_d & = 0.3 \quad \text{Drag coefficient [Unitless]} \\
    m & = 1250 \quad \text{Vehicle mass [kg]} \\
    L & = 2.5 \quad \text{Wheelbase length [m]}
\end{align*}
\]
The estimated variables are shown below. The $x$ term defines velocities (not the derivative operator) and $n$ is the discrete-time index.

$$
\hat{x}[n] = \begin{bmatrix}
\hat{x}_e[n] \\
\hat{x}_n[n] \\
\hat{x}_e[n] \\
\hat{x}_n[n]
\end{bmatrix}
$$

The form of the model in the Kalman filter design is provided below, where $x$ describes the state vector, $y$ the measurements, $w$ the process noise, and $v$ the measurement noise. It is assumed in this capstone design project that $w$ and $v$ are independent random variables (with a zero-mean). These independent random variables attain known variances.

$$
\begin{align*}
\hat{x}[n + 1] &= A\hat{x}[n] + Gw[n] \\
y[n] &= C\hat{x}[n] + v[n]
\end{align*}
$$

The $A$, $G$, and $C$ matrices are given (where $T_s = 1[s]$):

$$A = \begin{bmatrix}
1 & 0 & T_s & 0 \\
0 & 1 & 0 & T_s \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

$$G = \begin{bmatrix}
T_s/2 & 0 \\
0 & T_s/2 \\
1 & 0 \\
0 & 1
\end{bmatrix}$$

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From estimated north and east velocities, a saturation function, $Q[n]$, is constructed:

$$f_{sat}(z) = \min\left(\max(z, 25), 625\right)$$

$$Q[n] = \begin{bmatrix} 1 + \frac{250}{f_{sat}(\hat{x}_n^E)} & 0 \\ 0 & 1 + \frac{250}{f_{sat}(\hat{x}_n^N)} \end{bmatrix}$$
Chapter 4

Simulation Results

A kalman filter design and vehicle model is used to estimate position and velocity based on noisy position measurements (i.e. GPS sensor). The performance of the kalman filter is tested by simulating a designed scenario where the vehicle makes an intentional U-turn maneuver at a time of 260 seconds.

![U-Turn Maneuver (Curved Trajectory of 180 Degrees)](image)

Figure 4.1: U-Turn Maneuver (Curved Trajectory of 180 Degrees)

4.1 Turning Scenario

- $t = 0$ the vehicle is at $x_e(0) = 0$, $x_n(0) = 0$.

- The vehicle is stationary and heading east. It accelerates to 25m/s. It decelerates to 5m/s at $t = 50s$.

- At $t = 100s$, the vehicle turns toward north and accelerates to 20m/s.
• At \( t = 200\text{s} \), it makes another turn toward west. It accelerates to 25m/s.

• At \( t = 260\text{s} \), it decelerates to 15m/s and makes a constant speed 180 degree U-turn.

### 4.2 Simulation of Kalman Filter

To obtain results, the Simulink model is simulated (see Appendix B and C for documentation). The actual, measured and kalman filter estimates of vehicle position are attained and plotted in Figure 4.2.

![Plot of Actual, Measured, and Kalman Filter Estimates of Vehicle Position](image)

**Figure 4.2:** Plot of Actual, Measured, and Kalman Filter Estimates of Vehicle Position

Shown in Figure 4.3 is a plot of the error between the measured and actual position, in addition to the error among the estimates for the kalman filter and for
the actual position. Figure 4.3 demonstrates that the position measurement and the estimation error are normalized by the amount of data points. Estimates from the kalman filter have approximately 25 percent less of an error than the raw measurements do.

![Image](image.png)

**Figure 4.3:** Top: Error Between Measured and Actual Position, Bottom: Error Between Kalman Filter Estimate and Actual Position

The results in the top plot in Figure 4.4 show the actual velocity in the east direction for the kalman filter estimate. The bottom plot portrays the estimation error. One can see on the error plot in the legend that the east velocity estimation error is normalized by the number of data points.
4.3 Summary of Simulation

The Kalman filter estimates for velocity do track the actual velocity movements accurately. There is an observed decrease in noise levels when the vehicle travels at high velocities. This is consistent with the Q matrix where there is a variance of the process of noise and time-variation (see Section 3.3). The saturation function prohibits Q from becoming too big or too small. The value of 250 for the coefficient in the Q matrix is attained from a least squares fit to the acceleration time data for a vehicle.

Additionally, two large spikes are observed at $t = 50s$ and $t = 200s$. It is determined from the designed vehicle test scenario that the vehicle is going through...
sudden deceleration and a sharp turn during these two times. The changes in velocity at these moments in time are larger than the kalman filter estimates (which are based on the Q matrix input). Following multiple time-steps, the kalman filter estimates come closer to the actual values of the vehicle velocity.
Chapter 5

Conclusions

An analysis of automotive mechatronic control systems is conducted and the four-wheel steering system is considered for increasing the performance of lane-keeping control during a turning maneuver and for reducing the turning radius of a vehicle. This capstone design project looks at the performance of multiple different four-wheel steering systems for implementing design solutions to minimize the turning radius of a vehicle and to improve turning performance during critical maneuvers.

It can be concluded from this project’s research that by applying a steer-by-wire four-wheel steering system to a lane-keeping task such as a turning maneuver, the lateral and yaw dynamics during a turning maneuver may be effectively controlled to have desirable qualities, such as a reduced turning radius, compared to the two-wheel steering system. A four-wheel steering vehicle is capable of reducing the turning radius to a value half of what a two-wheel steering vehicle would produce.

From estimating positions and velocity of a vehicle using the kalman filter design in Simulink, the process noise dynamics of the vehicle model are found to be time-varying. The kalman filter performance is validated through simulation of various vehicle maneuvers (designed test scenario) and generated random measurement noise. The kalman filter improved the position measurements and presented velocity estimates for the vehicle model. Since road curvature is treated as a disturbance that reflects in steady-state error, the kalman filter is a good control algorithm for
estimating vehicle position and velocity, while also improving position measurements and providing velocity estimates for the vehicle model.

## 5.1 Future Work

To provide more results to analyze this project’s initial design problem, an additional control algorithm could be produced with application of a kalman filter for estimating road curvature by detecting only the vehicle’s lateral deviation, without requiring a feedforward of the road curvature. A lane-keeping controller for a turning maneuver would be created with a curvature estimation. The curvature could be included as a state variable in the state-space equation. The steering wheel angle could be calculated to determine feedback gains to minimize a performance index.

Lane-keeping control on a curved roadway could be simulated for advanced simulation results. The simulation could be under the condition that a vehicle runs in a straight-line at a constant speed of 100 km/h for 1s and then enters a curved trajectory (U-turn) with a constant radius of 400m. The future work would analyze the curved lane-keeping responses of three different types of vehicles. Looking at these new simulation plots, additional information could be provided regarding which four-wheel steering control system on a vehicle gives the best performance of the lane-keeping system with less steering effort and less deviation. The chosen four-wheel steering control systems could be chosen based off of the concept generation and selection work done in this capstone design project.

This project could also be extended by focusing on variations in road conditions in order to investigate robustness of proposed control strategy. Work to advance this project includes tracking a custom 4WS robotic platform and implementing a closed-loop control for executing a specific maneuver autonomously in GPS denied environment. Additionally, a petri-net mathematical modeling could be used for the safety critical selection of feasible driving maneuvers. Feasible and infeasible U-turn maneuvers are shown in Figures 5.1 and 5.2.

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5.1. Future Work

Figure 5.1: Feasible U-Turn Maneuver

Figure 5.2: Infeasible U-Turn Maneuver
Bibliography


Appendix A

Concept Generation and Selection Documentation

A.1 Concept Generation

A.1.1 Patent/Literature Search


A target vehicle path is corrected in accordance with the environment surrounding a vehicle during parking or turning. In order to achieve target vehicle path, target wheel speeds are set for respective wheels so as to generate a speed difference in the inside wheel and outside wheel of a turn. The target wheel speed is achieved by controlling braking force and driving force of each wheel. This makes it possible for the vehicle to turn with a smaller turning radius than that generated by a normal steering angle and to cause the vehicle to move accurately along the target vehicle path that avoids any obstacles that are present.

2. Passively Articulated Axle Steering

Implemented by adding a free pivot to one of the vehicle axles. Disadvantage of
single axle steering is that the wheels run in separate tracks when going around curves. Advantages includes mechanical simplicity, relatively low steering power, and moderate maneuverability. Velocity of front wheels is electronically controlled to maintain a desired angle of front axle. I have added a proportional controller to the concept and the output of this controller is subtracted from the front inner wheel and added to the front outer wheel. The output is based on the difference between the desired and actual steering axle angle.

3. Vehicle Speed Sensing Type Four-Wheel Steering (4WS)
Steering angle of rear wheels changes according to the vehicles speed. This system works in three principle phases: negative, neutral, and positive. At low speeds, the rear wheels turn in a direction opposite to the front wheels (negative phase). At high speeds, the rear wheels turn in the same direction as the front wheels (positive phase). At moderate speeds, the rear wheels remain straight (neutral phase). This concept also has an independent suspension model. Instead of using a steering mechanism, a speed sensor and controller are used at the rear wheels. This helps to avoid obstacles and in parking maneuvers where speed is very low. Crab steering is used as a method in the system where given a desired velocity and turn radius, the angular wheel velocity for each forward velocity can be calculated. This requires a controller in each wheel to sense the speed and position of each wheel. This concept assumes that the steering angles of the rear wheels are equal and opposite the front-steering angle.

4. Rack and Pinion Mechanism for Steering the Wheels
A simple rack and pinion mechanism is used for the steering of the vehicle’s wheels. Motion is transferred from the steering wheel to the steering column and then to the pinion. The pinion transfers motion to the front rack for turning of wheels. Another gear is meshed with the pinions and is connected to the shaft. On the other side of shaft, there is a pinion for the rear-steering assembly to provide motion to the rear rack. This method is simple in construction and the transmission of motion is smooth. The disadvantage is that this system

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A.1. Concept Generation

clashes the gears, which causes excessive wear and buckling of the connecting shaft. To fix this problem, this concept generated also includes an intermediate set of gears for switching from minimum turning mode to crab steering mode.

5. Motion of Steering Wheel Fed to Sensor (which is sent to controller to give motion to tie rods for turning of wheels)

The motion of the steering wheel is fed to the sensor and sends information to the controller. The controller sends the output signal to the front hydraulic steering assembly, giving motion to the tie rods for the turning of the wheels. An output signal is transmitted from the controller to the rear-steering assembly. This concept switches from crab steering to a minimum turning assembly mode by altering the signal from controller. The advantage of this concept is that there is less fatigue on the driver because the maximum force is transmitted through the hydraulic system. The motion of the vehicle’s wheels is always accurate and precise. The disadvantage of this system is the complexity of operation. If the system breaks down, it can only be repaired by someone extremely skilled with hydraulic systems. There is also a possibility that some hydraulic fluid leaks.

6. Bell Crank Lever and Toggle Disc System

The motion is transmitted from the steering wheel to the pinion and then to the rack. Next, the motion is transmitted from the rack to the tie rods and then to the front wheels. For the transmission of motion to turn the rear wheels, a bell crank lever and toggle disc is utilized in the concept. The main purpose of the toggle disc is to convert the system from a crab steering system to a minimum turning radius arrangement. The primary advantage of this system is that it is simple in construction and the transmission of motion is smooth. This is an ideal concept for achieving 4-wheel steering, which can reduce the turning radius of a vehicle much more than a 2-wheel steering vehicle.

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A.1.2 TRIZ

1. Concept of Skid SBW 4WS Conversion Kinematics with added point perception sensors

This concept includes stereo IR thermovision cameras, a laser finger, or a panoramic camera with the front axle. The sensor pointing increases the effective horizontal field of view resulting in a more robust autonomous navigation. The advantages of this solution include compactness, less weight, fewer parts, and exhibition of agility from point turning to line driving using only the motions, components, and swept volume needed for straight driving. The disadvantages include skidding, unpredictable mechanical energy requirements, terrain irregularities, nonlinear tire-terrain intersection, failing to achieve aggressive steering, failure to maintain maximum forward thrust during turn, limited motion of wheels to rotation about one axis, failure to work for vehicles that are longer than they are wide, and less accuracy in measurements for control and mechatronic coordination.

Contradiction:
1. Ease of manufacture, less reliability
2. Ease of manufacture, less versatility
3. Simple device complexity, less versatility
4. Simple device complexity, less versatility

2. Concept of Axle Articulated SBW 2WS Conversion Kinematics

This concept includes an added coordination of steer-by-wire four-wheel steering and drive-by-wire four-wheel drive to reduce effects of internal losses due to actuator fighting. The advantage includes less device complexity and the disadvantage includes an increase in the energy (propulsion) used by the moving vehicle.

Contradiction:

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1. Less device complexity, increase in energy used by moving object

3. Solution of Independent explicit SBW 4WS Conversion Kinematics

Includes added mechatronic control for angular velocity of front wheels to maintain a desired angle of the front axle. This allows for the steering configuration to have mechanical simplicity and maintain steering performance. Advantages are an increase in maneuverability of all-terrain vehicles, efficient maneuvering, reduced effect of internal losses due to actuator fighting, aggressive steering due to better dead reckoning, and less slip of wheels, lower power consumption. Disadvantages include actuation complexity, decrease in accuracy of coordination mechatronic control, higher actuator count, part count, and necessary swept volume.

Contradiction:
1. Less power consumption, more device complexity
2. Less power consumption, more loss of energy in system
3. Less power consumption, less measurement accuracy

A.2 Concept Selection

For the concept screening (shown in Figure A.1) the reference concept is chosen as the Independent Explicit SBW 4WS because for vehicles operating under normal circumstances, controlling the lateral dynamics using a SBW 4WS conversion mechatronic control system is desirable and a good benchmark solution. Here, the front and rear steering angles are the two control inputs. This is chosen as a reference because there is a substantial outline of RD works on the mechatronic control of SBW 4WS vehicles that exists and a variety of control structures that have been considered. Additionally, with regard to consumer truck and SUV platforms, many scientists and engineers have been working on a SBW 4WS conversion mechatronic control system that could be offered on select vehicles,
as well as full truck platforms. The considerable aspect is that this improvement in the 4WS SBW will allow larger vehicle platforms with a shorter turning radius for tight maneuvering and better road handling manners, especially under loaded and towing circumstances.

<table>
<thead>
<tr>
<th>Selection Criteria</th>
<th>Concepts</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Independent Explicit SBW 4WS</td>
<td>Coordinated Ackerman SBW 2WS</td>
<td>Frame Articulated SBW 4WS</td>
<td>Skid SBW 4WS</td>
<td>Axle Articulated SBW 2WS</td>
</tr>
<tr>
<td>Maneuverability</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>Low Mechanical complexity</td>
<td>0</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Low Control Complexity</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Low Propulsion power during steering maneuvers</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

| Sum +’s | 1 | 1 | 1 | 3 | 2 |
| Sum −’s | 0 | 1 | 0 | 1 | 1 |
| Net Score | 1 | 0 | 1 | 2 | 1 |
| Rank | 3 | 4 | 1 | 2 |
| Continue? | COMBINE | NO | COMBINE | YES | YES |

Figure A.1: Concept Screening

For the rating scales shown in Figure A.2, all four of the relevant criteria (maneuverability, low mechanical complexity, low control complexity, low propulsion power during steering maneuvers) use a 5-point scale, shown in Figure A.3, because there isn’t too much knowledge about the weights and ratings of each criteria. There are four criteria in total so this method seems to be a better option rather than a finer scale such as the 11-point scale.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Parameter</th>
<th>Independent Explicit SBW 4WS</th>
<th>Coordinated Ackerman SBW 2WS</th>
<th>Frame Articulated SBW 4WS</th>
<th>Skid SBW 4WS</th>
<th>Axle Articulated SBW 2WS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maneuverability</td>
<td>Rating</td>
<td>Good</td>
<td>Fine</td>
<td>Fine</td>
<td>Excellent</td>
<td>Fine</td>
</tr>
<tr>
<td>Low Mechanical Complexity</td>
<td>Rating</td>
<td>Fine</td>
<td>Good</td>
<td>Excellent</td>
<td>Excellent</td>
<td>Excellent</td>
</tr>
<tr>
<td>Low Control Complexity</td>
<td>Rating</td>
<td>Excellent</td>
<td>Passable</td>
<td>Fine</td>
<td>Excellent</td>
<td>Passable</td>
</tr>
<tr>
<td>Low Propulsion Power during Steering Maneuver</td>
<td>Rating</td>
<td>Good</td>
<td>Good</td>
<td>Fine</td>
<td>Passable</td>
<td>Excellent</td>
</tr>
</tbody>
</table>

Figure A.2: Rating Scale with Descriptions
A.2. Concept Selection

Translating into the 5-point scale:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Parameter</th>
<th>Independent</th>
<th>Coordinated</th>
<th>Frame</th>
<th>Skid</th>
<th>Axle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maneuverability</td>
<td>Rating</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Low Mechanical Complexity</td>
<td>Rating</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Low Control Complexity</td>
<td>Rating</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Low Propulsion Power during Steering Maneuver</td>
<td>Rating</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure A.3: Rating Scale with a 5-point Scale

For the weighted criteria for the concept selection, shown in Figure A.4, each criteria is weighted using rank (important number) because the rank of each and how important it is relative to one another based off relative priority, self judgement, and understanding of the design problem and requirements. The weights (sum = 1) are estimated for the chosen criteria.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maneuverability</td>
<td>0.45</td>
</tr>
<tr>
<td>Mechanical Complexity</td>
<td>0.2</td>
</tr>
<tr>
<td>Low Control Complexity</td>
<td>0.25</td>
</tr>
<tr>
<td>Low Propulsion power during steering maneuvers</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Figure A.4: Weighted Criteria

From the Concept Scoring matrix execution, shown in Figure A.5, the best concept is found to be the Skid SBW 4WS.

<table>
<thead>
<tr>
<th>Selection Criteria</th>
<th>Weight</th>
<th>Rating</th>
<th>Weighted Score</th>
<th>Rating</th>
<th>Weighted Score</th>
<th>Rating</th>
<th>Weighted Score</th>
<th>Rating</th>
<th>Weighted Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maneuverability</td>
<td>0.45</td>
<td>2</td>
<td>0.9</td>
<td>1</td>
<td>0.45</td>
<td>1</td>
<td>0.45</td>
<td>3</td>
<td>1.35</td>
</tr>
<tr>
<td>Mechanical Complexity</td>
<td>0.2</td>
<td>1</td>
<td>0.2</td>
<td>2</td>
<td>0.4</td>
<td>3</td>
<td>0.6</td>
<td>3</td>
<td>0.6</td>
</tr>
<tr>
<td>Low Control Complexity</td>
<td>0.25</td>
<td>3</td>
<td>0.75</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.25</td>
<td>3</td>
<td>0.75</td>
</tr>
<tr>
<td>Low Propulsion power during steering maneuvers</td>
<td>0.1</td>
<td>2</td>
<td>0.2</td>
<td>2</td>
<td>0.2</td>
<td>1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

| Total Score Rank | 2.05 | 1.86 | 1.4 | 2.7 | 1.90 |
| Continue? | DEVELOP | NO | NO | YES | NO |

Figure A.5: Concept Scoring

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A.2. Concept Selection

The sensitivity analysis done in Figure A.6 and Figure A.7 is where the weights of the relevant criteria are varied/swapped and require the new sum to still equal 1. The weighted scores and total scores for each concept are recalculated.

![Figure A.6: First Sensitivity Analysis with Weights Swapped](image)

![Figure A.7: Second Sensitivity Analysis with Weights Swapped](image)

Varying the criteria weights affected the scores for each concept as shown above in the second table. In the first table, the concept of the Skid Steer-By-Wire Four-Wheel Steering ranked the highest. However in second table, Axle Articulated Steer-By-Wire Two-Wheel Steering is ranked as the best concept with low propulsion power during steering maneuvers being the highest weighted criteria. By varying the criteria weights, the results were different than in the concept screening where maneuverability is weighted with the highest value. It is shown through this process that the there is a lot of sensitivity of the selection process due to the value chosen for criteria weights. The decision to weigh certain criteria a certain way can largely alter the concept selection process.

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The second best concept from the work in concept scoring is the Independent Explicit Steer-By-Wire Four-Wheel Steering, as shown in Figure A.8. If the individual ratings of this concept are slightly increased and everything else is kept the same, the recalculated weighted scores and total scores for this concept result in it still being only second-best.

![Concept Selection Table](Image)

The conclusion drawn about the sensitivity of the selection process to the concept ratings is that the selection process is not very sensitive to the concept ratings. If the ratings were to be increased even more, it would still increase the total score. Here it went from 2.05 to 2.25 when the individual ratings are increased by 0.2. Based on what is shown in the sensitivity analysis, the strategy that is recommended in order to increase confidence in the output from the concept scoring process is having more of an idea of the accuracy of the weighted value of the selection criteria. With direct input for these weights from a car company that produces automotive mechatronic control systems, more accurate weight values are be taken into accound and the confidence of the output from the concept scoring process will increase.

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Appendix B

Simulink

B.1 Vehicle Model and Kalman Filter

Figure B.1: Simulink Diagram for Kalman Filter Design and Vehicle Model
Figure B.2: Kalman Filter

Figure B.3: Measurement Noise
B.1. Vehicle Model and Kalman Filter

Figure B.4: Speed and Orientation Tracking

Figure B.5: Time-Varying Process Noise Covariance

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Figure B.6: Vehicle Model

Figure B.7: Avoid Division by Zero

Figure B.8: Sample Times for Simulink Kalman Filter

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Appendix C

MATLAB Code

%% State Estimation Using Time-Varying Kalman Filter
%
% This code shows how to estimate states of linear systems using
% time-varying Kalman filters in Simulink.
%
% The Simulink model consists of two main parts: Vehicle model and
% the Kalman filter.
open_system('kalmanfiltersim')
%
% Simulate the Simulink model. Plot the actual, measured and Kalman
% filter estimates of vehicle position.
sim('kalmanfiltersim');
figure;
%
% Plot results and connect data points with a solid line.
plot(x(:,1),x(:,2),'bx',... 
y(:,1),y(:,2),'gd',... 
xhat(:,1),xhat(:,2),'ro',... 
'LineStyle','--');
title('Position');
xlabel('East [m]');
ylabel('North [m]');
legend('Actual','Measured','Kalman filter estimate','Location','Best');
axis tight;
%
% The error between the measured and actual position as well as the error
% between the kalman filter estimate and actual position is:
% East position measurement error [m]
ne = y(:,1)-x(:,1);

% North position measurement error [m]
nx = y(:,2)-x(:,2);

% Kalman filter east position error [m]
e_xe = xhat(:,1)-x(:,1);

% Kalman filter north position error [m]
e_xn = xhat(:,2)-x(:,2);

figure;

% East Position Errors
subplot(2,1,1);
plot(t,ne,'g',t,e_xe,'r');
ylabel('Position Error - East [m]');
xlabel('Time [s]');
legend(sprintf('Meas: %.3f',norm(ne,1)/numel(ne)),
sprintf('Kalman f.: %.3f',norm(e_xe,1)/numel(e_xe)),
axis tight;

% North Position Errors
subplot(2,1,2);
plot(t,nx,t,xhat(:,2)-x(:,2),'r');
ylabel('Position Error - North [m]');
xlabel('Time [s]');
C. MATLAB Code

```matlab
legend(sprintf('Meas: %.3f', norm(n_xn,1)/numel(n_xn)));
legend(sprintf('Kalman f: %.3f', norm(c_xn,1)/numel(c_xn)));
axis tight;

c_ve = xhat(:,3)-x(:,3); % [m/s] Kalman filter east velocity error
e_vn = xhat(:,4)-x(:,4); % [m/s] Kalman filter north velocity error
figure;

% Velocity in east direction and its estimate
subplot(2,1,1);
plot(t,x(:,3),'b',t,xhat(:,3),'r');
ylabel('Velocity - East [m]');
xlabel('Time [s]');
legend('Actual','Kalman filter','Location','Best');
axis tight;
subplot(2,1,2);

% Estimation error
plot(t,e_ve,'r');
ylabel('Velocity Error - East [m]');
xlabel('Time [s]');
legend(sprintf('Kalman filter: %.3f', norm(e_ve,1)/numel(e_ve)));
axis tight;

bdclose('ctrlKalmanNavigationExample');
```

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Appendix D

Collaborations Poster

OPTIMAL VEHICLE CONTROL OF FOUR-WHEEL STEERING
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Background

- Four-wheel steering (4WS) is an advanced control technique which can improve steering characteristics.
- Compared with traditional two-wheel steering (2WS), 4WS moves the front wheels and rear wheels individually when cornering according to vehicle motion status speed, yaw velocity and lateral acceleration.
- 4WS can enhance the handling stability, improve the active safety for a vehicle, and allow a vehicle to turn in a significantly smaller turning radius.

Objectives

- Design a mechatronic control system to improve handling performance of the automotive vehicle under special steering circumstances and to reduce turning radius.
- Understand trade of vehicle dynamics, 4WS mechatronic control systems, steering kinematics, and Kalman Filter.
- Test a function for vehicle trajectory while minimizing turning radius (trajectory based on car’s center of mass, kinematics trajectory curve with vehicle kinematics, steering geometry, desired steering angle, and turning radius).

Design and Test

Kalman Filter Performance by simulating U-Turn closer to center of vehicle. When the turn radius of 900-2000 vehicles for same simulation in wheel heading.

Vehicle Model

- Designed a vehicle representation with inputs position in range of -1 and 1 (parallel) and steering angle (degrees) in Simulink model.

Simulink Model

- Vehicle States:
  - Vehicle Speed
  - Vehicle Acceleration
  - Vehicle Yaw Rate
  - Vehicle Yaw Angle
- Vehicle Parameters:
  - Vehicle Mass
  - Vehicle Length
  - Vehicle Width

Control Inputs

- Serpentine (front wheels)
- Straight (rear wheels)

Output

- Steered angle of front wheel

References