ALGORITHMS FOR A SPOT PRICE RESPONDING RESIDENTIAL LOAD CONTROLLER

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Abstract - Increased unbundling of electric utility services has become a major interest in the industry. This paper presents a description of the logic and structure for a set of spot price based algorithms designed for use in residential load control systems. The paper presents the functions to be fulfilled by such a price responding device and describes the end use devices available in residences and the control logic applicable to each. The paper concludes that there is a need to understand customer attitudes and acceptance in the design of the response strategies and in the design of the man machine interface.

Key Words: Demand Side Management, Spot Pricing, Thermal Storage Control, Price Response, Residential Customer Response

1. INTRODUCTION

Unbundling of electric service is a major topic of discussion in today's electric utility industry. One approach to unbundling is to charge the customers for electric energy as a function of the marginal cost of providing the energy (with revenue reconciliation to allow for recovery of capital costs). This leads to the establishment of a spot price based energy marketplace. This concept has been discussed in earlier papers and is presented in detail in Ref. [1]. Ref [2] discusses the relationship between spot prices and other structures such as interruptible rates.

This paper discusses algorithms for a residential load controller designed to operate in a spot price based energy marketplace. The algorithms are specially designed to be implemented in the EPRI supported hardware/software system called the Load Control Emulator. [Ref 3] The Load Control Emulator is a highly versatile, sophisticated tool designed to enable utilities to experiment with a set of demand side management techniques using a single programming system. Use of and response to price based signals is one of the control algorithms available within the emulator.

The algorithms presented in this paper are designed around but not limited to the Load Control Emulator. The presentation includes a wide variety of structures at a variety of levels of sophistication. This enables an individual utility to choose the particular algorithms that the utility feels best match its own needs with those of its customers.

Reference [4] contains a review of load management techniques leading up to the price responding class. A residential price responding energy management system can be viewed as just one component of a home automation system, Ref [5] reviews the state-of-the-art of this embryonic industry. Ref[6] summarizes existing experience with residential local and distributed load controllers. Ref [7] combines a general framework for price responding energy management systems with detailed optimization algorithms for HVAC. The present paper can be viewed as a realization of that framework. Ref [8] presents a planning tool for evaluating the inputs of price responding HVAC control. The Present paper can be viewed as a realization of that framework. Ref [9] and [10] and elsewhere are examples of residential price responding type energy management systems that have actually been implemented in the field.

The explicit subject of the paper is residential energy management. However, many of the concepts and even some specific algorithms presented are equally applicable to the small commercial customer. Price response is of course also applicable to the large commercial and industrial classes. Customers in these classes have also responded to spot price tariff structures. This is, however, beyond the scope of this paper.

Important issues related to a price responding demand side management system which are not discussed in this paper include how to specify the price variations, what class of price variations to offer the various customers, and cost benefit analysis from both the customers' and utility's point of view. Ref [11] addresses these issue in some but not complete detail.

The paper assumes one of two types of price signals are being provided:

- 1 Hour Update: Price changes each hour; specified 5 minutes before the hour.
- 24 Hour Update: Price changes each hour; specified at, say, 3pm of the preceding day for the period 2am to 2am starting the next day.

The price variation each hour depends on overall utility system conditions related to load, weather, contract constraints, equipment outages (generation through distribution) and fuel cost availability. It is assumed that forecasts of future hourly price changes and weather conditions are also provided. Time scales shorter than 1 hour could be implemented if desired.

Section 2 defines the four main functions. Section 3 discusses four stages of implementation sophistication. Section 4 categorizes end use devices. Sections 5 through 8 discuss the four main functions in more detail for each end use category. Section 9 discusses optimization issues. Section 10 provides a summary.

2. FOUR MAIN FUNCTIONS

A complete price responding energy management system has the following four main functions:

- Tactical Control: Provides the real time control of end use devices
- Behavior Modeling: Provides the behavior models that are needed by the Tactical Control
Figures 1 and 2 summarize the couplings between these four main functions and their operating time scales.

Figure 1 shows that the customer both provides information and receives it. The time scales of Figure 2 show that customer inputs are usually slow and infrequent while the outputs to the customer can be rapid.

Tactical Control, the real time heart of the overall system, is divided into two parts as shown in Figure 3. The Control Logic determines the actual signals to be sent to control the end use devices and/or to the customer. The State Estimator determines the on-off status of the various devices and measures the amount of thermal energy in storage, etc. The state estimator is essentially just a bookkeeping system and further discussions on its role are not provided.

There are two types of Tactical Control outputs, control signals to end use devices and information to the customer. The Control Logic can provide automatic control of end use devices with no customer involvement and/or can provide information or recommendations to the customer who then provides manual control of end use devices.

Behavior Modeling provides the basic models (structure of equations and parameter values for those equations) that are needed by Tactical Control. Four types of models are heat transfer models (which enable one to predict temperature given exogenous inputs such as air-conditioner outputs, outside temperature, etc.), hot water usage patterns, daily or weekly "on-time" requirements (such as for defrosting refrigerators and swimming pool filtration) and kWh/usage of individual end use devices (such as dishwashers, etc.).

Behavior Modeling develops these models by statistical averaging, etc. techniques applied to observed behavior over many weeks. The parameter values of the models are allowed to have seasonal changes and/or to adapt to sudden changes in household lifestyles.

The dotted line in Figure 1 implies that customer inputs (e.g. such as when someone is home) might be desired inputs into Behavior Modeling but it is not considered to be necessary.

Usage Diagnostic has no direct input into Tactical Control. Usage Diagnostic provides customer education by furnishing historical end use statistics such as kWh/end use device for the month and $/end use device for the month. An advanced Usage Diagnostic allows the customer to choose from a menu of different types of diagnostics.

Useful information for Usage Diagnostics could conceptually come from Behavior Modeling as shown by the dotted line in Figure 1.

Strategic Planning provides the limits and criteria which are used by Tactical Control; for example prior limits on individual devices above which the device should be cut off. Strategic Planning can be implemented in ways ranging from utility personnel talking with the customer to sophisticated computer customer dialogues of an expert system type. An advanced Strategic Planning logic uses adaptive techniques to learn what the customer really wants by observing what happens as the customer modifies the Strategic Planning logic at various times during the weeks, months and years.

Strategic Planning might receive information directly from Usage Diagnostics as indicated by the dotted line in Figure 1.

3. STAGES OF IMPLEMENTATION

Figure 4 defines four proposed stages of implementation. These are defined in terms of whether a particular function is being done automatically by the energy management system or whether it requires manual intervention of utility personnel. The stages are intended to be indicative of the type of functions that can be included in a process that would begin with testing and continue through implementation.

In Stage 1, only Tactical Control is entirely self-contained within the energy management system while in Stage 4 all four functions are automatic and operate without requiring utility personnel involvement. There are also various levels of sophistication within each of the stages.

The Stage 1 Behavior Modeling has all the basic algorithms needed for the Stages 2, 3 and 4 versions but relies on utility personnel to handle anomalies such as the detection and diagnosis of sensor failure and abrupt changes in customer behavior patterns. The implementation of automatic ways to handle anomalies involves straightforward but quite complicated logic to deal with all the special cases which can occur.

In Stages 1 and 2, Usage Diagnostic outputs are mailed to the customer while in Stages 3 and 4 they are made available to the customer by an in-house computer display system. The basic algorithms used do not change very much but in Stage 4 utility personnel are needed to handle and deal with anomalies and to perform the actual customer interviews.

Stage 4 incorporates the Strategic Planning function directly into the price responding control hardware/software. This is not a straightforward task. We have many ideas but they require more study and eventually field data before we can specify the most appropriate approaches.

4. CATEGORIES OF END USE DEVICES

Residential houses can contain a wide variety of different types of end use devices. The devices are placed into categories so that it is only necessary to discuss sets of Tactical Control Logic for each category. Figure 5 defines the six basic categories of interest.

To understand the "characteristics column" of Figure 5, it is important to realize that:

- An end use device uses electric energy to provide a SERVICE to the customer
- The three basic types of control actions which can be performed are to reschedule usage to different times; to reschedule service (and usage) to different times and to reduce service (and usage).

The first three end use categories of Figure 5 enable rescheduling of usage (i.e. the times at which the electricity is actually consumed) with an acceptable or no impact on the actual service being provided. There are three different categories of reschedulable usage because each one involves a
somewhat different set of control logics. The fourth and fifth categories include devices where it is only possible to reschedule the times at which the service is provided. The last category, Nonreschedulable Appliances, contains end use devices that cannot really be rescheduled either from a usage or a service point of view; hence the only way electricity consumption can be lowered is to reduce the amount of services provided.

5. TACTICAL CONTROL LOGICS

There are many possible Tactical Control Logics and no attempt is made to define the "best". The goal is to specify a large menu of candidate control logics from which a particular utility can choose for further evaluation based on a priori engineering judgments, results of "encounter group" interactions, results of previous field trials, etc. The Tactical Control Logics to be presented range widely in required Level of Sophistication, Sensor Inputs, Control Points, Customer Display Capability, and Type of Customer Response.

The discussion starts with the six end use decision categories of Figure 5.

Define

t: Time index, one hour step
P: Price of electricity during hour t, $/kWh

Thermal Storage: Temperature Controlled

The Thermal Storage category of Figure 5 involves temperature controlled devices. Define

\[ T_{\text{thermo}}(t) \]: Temperature setting of thermostat
\[ T_e(t) \]: "External" temperature, Outside house for HVAC, inside room for water bed, etc.
\[ W(t) \]: Other weather variables such as humidity and solar insulation

There are two basic types of heat transfer models. Dynamic models are differential (difference) equations relating temperature to electric energy usage and exogenous temperature, etc.; accounting for thermal storage. Static models are steady state versions of the dynamic models. Different Tactical Control Logics are used for each type.

Consider first the use of a static model. Define

\[ X_e: \text{Electrical energy (kWh/hour)} \]
\[ \text{required during hour } t \text{ to maintain the controlled temperature at } T_{\text{thermo}}(t) \text{ given } T_e(t), \text{ and, if needed } W(t) \]
\[ $/\text{Hour}: \text{Cost during hour } t \text{ of maintaining controlled temperature at its thermostat setting} \]

Then

\[ \text{$/\text{Hour}} = P_e \cdot X_e \]

Figure 6 summarizes three Tactical Control Logics for the Static Model. The values of Limit, \( T_{\text{Max}} \) and \( T_{\text{Min}} \) come from Strategic Planning and can be time varying. Comments on Figure 6 are:

TSD1: Turns off device when $/Hour exceeds some prespecified Limit. As a safety precaution, allows the device to run if actual temperature \( T(t) \) falls below \( T_{\text{Min}} \) (for heating) or rises above \( T_{\text{Max}} \) (for cooling). Operates by overriding or adjusting the thermostat.

TSD2: Similar to TSD1 except only warns the customer when $/Hour exceeds the Limit. The customer can then exercise control of the device either manually or through the computer. A display of next closest hour when the Limit will not be exceeded is provided to aid the customer's decision process.

TSD3: Same as TSD2 except the future costs $/Hour are provided for the next 24 hours to give the customer more information on which to base a control decision.

Now consider the use of a dynamic model. Define

\[ X_e: \text{Energy (kWh) used during hour } t \text{ to be determined by control logic} \]

\[ \text{Cost Over Time} = \sum_{t=0}^{t+N} P_e \cdot X_e(t) \]

\[ N: \text{Horizon time; determined by character of thermal storage and future price behavior; e.g. 12 to 48 hours} \]

\[ \text{Tradeoff Function: Some prespecified function of } \text{Tradeoff Function: Some prespecified function of } P_e, X_e, \text{ and resulting } T_x, k = t, t+n \text{ which expresses customer's tradeoff preferences between cost and temperature deviations from desired level.} \]

Figure 7 summarizes three Tactical Control Logics for the dynamic model. The values of \( T_{\text{Max}}, T_{\text{Min}}, \text{ and the definition and values for the Tradeoff Function come from Strategic Planning. Comments on Figure 7 are:} \]

TSD1: The \( X_e, k = t, t+n \text{ are chosen by an optimization logic to minimize "Cost Over Time" subject to constraints on allowable temperature deviations.} \]

TSD2: A variation of TSD1 which uses a more sophisticated criterion to determine the optimum \( X_e \).

TSD3: A different variation of TSD1 where the only criteria is to hit a given target temperature at a given target time which are specified by the customer.

Further discussions on the optimization logics themselves are provided in Section 9.

Thermal storage for HVAC is provided by the building (and its contents) and is available by special thermal storage devices such as "hot rocks," or ice. A basic assumption underlying the Tactical Control Logics of Figures 6 and 7 is that temperature is the quantity that determines the "quality of service" being provided to the customer. For the space conditioning HVAC end use, there are other factors such as inside humidity and rate of change of temperature which might be desirable to incorporate.

The HVAC control logics are satisfactory for electrical space heating and compressor driven airconditioners. Heat pumps could require more sophisticated logics.

Most of the airconditioning control logics, should be applied only to the compressor and not to the fans if possible.

Water Heating
Water Heating involves thermal storage and uses thermostats for control. However, it differs from the Thermal Storage category because, under normal operation, it is really the volume of hot water that is being stored and controlled; not its temperature. Hence Water Heating is viewed as a separate category.

The basic control logic exercises control of the lower heating element of a two element heater only. The upper element is always free to operate.

Figure 8 summarizes three Tactical Control Logics for water heating. Comments on Figure 8 are:

WH1: The lower element is allowed to operate only during time intervals during the day of minimum price. Hence the tank is filled up with hot water only once a day.

WH2: A variation on WH1 wherein the customer specifies usage times and the lower element is allowed to operate at the minimum cost time before a specified usage time. If there are N specified usage times, the tank is filled with hot water N times.

WH3: The lower element is turned on and off to minimize Cost Over Time (sum of price times kWh over time) subject to the constraints that a forecasted water pattern usage is met with an acceptable reserve margin.

WH1 and WH2 are simple logics based solely on timing and price. WH3 is a sophisticated optimization logic requiring Behavior Modeling to learn the customer hot water usage pattern.

Periodic Use Requirement

The Periodic Use Requirement category includes defrosting of refrigerators and swimming pool pumps; end use devices that require a certain number of hours of activity per day or week with little care needed as to the exact time.

Figure 9 summarizes two tactical control strategies; comments on Figure 9 are:

PUR1: Given the run time constraints (hours needed per time interval) as obtained from Strategic Planning, cost is minimized over time by running during the hours of minimum price.

PUR2: This is a variation of PUR1 that uses a more sophisticated optimization criteria based on a Strategic Planning provided Tradeoff Function between cost and run times.

Reschedulable Appliances

The Reschedulable Appliance category includes dishwashers, washing machines, and dryers.

Define

\[\text{kWh/Usage}: \text{Electrical energy (kWh) used during a single "average" run cycle of a given appliance}\]

\[\$/\text{Usage}: \text{Cost of a single "average" run cycle}\]

\[\$/\text{Deferred}: \text{Money saved if a single run-cycle is deferred from the present hour t to some future hour (during the next 24 hours) when the price is minimum.}\]

\[\$/\text{Deferred} = (P_t - P_{t+24}) \times \text{kWh/usage}\]

\[P_{\text{min}} = \text{minimum } k = \text{minimum of } P_t \text{ for } t = 0 \text{ to } 24 \text{ of } P_t.\]

Figure 10 summarizes six control logics; values of limits and \(t\) come from Strategic Planning. Comments on Figure 10 are:

RA1: A simple cut off whenever \$/Usage exceeds the prespecified limit.

RA2: Similar to RA1 except that instead of actually preventing usage, the customer is only warned when \$/Usage exceeds Limit. The customer can then, if desired, reschedule usage (either manually or via the computer) based on the information on the next hour when the limit will not be exceeded.

RA3: A variation on RA2 that provides the customer with more information on which to base any rescheduling decisions.

RA4: A variation on RA1 when the cutoff is based on \$/Deferral instead of \$/Usage; i.e. on savings from rescheduling rather than actual cost of operation. Also related to RA3 in that rescheduling information is provided to the customer who may or may not use it.

RA5: A variation on RA4 when the computer automatically reschedules usage to the minimum price time between the present time \(t\) and a prespecified fixed time \(t_{max}\) (e.g. 6 a.m.).

RA6: Instead of cutting off usage when \$/Deferral exceeds limit, the customer is just warned and can reschedule if desired. RA3 and RA6 are similar except RA3 uses \$/Usage while RA6 uses \$/Deferral.

Generally no appliance is cutoff after it has once started its cycle even if the price jumps during the cycle enough to exceed a limit. The exception to this might be a clothes dryer.

The kWh/usage number to be used for dishwashers and washing machines includes the electric energy used to heat the water (assuming it is an electric water heater). A more sophisticated set of Tactical Control Logics would couple the water heating control logics with the dishwasher, washing machine control logics.

Discretionary Activity Devices

Discretionary Activity Devices is a very broad category that encompasses a host of end use devices such as those associated with cooking (stoves, microwaves, toaster ovens, etc.) and hobbies/chores (vacuuming, ham radio, sewing machine, power carpentry tools, electric lawn mower, etc.) These are similar to the Reschedulable Appliances except that the electric energy consumed "per usage" is highly variable and depends on how much cooking is to be done, etc. This makes it more difficult to define "desirable" tactical control logics.

A control logic, like RA3 could be used but it has the undesirable property of being not very "user friendly". The customer is provided with a table of kWh/Usage for a list of typical discretionary activities. The customer must use this table (via Strategic Planning) to define a limit on price \$/kWh; above which a warning is given.

Non-Reschedulable Appliances
Non-reschedulable Appliance include lights, TV, etc. A logic similar to RA2 and RA3 based on $/Hour when device is on could be used except that information on future costs need not be given to the customer because rescheduling is not meaningful. The only control action is do without.

**Display of Total Uncontrolled Costs**

The Tactical Control Logics for Discretionary Activity Devices and Non-Reschedulable Appliances are based on displaying costs for individual functions or usage to the customer. An alternative approach is to display the costs per hour for all uncontrolled end use devices and appliances; lumped together.

**Emergency Load Shedding**

A utility needs to be able to shed load very rapidly, say within seconds due to a sudden unexpected loss of generation, tie line support, etc. If a suitable fast real time communication link with the customer exists, such emergency load shedding can be accomplished by sending a "super-high" price signal; say $10/KWh which is well above the price that would be expected even under major generation capacity constraints of a steady state type. The theory underlying such "dynamic real time" pricing is still evolving. However, implementation without a complete underlying theory is feasible.

The emergency load shedding Tactical Control Logic would inhibit usage of all possible end use devices except lighting and health related devices. Super-high prices would usually be expected to last only a short time; order of minutes. Customers would be educated to view their responses to super high prices as a "public service" to help maintain the integrity of the overall power system; not just as a way to save money. They would, of course, still have the option to override the control on any given end use devices.

6. **Behavior Modeling**

The discussions now turn to how parameter values for the mathematical models used by the individual Tactical Control Logics are obtained.

**Thermal Storage: Temperature Controlled**

Discussions are presented in terms of a dynamic heat transfer model for HVAC. Models for water beds, etc. can be simpler. Detailed considerations of swimming pool heating models have not yet been undertaken.

Many man years of effort have been devoted to development of detailed heat transfer models for residences and their corresponding HVAC systems. Such models require information on the thermal properties of the individual walls, windows, etc. and are often of very high order. In addition, the majority of these models are static in the time dimensions in which we are interested.

The basic approach being considered here is quite different. A relatively simple structure is assumed which includes only the most basic heat storage components in a lumped parameter representation. Statistical data processing techniques are then used to match the model parameters to the observed behavior.

The basic hypothesized physical structure consists of two thermal masses and is summarized using a thermal resistance capacitance network in Fig. 11. The estimation procedure is as follows. First, convert the differential equations into discrete time equivalents containing only directly observed variables. This results in a model with "black box parameters" which are non-linear functions of the physical parameters. Second, estimate the black box parameters by linear regression. Third, estimate the physical parameters by applying weighted least squares techniques to the estimated black box parameters.

The estimated physical parameters are allowed to track (adapt to) long term changes in customer behavior by use of finite memory, sliding window averaging techniques.

The equations for a static model follow from the dynamic equations by assuming all the time derivatives are equal to zero.

Note that Figure 11 represents only the structure itself. Specialized thermal storage devices such as ice or hot rocks are, of course, modeled separately.

**Water Heating**

Define:

\[ D(t,d): \text{Demand for hot water (gallons) during hour } t \text{ of day } d, \quad d = 1 \text{ week day}, \quad d = 2 \text{ Saturday, } s = 3 \text{ Sunday and Holidays} \]

Estimates of the \( D(t,d) \) parameters can be obtained by attaching a flow meter to the tank to measure directly the actual hot water usage or by trying to estimate the actual hot water usage by deducing the flows from indirect measurements of electrical energy usages, etc. The use of indirect deduction methods has advantages from a practical implementation point of view but is more difficult; both conceptually and algorithmically. When a flow meter is available, the estimates of the \( D(t,d) \) parameters are computed using finite memory, sliding window averages over past measured hot water usage for the corresponding hour \( t \) and day \( d \). The finite memory feature allows the parameter estimates to track (adapt to) long term changes in customer usage patterns.

**Periodic Use Requirements**

Two parameters needed to implement the Tactical Control Logics of Fig. 9 are number of hours of usage per time interval and kWh/hr. It is assumed that the number of hours required per time interval are exogenously specified by Strategic Planning. A value for kWh/hr is not really required for PUR1 but may be desirable for PUR2. It can be obtained indirectly from design data or can be the result of averaging of measured behavior.

**Reschedulable Appliances**

The key parameter to be estimated for the Tactical Control Logics of Fig. 10 is kWh/usage. This parameter depends on what type of cycle is used for the particular appliance (e.g. dishwasher; wash only or wash dry). The basic approach is to average the kWh usage over observed usage cycles using a sliding window over either the last month's data or something like the last 20 usages.

For dishwashers and washing machines, the kWh/usage should also include the electric energy used for hot water heating during the cycle itself and that which is lost in the pipes. Use of direct measures of hot water flow rates is tricky because of the possibility of simultaneous use of several appliances and other hot water usages such as showers, etc. Therefore the recommended implementation involves obtaining an approximate estimate of these parameters from published data or controlled tests.

**Discretionary Activity Devices and Non-Reschedule**
Appliances

It is recommended that the necessary parameters be obtained from published data and no attempt at direct measurement or estimation be done.

Modeling of Process Noise

The models discussed so far are deterministic in structure. More accurate models of the processes would include the presence of process noises representing uncertainty; for example in the solar intensity and outside temperature actually impacting the specific behavior for times $t$, $t+1$, etc. Step 3; implement the resulting control for time $t$. Step 4; At time $t+1$ go back to Step 1 and repeat.

A more elegant, less pragmatic approach is to explicitly model the uncertainty as realizations of stochastic processes and then choose a suitable cost function usually based on expectations. Probably the most powerful mathematical tool for this type of stochastic optimization is the use of stochastic dynamic programming. A stochastic dynamic programming algorithm was implemented for a very simple house heat transfer model. This work uncovered several misconceptions which we previously had on the relationship between stochastic and open loop feedback control. It showed us that, under some situations, stochastic dynamic programming can yield a quite different behavior than that obtained from an open loop feedback logic. However, we are not recommending the use of the stochastic optimization for the initial implementation for several reasons. We are not certain how important the differences between the optimal stochastic control and the open loop feedback will be in practice; the use of a stochastic control model requires the use of explicit models for the uncertainty, i.e. process noise; and the stochastic dynamic programming logic used in our investigations is too slow for actual implementation. Research is required to develop a practical approximation.

The third possible approach to handling uncertainty involves the use of known but bounded models for the uncertainty where in only upper and lower limits (i.e. bounds) on the uncertainty are assumed. An explicit probabilistic/stochastic structure is not required. One possible "Mini Max" approach is to choose an electric usage pattern (control) over future times to minimize some cost function, (related to the cost of electric energy) subject to constraints.

Uncertainty

Uncertainties exist in the behavior models and the forecasts of future weather and prices. Three ways to handle uncertainty are: open loop feedback control, stochastic control, and mini-max control.

Open Loop Feedback Control has the major advantage that it is not necessary to model the uncertainties. The basic approach is the following multiple step recursion. Step 1; at time $t$ make the best possible forecast of future prices, weather, heat losses, etc. Step 2; solve a deterministic optimization problem which ignores future uncertainties to get the "optimum" behavior for time $t$, $t+1$, etc. Step 3; implement the resulting control for time $t$. Step 4; At time $t+1$ go back to Step 1 and repeat.

Basic Algorithms to be Used

The deterministic versions of TSO1, TSO2, TSO3, W3 can all be put into a linear programming format (if a nonlinear tradeoff function is used several it is approximated by a piece wise linear function). The size of the resulting LP is not small, but also not necessarily beyond the capabilities of the Load Control Emulator/PC type hardware. A second deterministic algorithm is the two point boundary value iterations of Ref (7). It too is suitable for PC type hardware implementation. However we have developed a third approach which we presently believe is sufficient for most applications.

The algorithm is an iterative one which gives a feasible solution at each step, i.e. always satisfies
all constraints. For TSDI (minimization of HVAC subject to constraint on temperature deviations) the algorithm is basically as follows. Step 1; find an initial feasible solution assuming the spot prices are constant. Step 2; find the future time when the spot price is lowest and apply control (e.g. heating or cooling) until one of the constraints is violated. Step 3; find the time of the next lowest future spot price and repeat until no further improvement occurs.

We have not been able to prove mathematical optimality of the algorithm (except for some special cases) but we believe mathematical optimality is not a necessary or even sufficient condition for practical implementation. The iterative algorithm is well suited to the open loop feedback control approach to uncertainty as the iterations at hour \( t \) can start with the solution obtained at hour \( t \); unless a major change occurs in forecasted prices between hour \( t \) and hour \( t+1 \). The two point boundary value interaction can also make use of a good initial guess. For a more detailed discussion of the structure of this algorithm applied as a simple dynamic model and its conditions of optimality under specific circumstances see Ref [11].

## 10. CHOICE OF IMPLEMENTATION

The preceding algorithms were designed to be implemented in EPRI's Load Control Emulator. However, selected algorithms can be implemented directly using special hardware. A complete discussion of the tradeoffs between the costs and benefits of the utility and the customer is beyond the scope of this paper (and beyond the knowledge of the authors). However, a few simple discussions will be provided to illustrate some of the issues that must be considered when choosing a particular implementation. More extensive discussions of this type can be found in Ref. [11].

### Customer Attitudes

Economic theory says that customers are concerned with the tradeoff between the change in their electric bill and the inconvenience or benefits they receive from changes in their electric use pattern. Such theory is probably valid to some extent. However, residential customers may be more concerned with intangible properties such as how user friendly is the control system. The information provided by Usage Diagnostic might be valued more than the dollars saved.

<table>
<thead>
<tr>
<th>Characteristic of Utility</th>
<th>Resulting Spot Price Behavior</th>
<th>Types of Algorithms to be Implemented</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Fuel, Flat (say None)</td>
<td>None</td>
<td>Simple</td>
</tr>
<tr>
<td>Excess Capacity $0.08/kWh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>One Fuel Flat (0.08/kWh)</td>
<td></td>
<td>Simple</td>
</tr>
<tr>
<td>Capacity Limits A few hours/yr.</td>
<td></td>
<td>Simple</td>
</tr>
<tr>
<td>Sufficient Capacity $0.21 to .15/kWh)</td>
<td></td>
<td>Simple</td>
</tr>
</tbody>
</table>

**TABLE 1** Effect of Utility Characteristics on Choice of Algorithms

### Customer Characteristics

If electric HVAC is the dominant electric usage of the customer, it might be the only device under direct computer control. Control might be restricted to customers with external thermal storage devices that rocks ice.

### Utility Characteristics

The impact of the utility's characteristics on the choice of algorithms is illustrated by Table 1. A simple HVAC control logic for the second case of spot price spikes is to do just enough preheating or precooling to "coast through" the duration of the spike.

### Communications Media Used

Spot pricing is usually associated with direct utility to customer communication via telephone, power line carrier, radio, etc. However, this is not really necessary. For example, consider the second case of Table 1 with price spikes. The times of the spikes could be announced over TV, and in newspapers the previous day (i.e., a simple 24 hour update). If the customer exercised manual control, the only extra hardware needed is a recording demand meter at the house.

### Utility Characteristics

The paper does not contain the "last word" on price responding energy management systems for residential customers. It is only the beginning. A lot remains to be learned. The versatility of the EPRI Load Control Emulator makes it an ideal tool for obtaining answers (and not just for price responding approaches).

The major challenge lies in the area of customer attitudes and acceptance. In particular, the design of automated Strategic Planning and Usage Diagnostic and the associated "man-machine" interface is an unanswered critical area.

There are some specific technical issues which also require more work. For example, the effects of uncertainty on and/or the degree of required sophistication in the optimization algorithm must be explored. Further experience based on field measurements is needed to refine the Behavior Modeling algorithms. Although this paper presents a set of algorithms, past experience says that others will be developed in the future (or perhaps already exist).

There is no best approach. The choice depends on the characteristics and needs of both the utility and the customer.

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**REFERENCES**


Predicted Future Values

Present and Future Values

Present Value

Physical Loads Inside House

Behavior Modeling

Desired Augmentation

Strategic Planning

Constraints

Tactical Signals

Figure 1

Figure 1: Couplings between Four Main Functions
Tactical Control

Behavior Modeling

Usage Diagnostics

Strategic Planning

Continuous

Daily

Monthly

Once At Start Up

Then On Customer Demand

Warning That \$/Hour > Limit

Nearest Next Hour

Controls If Desired

Via Computer

Warning That \$/Hour < Limit

\$/Hour for Max 24 Hours

Figure 2

Operating Time Scales of Four Main Functions

Figure 3

Two Functions of Tactical Control

Stages

1

2

3

4

Tactical Control

Done Entirely by Price Responding

Demand Management Hardware

Software System

Behavior Modeling

Requires Some Involvement of Utility Personnel

Usage Diagnostics

Strategic Planning

Figure 4

Four Stages of Implementation

Figure 5

Six Categories of End Use Devices

Code Number

Control Signals

Displays to

Customers

Customer

Actions

TSS1

None

None

TSS2

Warning

When

Active

TSS3

None

None

Figure 6

Tactical Control Logics:
Thermal Storage, Static Model (TSS)

Figure 7

Tactical Control Logics:
Thermal Storage, Dynamic Model (TSD)
### Table 1: Control Logics for Water Heaters (WH)

<table>
<thead>
<tr>
<th>Code Number</th>
<th>Control Signal Sent</th>
<th>Display to Customer</th>
<th>Customer Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>WH1</td>
<td>Inhibit Lower Element Except During Times of Minimum Price During Day</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>WH2</td>
<td>Inhibit Lower Element Except During Times of Minimum Price Before Specified Usage Times</td>
<td>None</td>
<td>Define Specified Usage Times</td>
</tr>
<tr>
<td>WH3</td>
<td>Minimize Cost Over Time Subject To Reliability Constraints</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

---

### Figure 8: Tactical Control Logics for Water Heaters (WH)

- Only Lower Element Is Controlled

#### Table 2: Control Logics for Periodic Use Requirements (PUR)

<table>
<thead>
<tr>
<th>Code Number</th>
<th>Control Signal Sent</th>
<th>Display to Customer</th>
<th>Customer Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>PUR1</td>
<td>Minimize Cost Over Time Subject To Minimum Run Time Constraint</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>PUR2</td>
<td>Minimize Trade Off Function Between Cost and Run Times</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

---

### Table 3: Control Logics for Reschedulable Appliances (RA)

<table>
<thead>
<tr>
<th>Code Number</th>
<th>Control Signal Sent</th>
<th>Display to Customer</th>
<th>Customer Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA1</td>
<td>Inhibit When $$/Usage &gt; Limit</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>RA2</td>
<td>Warning When $$/Usage &gt; Limit</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>RA3</td>
<td>Warning When $$/Usage &gt; Limit</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>RA4</td>
<td>Warning When $$/Usage &gt; Limit</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>RA5</td>
<td>Warning When $$/Deferred &gt; Limit</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>RA6</td>
<td>Warning When $$/Deferred &gt; Limit</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

---

### Figure 9: Basic Heat Transfer Model Structure

- Temperature (°C)
  - $T_0$: External
  - $T_w$: Exterior Wall Mass
  - $T_a$: Air
  - $T_i$: Internal Wall, Etc. Mass

### Figure 10: Basic Heat Transfer Model Structure

- Thermal Resistances (°C/kW)
  - $R_{rew}$: $R_{ra}$

- Air Infiltration Resistance: (°C/kW)
  - $R_{in}$

- Thermal Masses: (kWH/°C)
  - $C_w$: External Walls
  - $C_i$: Interior Walls, Etc.

- Solar Heating (kW)
  - $Q_{s}(t) = B_s S(t)$
  - $Q_i(t) = B_i S(t)$
  - $S(t)$: Solar Insulation (lumens)

- House Heating (kW)
  - $Q_h(t) = R_h u(t)$
  - $u(t)$: Electric consumption (kW)

---

### Figure 11: Basic Heat Transfer Model Structure