# Modeling and Control of a Wind Turbine as a Distributed Resource

Bongani Malinga, Dr. John E. Sneckenberger, Dr. Ali Feliachi*
Department of Mechanical and Aerospace Engineering
*Lane Dept. of Comp. Science & Electrical Engineering
West Virginia University

***Abstract*** This paper is a part of a research project to study the dynamics and control of distributed resources (DRs) in the deregulated electric power industry[1]. It reflects the need to look at large wind farms as power plants, as a result of the increased penetration of wind energy in the power systems many places in the world. To obtain an optimal integration of high penetration of wind energy in the system, the wind farms must be able to replace other power plants, i.e. be able to participate in the control and stabilization of the power system. If e.g. a large wind farm trips due to a grid fault, the power system will suffer from a severe loss of supply. The main result of the project is a verified model of a wind farm, which can be used to study and improve the power plant characteristics of the wind farm, and a tentative assessment of the power quality of the wind farm, based on simulations with the developed model and verified by measurements.

This research provided a different approach to wind turbine modeling and control design methodology. All the results were in close agreement with results from other studies. The main strategy of the controller was to regulate the rotor angular speed and the power demand to match the required profiles. A simple wind turbine model was linearized about an operating point and it was used to systematically perform trade-off studies between minimization parameters. The robust nature of the PID controller was illustrated and optimal operating conditions were determined.

Continued research illustrates that the optimum wind turbine has not yet been build and most of the remaining work lies in how the wind turbine is controlled. Additional design and control strategy improvements can be expected as experience is gained with wind turbine operations.

***Keywords*:** Wind Turbine, Modeling, Controls, Distributed Resources, Power

## I. Introduction

The main sources of electrical power have been fuel-burning engines, which use the energy from non-renewable fuels to rotate a shaft connected to an electric generator [5]. These systems have seen vast improvements in the areas of efficiency, emissions and controllability because they have always been the primary power sources. The deregulation of electricity in the US has seen rise in research geared towards alternative energy sources. Some of the major sources being investigated include fuel cells, micro-turbines and wind turbines. Wind turbines will be the main focus of this research.

The most special feature about wind turbines is the fact that, unlike other generation systems, the power inflow rate is not controllable. In most power generation systems, the fuel flow rate, or the amount of energy, applied to the generator controls the output voltage and frequency. However, wind speed varies with time and so does the power demand. The fact that one has no control over the energy source input, the unpredictability of wind and the varying power demand are more than enough concerns to justify the need for a controller, which will regulate all the parameters that need to be controlled for a matched operation of the wind turbine.

## II. Wind Turbine Performance

### Wind Power Available

A packet of wind of mass m flowing at upstream speed u in the axial direction (x - direction) of the wind turbine possesses kinetic energy. The power, $P_w$ in the wind is the time derivative of the kinetic energy given by Equation 1 where $A_w$ is the cross-sectional (swept) area of the wind turbine and ρ is the air density. This represents the total power available for extraction.

$$P_w = \left(\frac{1}{2}\rho A_w u^3\right) \qquad (1)$$

As the wind passes over the turbine, the wind will lose power equal to the power extracted by the turbine. Since the upstream cross-sectional area is not physically measurable, the extracted power is usually conveniently expressed in terms of the wind turbine swept area A. Equation 2 shows that a turbine cannot extract more than 59% of the total power in an undisturbed tube of air with the cross-sectional area equal to the wind turbine swept area.

$$P_{ideal} = \frac{1}{2}\rho\left[\frac{8}{9}\left(\frac{2}{3}A_2\right)u_1^3\right] = \frac{16}{27}\left(\frac{1}{2}\rho A_2 u_1^3\right) = 0.59P_w \qquad (2)$$

### Wind Power Extracted

The fraction of power $P_m$ extracted from the available power in the wind $P_w$ by practical turbines is expressed by the coefficient of performance, $C_p$. The power extracted $P_m$ can then be expressed as shown in Equation 3.

[1]The project titled "Integrated Computing, Communication and Distributed Control of Deregulated Electric Power Systems" is sponsored by the USDOE EPSCoR and conducted by West Virginia University and West Virginia University Institute of Technology.

0-7803-7697-8/03/$17.00 © 2003 IEEE.

$$P = C_p P_w = C_p\left(\frac{1}{2}\rho A u^3\right) \quad (3)$$

The value of $C_p$ varies with the wind speed, the rotational speed of the turbine, the rotor pitch angle, and the turbine blade parameters. The tip speed ratio, $\lambda$, is a variable that combines the effects of the rotational speed and the wind speed. It is defined as the ratio between the rectilinear speed of the turbine tip, $\omega R$, and the wind speed u.

If the power performance of a wind turbine rotor is to be evaluated, its $C_p(\lambda,\beta)$ curve might be obtained from the wind turbine manufacturer and a look-up table can be created to evaluate the coefficient of performance for each tip speed ratio and blade pitch angle. If the rotational speed, $\omega$, and pitch angle, $\beta$, are known, ($\omega = \omega_o$ and $\beta = \beta_o$ for a constant rpm fixed pitch rotor), then the mechanical power output, $P_m$, at any upstream wind speed, u, can be found using Equation 2.6.

Sometimes the full $C_p(\lambda,\beta)$ data are not available so that Equation 3 cannot be used directly in power performance evaluations. According to Justus [8], for any operation pitch angle, a good approximation to $C_p$ as a function of the speed is found by using Equations 4a and 4b.

$$C_p = C_{pm}\left[1 - F\left(\frac{u_m}{u} - 1\right)^2 - G\left(\frac{u_m}{u} - 1\right)^3\right] \quad u_c \le u \le u_R \quad (4a)$$

$$C_p = C_{pR}\left[\frac{u_R^3}{u^3}\right] \quad u_R \le u \le u_F \quad (4b)$$

The coefficients F and G can be found using boundary conditions that the coefficient of performance $C_p$ is zero at the cut-in speed, and is $C_{pR}$ at the rated speed.

For each pitch angle, this approximation method requires knowledge of the cut-in speed, $u_c$, which is a function of the rotational moment of inertia of the rotor and the shaft, the maximum coefficient of performance, $C_{pm}$, the rated coefficient of performance, $C_{pR}$, the rated wind speed, $u_R$, the wind speed at which $C_{pm}$ occurs, $u_m$, and the rated power $P_m$ of the wind turbine. Fortunately, $u_c$, $u_R$, $C_{pR}$, $C_{pm}$ and $P_R$ are given for a wind turbine and the coefficients F and G can be found by applying the boundary conditions that $C_p(u_c) = 0$ and $C_p(u_R) = C_{pR}$.

The wind turbine rotor performance can also be evaluated as a function of the coefficient of torque $C_q$. The wind turbine power, $P_w$, is equal to the product of the torque, T and the rotational speed $\omega$. It follows that the torque coefficient, $C_q$, can be related to the power coefficient, $C_P$, through the relation shown in Equation 5.

$$C_p(\lambda,\beta) = \lambda C_q(\lambda,\beta) \quad (5)$$

Therefore, manipulation of the torque coefficient using $\lambda$ and $\beta$ will result in manipulation of the power produced by the turbine.

It is important to recognize the relationship between the aerodynamic torque $T_A$ and the torque coefficient $C_q$. Using Equations 3 and 5, the aerodynamic torque, $T_A$, that turns the rotor shaft is therefore represented by Equation 6.

$$T_A = \frac{1}{2}\rho A R C_q(\lambda,\beta) u^2 \quad (6)$$

## III. Wind Turbine Modeling

In deriving a wind turbine mathematical model, a specific constant-speed, pitch-control wind turbine was selected. One of the key factors is to find values for the constant parameters in the transfer functions representing the wind turbine system at operating conditions. The variation of the coefficient of torque $C_q$ with the pitch angle, $\beta$, and the tip speed ratio, $\lambda$, is highly nonlinear and unique for each wind turbine. It would be useful to obtain some experimental data for the variation of $C_q$ with $\beta$ and $\lambda$ because there are no general linear equations relating these parameters. Therefore, in choosing the wind turbine for the model, it would be better to choose a wind turbine whose experimental data $C_q(\beta,\lambda)$ is available. The geometry and aerodynamic characteristics of the simulated wind turbine resemble those of a Grumman Windstream-33, 10-m diameter, 20 kW turbine. The National Renewable Energy Laboratory's (NREL) National Wind Technology Center modified this wind turbine to operate at variable speeds using blade-pitch regulation [4].

### Wind Turbine Mathematical Model

The wind turbine plant model was divided into two main parts. The first part was the wind turbine, which included a turbine rotor on a low-speed shaft, a gearbox and high-speed shaft. The second part was the electric generator whose input was constant angular rotation from the turbine plant and whose output was electrical power. Figure 1 illustrates the general block diagram of the wind turbine system.

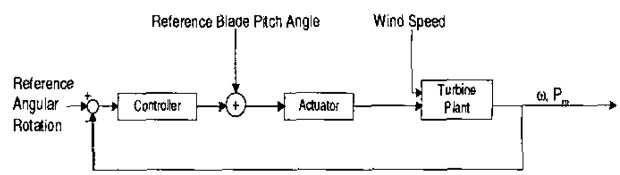


Figure 1: Block diagram of wind turbine system

Although the goal of this control sequence is to maintain a constant angular speed and constant power, $P_m$, only the angular speed is fed back to accommodate the wind speed fluctuations. This is because, controlling the angular speed automatically means that the aerodynamic torque, $T_A$ that causes the rotation, is controlled and hence the extracted mechanical power, $P_m$. This is derived from the fact that these three quantities, $P_m$, $T_A$ and $\omega$ are related by Equation 7.

$$P_m = T_A \omega \quad (7)$$

Therefore controlling $T_A$ and $\omega$ to remain constant will cause the power $P_m$ to remain constant as well. Throughout this paper, only angular speed control will be mentioned because its output profile will be closely related to that of the power Pm.

The aerodynamic torque, $T_A$, must be opposed by an equal and opposite load torque, $T_L$, for the turbine to operate at steady speed. If $T_A$ is greater than $T_L$, the turbine will accelerate, while if $T_A$ is less than $T_L$, the turbine will decelerate. Equation 8 gives this mathematical description, where $J_T$ is the equivalent combined moment of inertia of the rotor, gear reducer and both the low-speed and high-speed shafts.

$$J_T \dot{\omega}_T = T_A - T_L \tag{8}$$

$T_L$ is the mechanical torque necessary to turn the generator and was assumed to be a constant value derived from the wind turbine plant physical properties. The aerodynamic torque, $T_A$, is represented by Equation 6. The power extracted from the wind is shown in the Equation 9:

$$P_A = \frac{1}{2}\rho A C_p(\lambda, \beta) u^3 \tag{9}$$

Equation 9 is non-linear because it is a function of $C_p$, which is highly non-linear. A traditional approach is to design a commonly used linear controller, such as proportional-integral-derivative (PID) and linearize the non-linear turbine dynamics about a specified operating point.

Assuming, $T_A|_{OP} = T_L|_{OP}$, linearization of the Equation 6 results in Equation 10.

$$J_T\dot{\omega} = J_T\frac{\partial\dot{\omega}}{\partial u}\bigg|_{OP}\Delta u + J_T\frac{\partial\dot{\omega}}{\partial\omega}\bigg|_{OP}\Delta\omega + J_T\frac{\partial\dot{\omega}}{\partial\beta}\bigg|_{OP}\Delta\beta \tag{10}$$

Simplifying this equation yields Equation 11.

$$\dot{\omega} = \alpha\Delta u + \gamma\Delta\omega + \delta\Delta\beta \tag{11}$$

In this equation, $\Delta\omega_T$, $\Delta u$ and $\Delta\beta$ represent deviations from the chosen operating point $\omega_{TOP}$, $u_{OP}$ and $\beta_{OP}$. The parameters $\alpha$, $\gamma$ and $\delta$ represent the coefficients.

The parameters $\alpha$, $\gamma$ and $\delta$ represent the wind turbine dynamics at the linearization point. Their quantities depend on the wind speed and the partial derivatives of the coefficient of torque, $C_q$ with respect to $\lambda$ and $\beta$ at the operation point. The magnitudes of $\alpha$ and $\delta$ show the relative weight of the effect wind speed u and the pitch angle $\beta$, respectively, on the wind turbine angular speed. Equation 11 is the linear equation describing the wind turbine dynamics. Applying Laplace transforms and manipulating Equation 11 yields:

$$\Delta\omega = [\alpha\Delta u(s) + \delta\Delta\beta(s)]\frac{1}{s-\gamma} \tag{12}$$

After linearization about the chosen operating point for which the angular rotation speed is $\omega_{TOP}$, the wind speed $u_{OP}$, and the pitch angle is $\beta_{OP}$, Equation 12 is a linear equation that describes the wind turbine dynamics in the Laplace domain. It represents the change in rotor speed output from the wind turbine. The inputs $\Delta u$ and $\Delta\beta$ represent deviations from the chosen operating points..

The wind turbine is therefore represented by the first-order transfer function, $G_p(s)$ shown in Equation 13.

$$G_P(s) = \frac{\Delta\omega(s)}{\alpha\Delta u + \delta\Delta\beta} = \frac{\Delta\omega(s)}{(\Delta T_A / J_T)} = \frac{1}{s-\gamma} \tag{13}$$

**Actuator Model**

Equation 14 represents the actuator dynamics where $\Delta\beta_C(s)$ is the Laplace transform of the input pitch angle change from the controller and $\Delta\beta(s)$ is the Laplace transform of the output pitch angle change.

$$G_A(s) = \frac{\Delta\beta(s)}{\Delta\beta_C(s)} = \frac{k_A/s}{1+k_A/s} = \frac{k_A}{s+k_A} \tag{14}$$

**Controller Model**

The transfer function $G_C(s)$ for the PID controller between the input rotational speed error and the output pitch angle change is:

$$G_C(s) = \frac{\Delta\beta_C(s)}{\Delta\omega(s)} = \frac{k_D s^2 + k_P s + k_I}{s} \tag{15}$$

**Wind Turbine Aerodynamic Data Specifications**

The complete $C_p(\lambda,\beta)$ data was not available for the Grumman Windstream-33 wind turbine but a few key data points relating $C_p$, $\lambda$, and $\beta$, were available and they were sufficient to evaluate the wind turbine performance. The available data was obtained from the study that was conducted in [14]. Based on the available data, the linearized operating point was chosen to be $\beta_{OP} = 9°$, $\lambda_{OP} = 7$, $u_{OP} = 7.5$ ms$^{-1}$ and $C_{pOP} = 0.2$. This operating point was chosen because it represents operation at relatively aerodynamically stable conditions.

The coefficient of performance, $C_p$ was evaluated at the linearized wind turbine operation blade pitch angle, $\beta$, of 9°. The variation of the coefficient of performance with the tip speed ratio was obtained using the method outlined in earlier.

The coefficient of performance as a function of the pitch angle was derived using the three $C_p(\beta)$ data points that were given at the operation (linearization) tip speed ratio of 7. A third-order polynomial was used to fit the data in order to derive the variation of the coefficient of performance, $C_p$, with the pitch angle, $\beta$. The constants at the operation point, the derived $C_p(\lambda,\beta)$ curves and the relation between $C_p$ and $C_q$ were used to calculate the wind turbine dynamic constants $\alpha$, $\delta$ and $\gamma$.

**Electric Generator Model**

For the purpose of this research, the generator was modeled using the Power System Blockset in Simulink [33]. In modeling the synchronous generator, the Matlab Power System Blockset has the mechanical power as its input. The voltage output is controlled through the excitation field voltage $v_{fd}$, which produced the magnetic flux that induces the emf on the coils.

This research focused more on the mechanical control of wind turbines. Also a lot of studies have been done in the modeling and control of synchronous generators [2,12]. Therefore, the existing generator models that are built within the Power System Blockset were used. The modeled synchronous generator was a 150 kW generator, which was extensively studied by Jadric [14].

## IV. Controller design methodology

Gain selection for PID controllers has generally been a trial-and-error process relying on experience and intuition from the field control engineers. A systematic approach to gain selection provides visualization of the potential performance enhancements to the system control. This work presents a methodology for selecting gain values for a PID controller that regulates the rotor speed of a constant-speed wind turbine by adjusting the blade-pitch angle.

Visual inspection of the rotor speed response and the pitch angle response may be used to determine the best combination of $k_P$, $k_I$ and $k_D$ gains to achieve appropriate damping of the system. However, when the third gain is introduced, this trial and error method becomes much more tedious and complicated. This method does not provide the control designer with a feel for the sensitivity of the controller to slight variations in the gain values, and a best possible range of gain values is not easily identified. Therefore a different PID tuning method was used.

**Controller Design: Gain Selection**

The main controller requirement is to compensate the wind speed deviations by changing the pitch angle, β, to keep the rotor angular speed, ω, constant. A Matlab code was written in order to determine the best combination of the controller gains, $k_P$, $k_I$ and $k_D$. The design was based on the minimization of two parameters. The first parameter was the root mean square (RMS) of the error between the actual rotational speed and the desired rotational speed. The root mean square of the error indicates the capability of the controller to reject the wind speed fluctuations. The second parameter was the actuator duty cycle (ADC), which was proposed by Kendall, et al. (1997) as a measure of actuator motion during a simulation run. It is simply the total number of degrees the blades are pitched over the time period of the simulation. In order to prevent over-heating of the hydraulic fluid, this value must remain below a certain value provided by the manufacturer. For each simulation run, these two parameters were analyzed, and both were considered in determining acceptable operating conditions.

In order to systematically determine combinations of the three gains that produce acceptable operating conditions, the simulation was used repeatedly. Each of the gains was varied over a wide region, and the two minimization parameters were analyzed for each run. Contour plots for both parameters were created while the $k_P$, $k_D$ and $k_I$ gains were varied. Trade-off studies between the series of surfaces were performed to determine the region where acceptable operating conditions existed. Lastly, time-series traces of rotational speed and pitch angle for gain combinations within this region were produced to verify acceptable operation.

Figures 2 depicts surface for a fixed value of $k_I = 10$. All of the contour plots indicate wide, flat surfaces for the RMS of the rotational speed error. This surface illustrates that a wide range of gain value combinations may be chosen with similar results. Thus, the controller is robust and relatively insensitive to changes in the values of the gains. However, choosing the most acceptable operating set points for the gains requires a closer examination of the surfaces.

The output angular speed satisfactorily tracks the reference angular speed as shown in Figure 3. This shows the robustness of the controller and the fact that wind turbine operation heavily depends on the control strategy.

Figure 4 shows the wind turbine output torque for the selected gain combination.

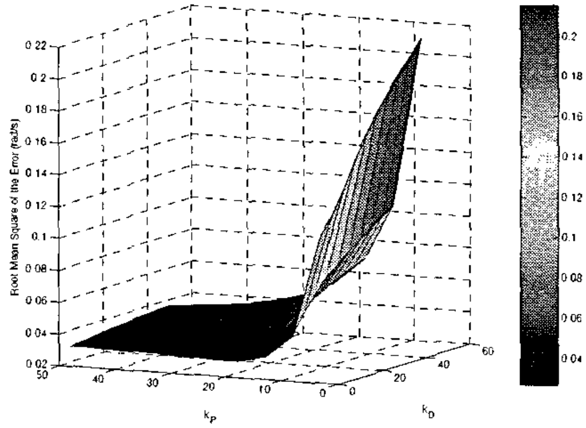


Figure 2: Graph of $k_D$, $k_P$ versus RMS of angular speed error for $k_I = 10$.

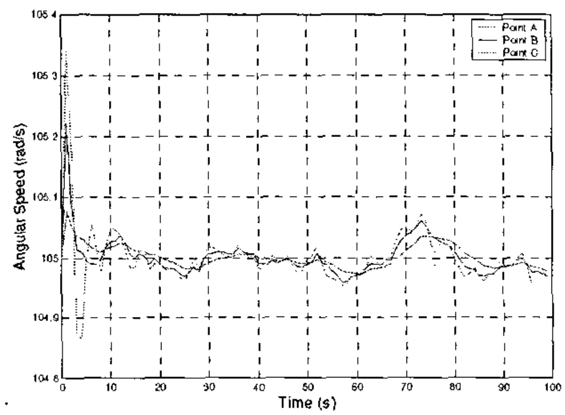


Figure 3: Angular speed for three gain combination

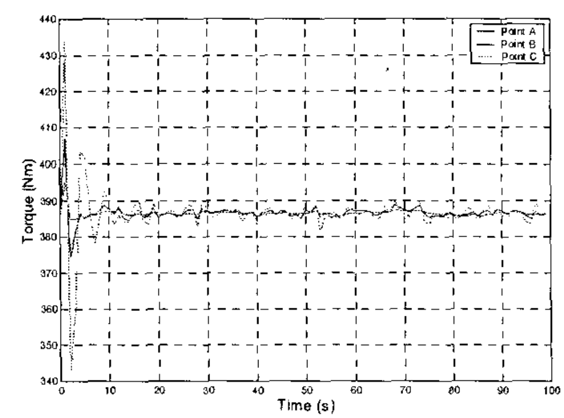


Figure 4: Torque for selected three-gain combination

**Synchronous Generator Control**

The wind turbine system in this research has already been controlled to track the desired angular speed and the mechanical power profile. Therefore, the voltage controls only depended on the exciter of the synchronous generator. The inbuilt excitation system was used for the control of the generator. This excitation system implements an IEEE type 1 synchronous generator voltage regulator combined to an exciter [2].

The control strategy was simpler for the generator because the mechanical power input and the angular rotation of the shaft were already controlled at the wind turbine. Synchronous generators are a common platform for most electricity production. The only difference between generation devices is the reliability and controllability of the mechanical power system. This research has shown that a wind turbine can replace a diesel engine and electric power production will continue satisfactorily, using the same generator. This also shows the extent of wind turbine applicability. Wind turbines have reached a stage where their operation and controls have matched that of traditional electric generators.

## V. Conclusions and Recommendations

### Conclusions

This research contributed a method of wind turbine performance estimation, modeling, linearization and control.

The wind turbine model was derived using a linearization technique coupled with the approximation of the wind turbine dynamics. Given that there was no readily available data for a straightforward data reduction procedure, the method developed by Justus [8] was used and it proved to be successful based on the output profiles that were obtained for all the simulations. All of them were comparable to the profiles of other wind turbines that have been studied [5]. This was a different and useful approach to wind turbine modeling.

The systematic approach to PID-controller design that was used provides a means of visually observing the effect of gain changes on both RMS speed error and actuator duty cycle, which is also tied with the size of the gains and the cost. While these parameters were in opposition by nature, the surfaces permitted selection of gain values that produce favorable results for both of the parameters. This visualization of the effect of gain permits selection of the best possible combination of controller parameters without requiring a lengthy trial-and-error process.

The level of connectivity of wind turbines to the electric grids was reflected by the ease with which the mechanical wind turbine was connected to the synchronous generator without having to apply any special control scheme to the generator. It is the reliability and the controllability of the mechanical power output of the wind turbine that determines its connectivity to the grid.

### Future Research and Recommendations:

The non-linear dynamics simulated with this simple model are easily linearized, but several considerations must be made in order to design a PID controller using a linear model. The optimal region based on the balanced performance of the two minimization parameters shifts with the linearization point selection. Operating point selection for a linear model is critical to obtaining the best possible performance from this highly non-linear system. Therefore, other operating points can be selected and analyzed.

## VI. REFERENCES


***Papers Presented at Conferences:***

[1] Muljadi, E. and Butterfield, C.P.; "Pitch-Controlled Variable-Speed Wind Turbine Generation," Presented at the IEEE Industry Applications Society Annual Meeting, Phoenix, AZ, October. 3-7, 1999.

[2] Muljadi, E., Butterfield, C.P., Sallan, J. and Sanz M.; "Investigation of Self-Excited Induction Generators for Wind Turbine Applications," Presented at the IEEE Industry Applications Society Annual Meeting, Phoenix, AZ, October. 3-7, 1999.

[3] Hinrichsen, E.N.; "Controls for Variable-Pitch Wind Turbine Generators," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-103, No. 4, April 1984, pp 886-892.

[4] Hand, M.M. and Balas M.J.; "Non-Linear and Linear Model Based Controller Design for Variable-Speed Wind Turbines," Presented at the 3rd ASME/JSME Joint Fluids Engineering Conference, San Francisco, CA, July 18-23, 1999.

***Books:***

[5] Johnson, Gary L.; Wind Energy Systems, Englewood Cliffs, NJ: Prentice Hall, 1985.

[6] Le Gourieres, D.; Wind Power Plants: Theory and Design, Pergamon Press Inc, NY: 1982.

[7] Eggleston, D.M. and Stoddard F.S.; Wind Turbine Engineering Design, Van Nostrand Reinhold Company, NY: 1987.

[8] Justus, C.G., Winds and Wind System Performance, The Franklin Institute Press, Philadelphia, PA, 1978.

[9] Nise, Norman S.; Control Systems Engineering, 2nd Edition, Addison-Wesley Publishing Company, 1995

***Technical Reports:***

[10] March, F., Dlott, E.H., Korn, D.H., Madio, F.R., McArthur, R.C. and Vachon, W.A.; "Wind Power for the Electric-Utility Industry", Arthur D. Little Book, Lexington MA, 1982.

[11] Mullane, A., Lightbody, G., Yakamini, R. and Grimes, S.; "The Simulation and Control of a Grid-Connected Variable Speed Wind Turbine", Department of Electrical Engineering, National University of Ireland, Cork, Ireland.

[12] Machowsi, J., Bialek, J., Robak, J. and Bumby, J.; "Excitation Control System for Use with Synchronous Generators.", IEE Procedures in Generation Transmission and Distribution, Volume 145, No. 5, September 1988, pp537 - 546.

***Dissertations:***

[13] Malinga, B.; "Modeling and Control of a Wind Turbine as a Distributed Resource in an Electric Power System," Master's Degree Research, West Virginia University, Morgantown, WV, 2001.

[14] Jadric, I.; "Modeling and Control of a Synchronous Generator with Electric Load," Master's Degree Research, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1998.

***Word Wide Web:***

[15] www.winddata.com


## VII. BIOGRAPHIES


**Dr. John E. (Ed) Sneckenberger** performs teaching, research and service as a professor of mechanical engineering at West Virginia University (WVU), where he directs the **Glass Industry Assistance (GIA) Program** of the **Projects with Industry (PWI) Program.** The Energy Efficiency Program of the **West Virginia Development Office** (WVDO) in Charleston, WV sponsors the GIA Program. Ed has been a faculty member for 35 years and Director of the GIA Program for eight years. Ed was also Director of the **Industrial Energy Efficiency (IEE) Program** of the Projects with Industry (PWI) Program for seven of the past eight years.

**Bongani Malinga** is currently a development engineer at International Truck and Engine Corporation at Melrose Park, IL. His education includes a bachelor's degree in Aerospace Engineering and a master's degree in Mechanical Engineering from West Virginia University. This paper reflects the thesis for his master's degree under the direction of the co-author, John E. Sneckenberger.