Security Analysis and Optimization

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Invited Paper

An operationally "secure" power system is one with low probability of blackout or equipment damage. The power system control processes needed to maintain a designated security level at minimum operating cost are extremely complicated. They increasingly depend upon on-line computer security analysis and optimization.

This on-line technology is still relatively new, with enormous further potential. Since security and optimality are normally conflicting requirements of power system control, it is inappropriate to treat them separately. Therefore, they are slowly becoming coalesced into a unified hierarchical mathematical problem formulation: one that is, however, far too complex to afford anything but an approximate, near-optimal solution. The practical validity of this unifying trend relies on being able to incorporate all significant security constraints within the process.

The main two current computational tools in this field are contingency analysis and special operations-oriented versions of optimal power flow (OPF). Contingency analysis identifies potential emergencies through extensive "what if?" simulations on the power system network. OPF is a major extension to the conventional dispatch calculation. It can respect system static security limits, and can schedule reactive as well as active power. Moreover, the advanced versions of OPF include or interface with contingency analysis.

This paper reviews present formulations and methods, and tries to point out areas of difficulty that constitute the main challenges for successful practical on-line implementations over the coming years.

I. Introduction

A. General

A key requirement of any modern society is the economic secure operation of its electric power system. Such an important objective naturally demands the use of advanced large-systems analysis, optimization, and control technology. This technology is being incorporated into the economy-security functions of the utility-company Energy Management System (EMS). The relevant security analysis and optimization is the subject of the present review paper.

The security-constrained optimal control of an electric power system generation-transmission network is an extremely difficult task. This difficulty tends to increase with growth in system size, interconnection, and other operating problems. The modern EMS is the power industry's major response to the challenge. This very large and complex hardware-software system is based in the utility company's main control center. It performs extensive on-line monitoring, assessment, and optimizing functions for the network, to forestall or correct operational problems while maintaining economy. The economic and social benefits of these functions are intuitively great, although they are not easy to quantify with any precision.

Ideally, the security calculation results should be dispatched automatically by the EMS, without further burdening the human power-system operator. At present, however, there are very few examples of security-constrained scheduling calculations working in a closed-loop manner. Most installations are initially adopting an interactive approach; the results are presented in the form of data and advice to the operator, who accepts, modifies, or ignores them using his engineering experience. The number of automated implementations will naturally increase in step with the industry's confidence in the reliability of the security control calculations.

Power system economy-security control is still a young field, many of whose practical and even conceptual aspects are yet to be fully worked out. Nevertheless, the references quoted in this review represent a very small percentage of the publications in the area. An attempt has been made to capture as much of the field as possible by quoting the more recent references on a topic, rather than crediting the originators. Many other works are referenced in the general and review papers [1]–[10] and [11]–[22], respectively.

B. Security Concepts and Terminology

The economy-security field is still so evolutionary that a common set of concepts and terminology has not been fully established. This is a frequent source of confusion. Words such as "monitoring," "analysis," and particularly "security" itself have different usages. This paper defines its terms as it goes along. It uses "security" in a very general sense as pertaining to maintenance of supply (i.e., avoidance of loss of load). In certain cases, "control," "scheduling," and "dispatch" are used interchangeably.

Today's on-line security-related functions deal with static "snapshots" of the power system. They have to be executed

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at intervals commensurate with the rate-of-change of system state, which is greatest when network switching occurs. This quasi-static approach is to a great extent the only currently do-able one, since dynamic analysis and optimization are orders of magnitude even more difficult and computationally more time-consuming.

The overall aim of economy–security control is to operate the system at lowest cost, with the guaranteed avoidance or survival of emergency conditions. This of necessity means operating the system as close as possible to its security limits.

A power system is in an emergency condition of varying severity when operating limits are violated. The most severe, least predictable violations result from contingencies. An important part of the security concept, therefore, revolves around the power system's ability to withstand the effects of contingencies. A particular system state can be pronounced secure only with reference to one or more specific contingency cases, and a given set of quantities monitored for violation.

A formal classification of power system security levels is necessary in order to define the relevant EMS functions [2]–[4]. Fig. 1 illustrates the authors' version, in which the arrowed lines represent involuntary transitions between levels 1 to 5 due to contingencies. The classifications, of course, remain valid if the transitions are due to load and generation changes or control actions. The specific inclusion of levels 2 and 4 represents an extension to the more traditional classifications.

The removal of violations from level 4 generally requires EMS-directed "corrective rescheduling" or "remedial action," bringing the system to level 3. Once level 3 has been reached, further EMS-directed "preventive rescheduling" must be performed to return the system to either level 1 or 2, depending on the operational security objectives.

If the power system has reached level 5, load will be lost by automatic local switching control actions or by directives from the control center. In some cases, optimal amounts and locations of the control actions can be calculated.

Levels 1 and 2 in Fig. 1 represent normal power system operation, in the sense of being acceptable operational states. Level 1 has the ideal security: the power system survives any of the relevant contingencies without relying on any post-contingency corrective action. Level 2 is more economical, but it depends on the EMS corrective functions to successfully remove nonsevere violations without loss of load, within a specified period of time. Note that the level-2 concept applies primarily to post-contingency active-power control.

The post-contingency operating limits might be different from their pre-contingency values. Hence, for instance, the long- or medium-term limits at level 4 could be violated, but not the short-term limits. The precise usage will vary from utility to utility.

C. EMS Security Analysis and Optimization Functions

The EMS economy–security functions dealt with in this paper are **Security Assessment** and **Security Control.** The former determines the security level of the system operating state. The latter calculates the appropriate security-constrained scheduling needed to optimally achieve the target security level.

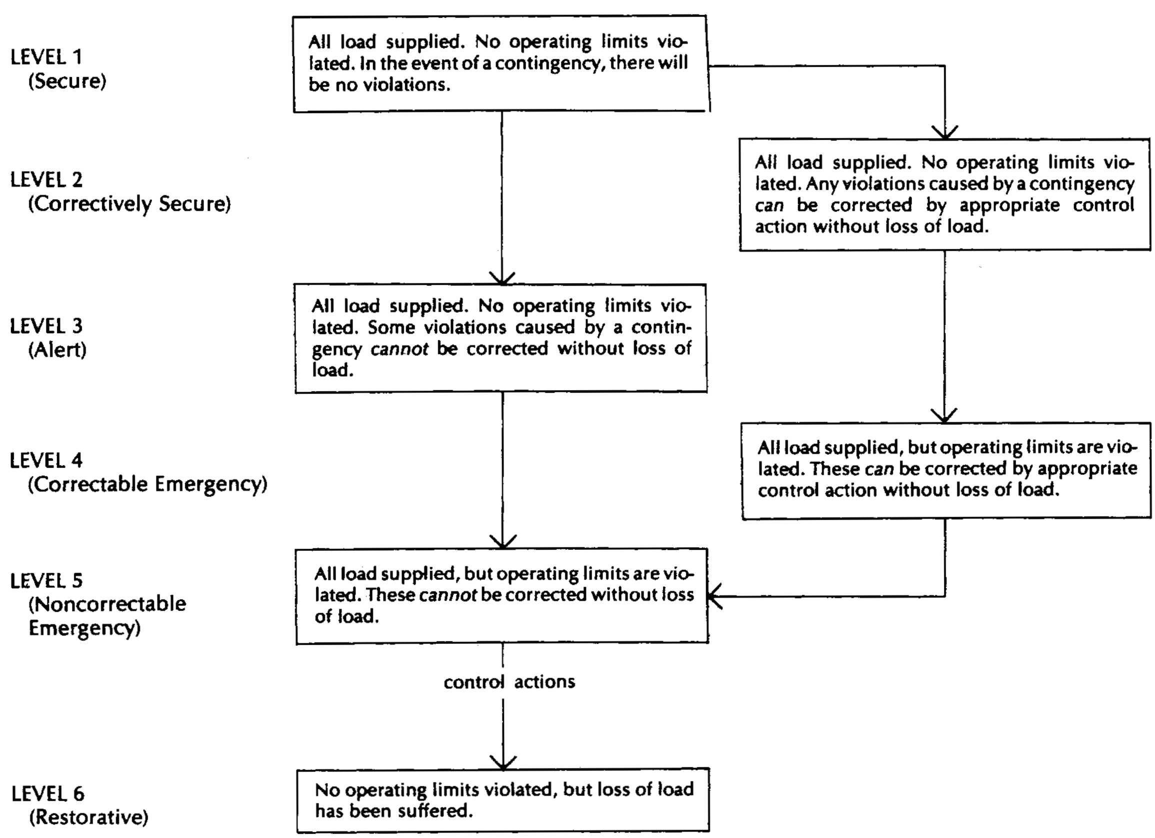

Fig. 1. Power system static security levels.

The security functions in an EMS can be executed in two ways: in "real-time" mode and in "study" mode. In the real-time mode, the static model of the power system under observation is generally derived from the output of the state estimator, with any model of the unobserved (or external) system attached to it [23]. It represents the power system operating state at the current snapshot instant in time. If security monitoring detects real-time violations, security control calculations for immediate implementation are needed. Real-time application functions have a particular need for computing speed and reliability.

In the study mode, the static model of the entire power system usually represents a forecast operating condition, generated automatically from stored historical patterns, recent trend information, and specific knowledge or hypothesis. The main objective of study-mode functions is to plan (over however short a time frame) for future security and optimality of power system operation. Saved previous operating states may also be studied.

II. Security Assessment

As indicated by Fig. 1, the static security level of a power system is characterized by the presence or otherwise of emergency operating conditions (limit violations) in its actual (pre-contingency) or potential (post-contingency) operating states. System security assessment is the process whereby any such violations are detected.

Security assessment has two functions. The first is violation detection in the actual system operating state. In its simplest form, this just entails monitoring actual flows, voltages, etc., and comparing them against prespecified limits. It may also include trend analysis, to apprize the operator of any tendency towards violation. In the case of line and transformer loadings, it may involve on-line refinement of limits, using data such as ambient temperatures, wind factors, and loading history [24].

The second, much more demanding, function of security assessment is contingency analysis. The entire Section II of the paper is devoted to it.

A. Purpose of Contingency Analysis

Contingency analysis is performed on a list of "credible" contingency cases (single or multiple equipment outages). Those contingencies that, if they occurred, would create steady-state emergencies must be identified and ranked in order of severity. The power system operator and/or an automated security-constrained scheduling function can then respond to each insecure contingency case, usually in decreasing order of severity, by:

a) altering the pre-contingency system operating state to mitigate or eliminate the emergency resulting from the contingency, or
b) developing a control strategy that will alleviate the emergency, should it occur, or
c) deciding to do nothing, on the basis that the post-contingency emergency is small and/or very unlikely.

B. Approaches to Contingency Analysis

The traditional concept of contingency analysis is that each contingency should be simulated on the base-case model of the power system. Then the calculated post-contingency operating state is checked for operating-limit violations. In principle, this is straightforward: a routine power-flow solution must be run for each contingency case [25].

In practice, there are three major difficulties. The first is to establish the appropriate power system model. This depends on what post-contingency state is to be represented, and how accurate the results need to be. The second is to determine which contingency cases to consider. The third difficulty is the fact that processing power-flow solutions for large numbers of contingency cases, usually at frequent intervals of time, requires an enormous computational effort. In fact, contingency analysis is often the most time-consuming function in an Energy Management System, with a significant and very unwelcome impact on computer sizing.

The general approach now widely adopted is to separate on-line contingency analysis into three distinct stages: contingency **definition, selection,** and **evaluation.**

Contingency definition is the least time-consuming function. The contingency list to be processed within the EMS comprises those cases whose probability of occurrence is deemed sufficiently high, and is specified by the utility company at system element level. This list may vary with system topology and load, and may include secondary switching (where one contingency results in further contingencies). The list, which is normally large, is automatically translated into electrical network changes: normally injection and/or branch outages.

Contingency selection is the process that offers the greatest potential for computational saving, and has received most development effort. Its purpose is to shorten the original long list of contingencies by eliminating that vast majority of cases having no violations. It invariably uses an approximate (where possible, linear) power system model with appropriate computational techniques, to give relatively rapid but limited-accuracy results. On the basis of these results, the contingency cases are ranked in rough order of severity.

Contingency evaluation using ac power flow is then performed on the successive individual cases in decreasing order of severity. The process is continued up to the point where no post-contingency violations are encountered, or until a maximum number of cases has been covered, or until a specified time has elapsed.

In some cases, contingency selection and evaluation become merged into one process. A single set of simulations on the contingency list can be performed when either a) the accuracy of an approximate selection-type model/solution is adequate throughout, or b) when fast selection cannot be performed reliably, and the more accurate evaluation-type models/solutions are needed throughout.

C. Modeling for Contingency Analysis

Accurate contingency-analysis modeling is the same as for normal power flow, requiring the iterative solution of nonlinear equations and the representation of various control devices. However, in contingency selection and sometimes in contingency evaluation, approximate formulations and solutions are made in return for computational speed.

The power system limits of most interest in contingency analysis are those on branch flows and bus voltages. Both these and other types of operating limits are very "soft," in the sense that their values are neither precise nor to be rigidly enforced. This softness lends justification to the use of limited-accuracy models and solutions.

At the same time, the permissible levels of approximation in contingency analysis are very system- and case-dependent. Far too little work has been performed in the industry on quantifying the tradeoffs between computational speed and validity of results. Speed-versus-accuracy compromises are frequently based largely on engineering judgment and sheer expediency.

The most fundamental approximate power-flow model is the familiar Newton–Jacobian matrix equation [26], representing linearization about a given operating point:

$$\begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = - \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}. \tag{1}$$

In the past, considerably more emphasis has been placed on branch flows than on bus voltages. This has given rise to the very extensive use of linearized active-power models. The original dc power-flow model is too dubious in accuracy for use on most power systems. Its incremental version is normally preferred. It can be expressed in "fast-decoupled" power-flow form [27] as

$$B' \cdot \Delta\theta = \Delta P \tag{2}$$

where matrix B' is a symmetric approximation to the unsymmetric submatrix H above. Alternatives to B', including H itself, do not seem to provide generally superior stand-alone linear P–θ models.

The use of an active-power model makes the assumption that voltages and reactive flows change very little after a contingency, and that the latter are relatively small. This assumption is most valid for strong high-voltage transmission systems, where branch R/X ratios are small. The monitored portions of EMS power system network models have tended to fall into this category. As EMS network models become extended into parts of the lower voltage system, the number of exceptions is bound to increase.

Generator outage contingencies tend to place the greatest strain on the active-power model validity. In addition, the utility has to prespecify whether it wants to monitor post-contingency "steady-state" conditions immediately after the outage (system inertial response), or after the automatic controls (governor, AGC, economic dispatch) have responded. Depending on the decision, different participation factors are used to allocate the lost MW generation among the remaining units. The same applies to load outages, and network islanding involving reallocation of generation to restore the power balance.

In some power systems, particularly those where active-power transfers are limited by voltage drops, emphasis on contingent voltage monitoring is now increasing considerably. The scope for fast approximate modeling is much less than in the active-power case. The reactive problem tends to be more nonlinear, and voltages are strongly influenced by active-power flows. Voltage/VAR counterparts of (2) are available

$$B'' \cdot \Delta V = \Delta Q/V \tag{3}$$

where B'' is an approximation to the Jacobian submatrix L from (1), or even L itself [27]–[30]. Such an equation is rarely acceptable as a stand-alone linearized model for the large perturbations caused by contingencies. It is necessary to evaluate ΔQ and possibly B'' using the post-contingency voltage angles obtained from the active-power model. Representing automatic voltage/VAR controls can be very important. In particular, failure to model generator VAR limiting can lead to extremely unreliable voltage values.

In strong, relatively linear high-voltage power systems, the coupled Jacobian model of (1) might be adequate for approximate results. On the other hand, it is usually less accurate and more time-consuming than (2) with (3), and is rarely used.

D. Contingency Selection

The two main approaches to contingency selection are as follows:

a) **Direct Methods.** These involve screening and direct ranking of the contingency cases. Screening involves the fast approximate power-flow simulation of each contingency case. By monitoring the appropriate post-contingent quantities (flows, voltages), the case's severity can be quantified directly in some heuristic manner for ranking purposes. The severity measure is often a single number, the Severity or Performance Index.

b) **Indirect Methods.** These produce the values of the contingency-case severity indices for ranking, without calculating the monitored contingent quantities.

Much of the development work on contingency selection has been performed for the active-power problem, using dc power-flow-type models. Although many of the methods have employed the typically inadequate nonincremental-dc version, incremental-dc model versions of these methods can in most cases be derived.

E. Active-Power Contingency Screening

The following subsections deal with a series of related methods for screening of contingent branch overloads. Many of the techniques introduced here also apply to the other areas of contingency analysis.

The Inverse Matrix Modification Lemma (IMML) is used explicitly or implicitly throughout the contingency analysis field. Specific versions have been called "compensation" methods [31], [32]. The IMML provides a rapid way of calculating the effects of network changes due to contingencies, without reconstructing and refactorizing or inverting the base-case network matrix.

The independent vector ΔP in (2) is very sparse for branch outages. The IMML is particularly convenient and fast for these cases, which have been duly emphasized in the literature. This should not obscure the fact that generator and bus outages and, in some systems, bus splitting are very important contingency cases, which can also be handled by IMML but not so efficiently.

Contingency analysis methods are able to take considerable advantage of modern sparsity techniques [33]–[35]. Among these are sparse vector solutions, in which fast-forward and fast-backward substitution avoid processing many of the elements of the sparse matrix triangular factors.

a) The most basic active-power screening method explicitly inverts matrix B' in (2) [36]. The best complete inversion method seems to be one that successively generates the upper triangular inverse columns in reverse order using fast-forward and normal backward substitution on the sparse triangular factors. The effort of obtaining the non-sparse inverse grows by little more than the square of the number of network buses.

It is also possible to economize by obtaining only local solutions, by calculating the inverse elements in the vicinities of the contingencies. These elements may be obtained using fast-backward substitution. Alternatively, the desired buses can be ordered last (at some sacrifice of factor sparsity). A problem with local solutions is to know how far from the contingency its effects propagate. Some form of *a priori* sensitivity analysis might be employed.

Once the matrix inverse has been obtained, the IMML permits the rapid calculation of the changes $\Delta\theta$ in (2) due to a single or multiple branch outage. For example, the outage of a branch between buses i and k requires not much more than the subtraction of columns i and k of the inverse. The resulting vector is sometimes called the compensation vector. The changes in the monitored branch MW flows are then calculated from $\Delta\theta$, and compared against limits derived from the branch ratings in amperes, megavolt amperes, etc. In practice, it is more economical to pre-process the base case, transforming each branch flow limit into limits on the change in θ across the branch.

b) A popular very minor variant of the above derives "distribution factors" from the matrix inverse elements [13], [37], [38]. Following a single-branch outage, the flow change in any other branch is given by its distribution factor multiplied by the pre-contingency flow in the outaged branch. The method generalizes to more complex contingencies. At least one implementation uses the complex Y matrix instead of B'; but the basis for this approximation is not too clear.

c) The expensive component of the active-power screening process is the matrix inversion. Moreover, the inverse must be updated or recalculated each time the network toplogy changes. When the number of contingency cases is not large relative to the network size, or if storage is a problem, it may not be economical to precompute the inverse. Instead, the above-mentioned compensation vector can be generated only as and when needed for each contingency case. Fast-forward and normal or fast-backward substitution on the matrix sparse triangular factors is used, depending on whether the entire system is to be monitored. As before, this provides $\Delta\theta$, from which overload monitoring is performed.

d) Theoretical and practical work has been undertaken to define and compute steady-state security regions [39]–[41]. Following these lines, a reduction of up to half the overall work for branch-outage cases in method c) above can be obtained with a simple but nonobvious modification [42]. First, those relatively few elements of the compensation vector that are needed to obtain the bus voltage angle change δ around the terminals of the outaged branch are calculated by fast-forward and fast-backward substitution on the matrix factors. The rest of the network as seen from these buses acts as a voltage divider of δ, and it is reasoned that the angle change across any branch cannot be larger than δ. Applying this principle, it is now only necessary to compare δ with the pre-prepared maximum angle change for each branch, in order to detect and eliminate contingency cases with no overloads.

e) Sparse factor updating [34], [51] can be competitive for complicated multiple-outage cases that require a large number of compensation vectors or the creation of new buses by bus splitting. In this technique, the base-case factors become modified, therefore an extra copy of them must be stored.

f) A technique that can be used in conjunction with the previous methods is to eliminate the unmonitored parts of the network [43], [44]. Elimination is beneficial only while it decreases the "computational size" of the network. Size can be measured in various ways. It is a function of the reduced network's number of buses, branches, and network-matrix factor elements. Elimination of all the unwanted buses is frequently self-defeating, since an enormous number of equivalent branches can be produced when the number of boundary buses is large. An alternative approach [45] prematurely terminates the reduction process, thereby retaining some unneeded buses, as soon as the size of the reduced network starts to increase. A more recent approach [46] also retains unneeded buses, ensuring the sparsity of the reduced matrix factors. With all reduction methods, high-impedance equivalent branches can be discarded with little loss of accuracy. The less sparse reductions tend to have many such branches.

g) Another noteworthy approach introduces "concentric relaxation" [47], which is outlined as follows. The idea is that the algorithm should adaptively determine how much of the system in the vicinity of the contingency needs to be solved. At the beginning, the voltages of the second-neighborhood buses around the contingency are assumed fixed, and the first-neighbor voltages are solved by an iterative Gauss–Seidel technique. The flows into the second-neighbor buses are then calculated and compared with their pre-contingency values. If there is sufficient discrepancy, the second-neighbor buses are included in the local subsystem of interest, keeping the third-neighbor bus voltages fixed at their pre-contingency values. The enlarged subsystem is then solved iteratively. The enlargement process is repeated successively until acceptably unchanged boundary flows have been arrived at.

h) A few power systems exhibit such nonlinearity that the accuracy of the incremental dc power-flow model is inadequate even for active-power screening. In these cases, it is necessary to revert to an ac approach of the complete contingency-evaluation type. Some of the above techniques, such as sparse vector methods, reduction, and concentric relaxation can still be used.

F. Reactive-Power Contingency Screening

Since no stand-alone linearized voltage/VAR network models are valid for large-perturbation analysis, there are no direct counterparts to the active-power screening methods of the last section. Even when simulating a contingency approximately, it becomes necessary to include the change in active-power conditions in calculating the change in voltages. Nevertheless, this coupling is not necessarily a great burden, since active-power screening will generally accompany reactive-power screening anyway.

a) The most straightforward approach is to first calculate the bus angular changes for the contingency case by an active-power screening technique. The updated angles are then used in calculating the voltage-normalized reactive mismatches $\Delta Q/V$ in (3).

Equation (3) is then solved, using the symmetric factors of the base-case matrix B'', and including the effects of the contingency by the IMML (compensation) or factor updating. The whole process is effectively a one-iteration fast-decoupled power flow [27]. Unsymmetric substitutes for B'', such as the Jacobian submatrix, might give improved accuracy at extra computational cost.

Such a one-iteration solution has been used for reactive-power screening, and it may be adequate on certain power systems. However, it cannot be regarded as generally reliable, because of problem nonlinearity and/or failure to represent voltage-control features. Of these controls, VAR limiting is the most clearly essential. It can be implemented following the solution of (3) using factor updating methods.

b) Some of the techniques described for the active-power problem may be used to decrease the computing effort of solving (3). The scope is more limited, however, because the independent vector $\Delta Q/V$ is not sparse. Network reduction is one of the available techniques. A reduction that preserves some of the reactive response of the eliminated system should be employed. Among equivalents in normal power-flow model form, Extended Ward and Ward-PV are currently favored [43].

c) It is also possible to obtain solutions only in the vicinity of each contingency. Sparse vector methods offer one approach. Another requires the inverse elements of B'' for the buses of interest. Then a nonsparse but small subset of (3) can be solved in the vicinity of the contingency, with local generator VAR limits modeled where necessary. As in the active-power problem, a difficulty is knowing *a priori* how far the contingency's effects propagate in the system.

d) An approach to the last mentioned difficulty is concentric relaxation, as outlined in Section II-Eg) above.

G. Direct Severity Ranking

Given a numerical measure for the severity of each contingency case, direct ranking of the cases in order is a relatively simple process. There is a wide choice of heuristic definitions for the Severity Index. A popular one is

$$\sum w_i(x_i/X_i)^{2m} \tag{4}$$

where x is a monitored quantity such as a branch flow or voltage, X is its upper limit, and m is a positive integer. A small modification is made to accommodate lower limits also, where needed. Large violations make a large contribution to the value of the index.

The choice $m = 1$ is often found to be very unsatisfactory, since it fails to give sufficient discrimination. For example, the index for a case with many small violations can be comparable in value to the index for a case with one huge violation. By most operational standards, the latter case is much more severe. This lack of discrimination has been termed the masking effect. It can be improved by increasing m to 2 or higher.

Since the monitored quantities x have been explicitly evaluated by the screening process, the severity index does not need to be a pure algebraic formula [49]. For instance, the index might incorporate the largest violation, a non-analytic quantity.

H. Indirect Contingency Selection

Indirect ranking is a special, mathematically elegant, but restrictive approach to contingency selection. Useful methods can be derived only for specific power system models and severity indices. The approach does not explicitly calculate individual monitored quantities. It only produces the numerical value of the severity index for each contingency case, to be ranked in decreasing order, as usual. The sole attraction is its spectacular speed in calculating these indices.

There have been various different algorithmic derivations based on a linear power-flow model [50]–[54]. The currently most accepted derivations use the familiar IMML, in conjunction with a severity index of the form (4) above. Research efforts have refined the early versions of the approach, by eliminating their approximations to the dc power flow and by catering for multiple outage cases including generators. Unfortunately, however, it has not yet been possible to produce usable versions with severity-index exponents m larger than unity. Thus the approach frequently gives unsatisfactory discrimination between contingency-case severities. This masking problem can be somewhat reduced by applying the approach separately to different subnetworks of the power system, but with correspondingly reduced computing speed advantage.

For voltage-severity ranking, applications of the same approach to stand-alone linear Q–V models as in (3) are predictably unsatisfactory, in view of the inadequacy of those models for large perturbation analysis. It might be possible to derive versions using the coupled model of (1), but the results are still in danger of being unreliable without generator VAR limit representation.

The future prospects for the indirect approach do not seem promising, unless breakthroughs occur that enable higher order severity indices to be used while preserving computational efficiency.

I. Contingency Evaluation

The selection process ranks the contingency cases in decreasing order of estimated limit violation severity. As previously described, contingency evaluation then takes place on successive cases in the ranked list, terminating when there are no more violations or when a preset number of cases has been processed, or when a certain time has elapsed. Evaluation of a case typically involves a conventional fast-decoupled power flow solution, using IMML (compensation) techniques and perhaps factor updating on the base-case matrices B' and B''. Convergence to high accuracy is rarely needed.

Most screening methods calculate the post-contingency bus voltage angles. Some also calculate their magnitudes. Fast-access storage permitting, these values and the compensation vector(s) (if available), may be saved to start the corresponding evaluation solutions later.

When contingency screening and evaluation both use the fast-decoupled power flow, the selection and evaluation solutions can sometimes be merged together. For each contingency case, the first half-iteration (MW monitoring only) or more of the fast-decoupled power flow is performed, and then the limit violations are monitored. If there are no violations, the case is abandoned. Otherwise, evaluation is performed merely by continuing iteration to higher accuracy. Severity ranking of cases takes place at the very end. The main drawback of such a scheme is the unpredictability of the time taken to find the most severe contingency cases.

J. Concluding Remarks on Contingency Analysis

The foremost challenge in EMS contingency analysis is the excessive number crunching required to process a large number of cases on a large power system at frequent intervals of time. This is becoming the most serious computational bottleneck in the application functions of an EMS. The problem becomes even more critical when it is realized that contingency-constrained OPF usually needs to *iterate* with contingency analysis.

The chief approach for both contingency selection and evaluation is still direct power-flow simulation. Major approximations aimed at enhancing computing speed are prevalent in the selection process. These approximations are not always consistent with reliable security classifications. The problem is by far the greatest for voltage/VAR contingency conditions. It appears that the general trend must move in the direction of less inaccurate simulations. This will further increase the considerable computing load imposed by contingency analysis, and the sizes and costs of control center computers.

Unless some radically different contingency-analysis approaches emerge, potential for major improvements in the best existing analysis and computing methods seems rather limited. In fact, the pressures to meet EMS contingency analysis timing specifications may already have resulted in some oversophistication, in the sense that certain methods unduly sacrifice accuracy and flexibility in return for speed.

Today and increasingly in the future, the tradeoffs tend to favor not only faster software and methods, but also hardware exploitation. The modern EMS is tending towards more open architectures that permit the easy connection of auxiliary computing devices, on to which self-contained but computation-intensive calculations can be downloaded. Contingency analysis has the ideal characteristics for distributed processing. The separate cases in the contingency list can easily be shared between several or many small, simple, powerful, and very inexpensive processors, which are now becoming commercially available. Parallel, as opposed to distributed, processing seems ultimately less suitable and cost-effective for such an application.

III. Security-Constrained Optimal Scheduling

A. Introduction

This part of the paper deals with the EMS functions that optimally schedule the system controls, constrained by the network power flows and system operating limits. By definition these scheduling functions all fall into the generic category "optimal power flow" (OPF). However, they take a variety of different forms which, due to the absence of common terminology, are referred to within the industry by a range of different names. Unit commitment and hydro scheduling are beyond the scope of this paper, and are not included.

The purpose of an on-line OPF function is to schedule the power system controls to achieve operation at a desired security level (normally level 1, 2, or 3, or very exceptionally 6, in Fig. 1), while optimizing an objective function such as cost of operation. Security levels 1 and 2 are "contingency constrained." The new schedule may take system operation from one security level to another, or it may restore optimality at an already achieved security level.

Any specific on-line OPF scheduling function, then, is designed to:

- operate in real time or study mode,
- schedule active- or reactive-power controls or both,
- achieve a defined security level,
- minimize a defined operational objective.

Most OPF computer programs are able to perform more than one specific function: for instance, they might handle various security levels, several objectives, and possibly both real-time and study modes. However, as described later, there is no single solution method that is well-suited to all on-line OPF problems.

In real time, the emergency state represented by level 4 in Fig. 1 will typically first require corrective rescheduling to bring it to level 3. Then contingency-constrained scheduling can be undertaken to bring the system to the more acceptable security represented by levels 1 or 2, as appropriate. In real-time or study mode, the choice between levels 1 and 2 depends on the utility's overall economy-versus-security policy, and its means for applying corrective real-time control.

Most security-constrained scheduling calculations are being designed to communicate interactively with the power system operator, in both real-time and study modes. In the real-time mode, the calculated schedule, once accepted, may be implemented manually or where possible automatically: generations and interchanges can be fed into the ED/AGC subsystem, while other controls are handled by the appropriate supervisory control mechanisms.

The ultimate real-time goal is to have the security-constrained scheduling calculation initiated, completed, and dispatched to the power system entirely automatically, without human intervention. Some experienced utilities insist that this is the only effective mode of use. There are few examples of closed-loop implementation presently in routine operation, and then only in simplified forms. Nevertheless, the feature is being designed as an option in many modern energy management systems. The utility company will be able to switch over to closed-loop execution when enough confidence has been gained in the reliability and accuracy of the security-constrained scheduling functions.

B. OPF Problem Formulation—Generalities

Optimal power flow is formulated mathematically as a general constrained optimization problem thus

Minimize

$$f(u, x) \tag{5a}$$

subject to

$$g(u, x) = 0 \tag{5b}$$

and

$$h(u, x) \geqslant 0 \tag{5c}$$

where u is the set of controllable quantities in the system, and x is the set of dependent variables. Objective function (5a) is scalar. Equalities (5b) are the conventional power flow equations, occasionally augmented by a few special equality constraints. Inequalities (5c) are the limits on the control variables u, and the operating limits on the power system. The former are usually treated as "hard," and the latter may be regarded as "soft" (imprecisely known, and for which small violations are tolerable).

At this stage of the paper, it is convenient to regard the variables u and x in (5) as belonging to a single power system operating state: either pre- or post-contingency. The specific treatment of contingency constraints is deferred until later, in Sections III-O to III-R.

The original "classical" OPF formulations were pioneered by Carpentier [55] and Dommel and Tinney [56]. Much of the OPF research since then has addressed similar formulations, without considering the additional requirements needed for practical on-line applications. There are several reasons for this. One is that OPF problems are mathematically and computationally very different, and are still stretching applied optimization technology to its limits. It is not easy to investigate complicated practical OPF fomulations with small-scale research. Another is that up to now very little industrial application of on-line OPF has taken place, with a consequent lack of iteration between users, researchers, and software developers on the utilities' detailed practical requirements. A third reason is that since OPF also has major applications in system planning, there has been a tendency to focus on the common subset formulation.

The real-life problem is not in the continuous smooth form associated with text-book nonlinear programming [18]. All types of power system devices have to be accommodated. Many control variables move in discrete steps, which are sometimes very large and/or irregular. Some objective functions and even constraints are not algebraic or differentiable. Multiple solutions are likely to exist: on present evidence, the tendency towards multiple local optima seems to be greatest when there are many reactive-power controls in network loops.

Unfortunately, there are no solution approaches that can handle these types of problems efficiently in a reasonably rigorous form. Universal practice is, therefore, to approximate the formulation and modeling to make the problem more solvable with available optimization techniques.

Compromises between OPF formulation and algorithm are inescapable. Even so, no individual solution approach offers a combination of flexibility, speed, reliability, and other desirable properties for all types of on-line OPF problems.

In real-time mode, OPF formulations and their corresponding solution approaches are strongly influenced by the need to implement a calculated control schedule immediately. Computational speed and reliability are, of course, at an absolute premium. In many cases it is difficult, undesirable, or impossible to move large numbers of system controls all at the same time. Likewise, it is frequently inappropriate to try to reschedule active-power controls simultaneously with reactive-power controls. Grossly approximating or neglecting problem discreteness may be intolerable. Time limits on the correction of violations may be imposed, modeling simplified control-movement dynamics. Since real-time control is essentially the tracking of a moving target, pinpoint optimality of the calculation may not be essential, and in any case is subordinate to enforcement of operating limits.

In study mode, where operating decisions, targets, and trajectories are established, there is more emphasis on determining global optimality, fully utilizing the operational capabilities of the power system. In this case, many of the factors emphasized for the real-time mode assume less or no importance. In addition, however, it is nevertheless desirable to be able to duplicate real-time scheduling calculations in the study mode, to develop *a priori* emergency strategies, or to make *a posteriori* investigations of operational situations.

In formulating OPF problems, it is important to adhere to the normal engineering principles of power system operation. There are plenty of opportunities, particularly in using a general-purpose OPF tool, for creating badly posed problems. An example is to try to minimize losses with generator MWs as variables. Or to impose limits on MW reserves with only voltages and taps as controls. It is helpful to associate each control, constraint, and objective with the active-power or reactive-power subproblem, or both. This provides a rule of thumb for initially testing the reasonableness of a formulation. Nonconforming situations, such as where voltage limits are to be enforced with active-power controls, need to be examined on their individual merits.

C. The Power System to be Optimized

Security-constrained scheduling is sometimes applied to the relevant controls throughout an entire interconnected power system. Much more often, it is applied to the controls within only a portion of the system, comprising one control area or a group or control areas such as a power pool. The general situation is therefore that of an internal "optimized" area surrounded by external "nonoptimized" control areas.

In trying to optimize only a portion of an interconnected system, there is always the fundamental problem of how much the scheduling is allowed to affect the neighbors. One approach to the problem is to require that any rescheduling within the internal system shall not alter the flows or voltage magnitudes at the boundaries with the external system. In this case, the external system is simply not modeled. Pre-optimization tie-line flows are treated as fixed loads, and the boundary-bus voltages are constrained to be constant.

This approach is extremely conservative, and unless the internal system is large with a high degree of freedom, it may overconstrain the optimization to the point where little or no rescheduling is possible. It does not take advantage of the mutual-assistance benefits of interconnection. A somewhat less restrictive approach is to model the boundary buses as generators with fixed MWs. Limited ranges are imposed on the voltages and MVARs of these fictitious sources, and possibly on their total reactive generation (the VAR interchange). It may be difficult to predefine realistic values for these ranges.

By far the most common and straightforward approach is to provide detailed power-flow modeling of the external network in a "buffer zone" around the optimized part of the system. This typically coincides with the main EMS model used for operator power-flow and contingency analysis, and sometimes even for state estimation. Individual or summated limits can now be imposed directly on any buses and branches to prevent external operation from being badly affected by internal scheduling.

In practice, it is very convenient for the same power-flow models to be adopted for real-time and study modes, particularly because study cases are usually derived from saved real-time cases.

D. Control Variables

The following is a list of the most common power system controls, categorized according to whether they primarily affect active- or reactive-power subproblem operating conditions, or both.

1) Active-Power Subproblem:

- generator MW outputs
- phase-shifter taps
- MW interchange transactions
- HVDC link MW transfers.

2) Reactive-Power Subproblem:

- generator voltages or reactive powers
- in-phase transformer taps
- shunt reactors and capacitors.

3) Active- and Reactive-Power Subproblems:

- transformers with varying complex turns ratios
- generating unit start-up/shut-down
- load reduction or shedding
- line switching.

The interchange transaction referred to above is the MW amount that the internal system buys from or sells to an individual external company. It needs the external network to be modeled explicitly or with a good equivalent, in order to represent the associated changes in power flow.

Synchronous condensers are regarded as zero-MW generators. Shunt devices include saturable reactors.

Line switching is a type of control for which continuous methods are not at all appropriate. Switching optimization is an important future feature of scheduling methods. Although it is not described in detail in this paper, [57], [58], and [90] represent some of the most recent work. Switching is treated nonrigorously by large-perturbation sensitivity techniques similar to those used in contingency analysis. A simple illustration of the idea is that by switching out an overloaded line, the power may be rerouted through lines that have adequate capacity.

E. Operating Constraints

Apart from the equalities (5b), most operating constraints are upper and lower limits on system quantities, which are separated here into subproblems according to whether they are affected mostly by active-power control, reactive-power control, or both. Some of the most common "soft" operating limits are on the following:

1) Active-Power Subproblem:

- branch MW flows
- spinning MW reserves
- area MW interchanges
- branch-group MW transfers
- bus voltage angle separations.

2) Reactive-Power Subproblem:

- bus voltages
- branch VAR flows
- spinning MVAR reserves
- area MVAR interchanges
- branch-group MVAR transfers.

3) Active- and Reactive-Power Subproblems:

- branch current and MVA flows
- branch-group MVA flows.

The above subproblems represent part of the inequality constraint set (5c). The remainder of this set comprises the upper and lower limits on all control variables, which are usually "hard," corresponding to the ranges of physical apparatus.

Another type of control limit imposes a maximum time on the control action. This can be translated into limits on the control variables, determined from their rates of change. Still other operating constraints can be represented through equalities, limits, or modeling. These include generator MVAR sharing, tap ganging, and MW wheeling.

F. Operational Objectives

The classical OPF formulation has a single objective. More often than not, this is an inadequate statement of the on-line operational problem. In practice, it is usually required to optimize more than one power system attribute simultaneously. Many such cases are resolved by making the subsidiary attributes constraints on the primary objective. Where necessary, however, composite-objective OPF calculations can be undertaken. These must be formulated with great care, since they easily lead to poorly posed engineering and mathematical problems [91].

The four most common objectives are discussed as follows. It will be seen that these are rarely simple. Many other objectives can be defined, and will be incorporated into OPF programs as the engineering needs are identified and clarified.

1) Minimum Cost of Operation: This objective comprises the sum of the costs of the controlled generations, plus the costs of any controlled interchange transactions. All system control variables are eligible to participate in minimizing this objective. If only active powers are controlled, the calculation is referred to as security-constrained economic dispatch (SCED) or scheduling. Where control variables without direct costs, such as voltages and transformer taps, are included, they participate by coordinating transmission losses in the overall cost minimization.

Cost minimization with both active- and reactive-power controls represents the classical "full OPF" problem. It is sometimes confusingly called the "cost plus losses" problem, which term might more appropriately be applied to SCED followed separately by loss minimization.

The most critical factor in cost minimization is the modeling of the generator cost-versus-MW curves. This not only has an effect on overall optimality, but is intimately bound with the solution method used. Costs for thermal units are derived from the heat-rate curves, which are far from smooth or convex for turbines operated in a multiple-valve mode. However, convexity and smoothness are prerequisites with many OPF methods. One approach is to approximate the cost curve as a convex polynomial or exponential. A more flexible approximation is to model each intervalve segment as a quadratic, but maintaining overall convexity. More flexible still, in terms of fitting the "correct" curve, is to use an arbitrary number of linear segments, again maintaining convexity [59]. Various computer programs can accept cost curves in different forms, but may by constrained by the solution approach to convert them into a form compatible with the algorithm.

It is fairly common to have regions of prohibited operation on the cost curves. These are normally accounted for similarly to other discretizations by post-processing. However, they can be represented rigorously in the optimization by separating the curves into a larger number of smaller curves.

The cost curve associated with an MW interchange transaction is normally linearly segmented, corresponding to interchange block tariffs. Load shedding can only be incorporated as a control as long as it is given an artificial high cost (otherwise, the cheapest solution would be to shed as much load as possible).

An objective whose formulation and solution is closely related to minimum cost is mimimum emission [60], [61].

2) Minimum Active-Power Transmission Losses: The controls that can address this objective are all the ones without direct costs, that is, all except MW generations and interchanges. At the same time, it should be recognized that some controls, such as phase-shifter taps and dc line flows, will usually not be scheduled for loss minimization because they are more useful for active power control. Therefore, loss minimization is normally associated with voltage/VAR scheduling. It is a fine-tuning objective for system operation. It tends to reduce circulating VARs, thereby promoting flatter voltage profiles. In some systems the MW generation saving is appreciable.

Losses can be minimized in any designated part of the power system, generally either in the internal "optimized" area, or else in the entire interconnected system. In the former case, the solution sometimes (but by no means necessarily) decreases the internal losses at the expense of those of the neighbors. In the latter case, the internal-system losses could conceivably increase. There is no general rule.

A very similar but potentially competitive objective is to minimize series reactive-power losses. This objective has several beneficial effects. It near-minimizes active power losses, keeps voltage profiles flat, and near-maximizes generator VAR reserves.

3) Minimum Deviation from a Specified Point: This frequently quoted objective is usually defined as the sum of the weighted squares of the deviations of the control variables from their given target values. The target values correspond to the initial or some other specified operating point. Such an objective may be used for corrective rescheduling. It also offers a practical way of interfacing a scheduling function with a hierarchically superior one. For instance, a study-mode calculation may define a desirable (e.g., loss-minimizing) voltage/VAR schedule, and a real-time calculation may maintain violation-free operation with minimum deviation from the schedule.

This objective is really a composite or multiple objective, comprising the weighted sum of dissimilar quantities. It deceptively appears to be straightforward, but in fact it must be used very carefully, especially with a mix of control variable types. The relative values of the weightings are at once arbitrary and critical. These points may be sufficiently nonobvious, and of more general interest in the context of composite objectives, that some illustrations are given as follows.

Suppose that the objective consists of the squared deviations of generator powers (in MW) and voltages (in per-unit), weighted equally. Then the OPF program finds it just as expensive to reschedule the power of each controlled generator by 0.1 MW as to change its voltage by 0.1 per-unit. If the violation is a largely MW branch overload, the result is a massive unnecessary rescheduling of voltages that will lead to bizarre and possibly unstable reactive-power conditions in the network.

The above relative weightings were clearly inadequate. Suppose that the generation weighting is instead reduced by a factor of 1000. Now the cost of moving each generator by 100 MW is the same as rescheduling its voltage by 0.1 per-unit. There will be more participation by the generator MWs and less by their voltages in alleviating the branch (or any other) violation. What basis do we have for determining that these weightings now give satisfactory engineering answers? How reasonable are they if the violation is a bus voltage instead of a branch flow?

In fact, it seems that there are no general analytical criteria whereby the relative weightings can be automatically assigned. So the issue comes down to whether the power system operators eventually become able to assign them on the basis of their experience in using the objective. Examples of the problems are: finding suitable relative weightings for transformer phase-shift angles and in-phase taps; weighting generators by MW shift, or nameplate rating, or current incremental cost, or participation factor, etc; weighting shunt capacitors and reactors by MVAR, or rating, or voltage level, etc.

To compound the difficulty, it appears that the suitability of a set of weightings may depend on the types and locations of the limit violations, and the particular loading and configuration of the system.

Issues of the above types need to be resolved before applying this and other composite objectives to a specific power system.

The above examples demonstrate that the active- and reactive-power subproblems should, in any case, be solved separately. In each subproblem, the control weightings can then be related to a common reference. For instance, all active-power controls can be weighted according to the MW shifts that they produce: approximate sensitivities of phase-shifter angles to their MW flows are needed. Likewise, all reactive-power controls are weighted according to the per-unit voltage shifts that they produce, requiring voltage/VAR sensitivities for shunt devices.

4) Minimum Number of Controls Rescheduled: Some practical approximation to this objective is essential in the many cases when it is impossible or undesirable to reschedule a large number of controls at the same time. That is, the objective applies when there are limited means for the automatic dispatch of many controls simultaneously from the control center, and/or frequent moving of tapped apparatus is to be discouraged.

Unlike the previous objectives, this objective defies rigorous formulation or solution. The definition of "minimum number of controls" can be very loose. The intent is simply to reschedule a manageably small number of controls, where "small" depends on the particular EMS and power system.

Given that restrictions on the number of controls moved are real-time in nature, the objective is rarely avoidable in real-time voltage/VAR scheduling. One application is corrective control (remedial action) with the aim of staying close to a given (usually the present) target point, for instance, a previously obtained loss minimization schedule.

In such cases, a semi-rigorous minimization of the number of controls can be achieved with solution methods that permit each control to be given a dummy linear "V" shaped cost curve, centered on the control's target value. If all these curves have the same fixed incremental cost (i.e., the same absolute value of slope on either side of the "V"), the controls will be scheduled in order of their electrical sensitivities to the violation(s) being alleviated [73].

Another approach, falling within the grey area of composite objectives, combines the minimum deviation objective with that of loss minimization [72], [93]. After first minimizing losses on their own, the controls that have moved least are identified, and are successively penalized to encourage them to return to their original values.

A somewhat similar composite objective is described in [62], but its intent and effect are different. In this case, the loss minimization objective is augmented with a minimum-deviation term that allows all controls to move, but less than they would otherwise do. Such a scheme will tend to reduce the computational expense of control limit enforcement, which in some algorithms is great, but it is not oriented much towards practical operating requirements.

G. Suppression of Ineffective Rescheduling

For on-line applications, a special constraint on any objective is the suppression of ineffective rescheduling. The requirement is to prevent the movement of those controls that, if rescheduled, would have little effect on the objective or limit enforcement. This constraint, which is difficult to state analytically, is related to but not identical with the minimum-number-of-controls objective.

The constraint can be imposed heuristically in different ways. The easiest is when the optimization method, such as dual linear programming, moves the most cost-sensitive controls, one at a time. Each move can then be tested in advance for effectiveness and if necessary abandoned [73]. An alternative approach is to first solve the complete original OPF problem, and then backtrack, successively inhibiting ineffective control moves.

H. OPF Problem Infeasibility

An important aspect of an on-line OPF function is how it performs when the problem is mathematically infeasible, i.e., when all the operating limits cannot be respected. Rather than terminate as unsolved, the function should provide the "best possible" solution, with or without interactive guidance. This should be a major consideration in the overall function design and in the choice of optimization methods.

The OPF algorithm needs to be able to identify problem infeasibility decisively and rapidly. Unfortunately, the majority of OPF methods are deficient in this regard.

Once an OPF problem has been pronounced infeasible, it can be altered and resolved in two alternative ways:

a) With the OPF controls and/or constraints modified. The candidate modifications include:

- switching in additional controls (freeing previously fixed controls, connecting extra generators, etc.);
- switching operating limits to more expanded values (e.g., from long-term to medium-term values, or ultimately to sufficiently wide limits that the constraint becomes ignored);
- network topology change;
- load reduction or shedding (normally last resorts).

b) With the OPF objective modified to provide a solution in which those operating limits causing the infeasibility are minimally violated. The "minimal-violation solution" can be defined in different ways. The most common is weighted least squares violations. This means augmenting the existing objective, such as cost or losses, with a series of weighted minimum-deviation functions. All the weighting arbitrariness and associated difficulties described for the minimum-deviation objective can be encountered.

Approaches a) and b) are not mutually exclusive. For instance, if strategy a) fails to provide a feasible solution, strategy b) may be reverted to, or *vice versa*. Note that uniformly expanding limits can give quite different solutions from those designed to minimize violations.

I. Control and Constraint Priority Systems

A versatile on-line OPF function will cater for all the problem modifications in a) and b) above, in a predefined "priority" sequence [73]. It should be possible to specify this sequence in a highly flexible manner, to accommodate the utility's varying operational requirements. Different sets of priorities are likely to be needed for different OPF problems on a given power system.

In perhaps the most basic approach, the set of activated controls and constraints (including any limit expansions) is explicitly defined, by type or individually, for each separate priority level. The priority levels are numbered so that system operation successively becomes either:

a) more feasible, or
b) less feasible.

At present, alternative a) seems to be more common. It starts from the premise that the overall defined set of operating limits is realistic, and can normally be satisfied early in the priority sequence. One version of the scheme, ensuring that each successive priority level is more feasible than the last, assigns priority-level numbers to the various controls and expanded operating limits themselves. Such a scheme can be summarized by the following steps:

1) Attempt to solve the original (complete) OPF problem. If solved, exit.

2) Increment the priority level. If all priority levels are exhausted, switch to a minimum-violation OPF problem, solve, and exit.

3) Include in the problem all limits (expanded or original) and controls as in Section III-Ha) above with priority numbers up to and including the current level. Attempt to solve the new OPF problem.

4) If solved, exit. Otherwise return to step 2).

The time taken for the complete priority sequence depends on the speed of each individual OPF solution, how many priority levels must be passed through, and how quickly the optimization method can detect infeasibility. In particular, the more stressed the power system operating state, the more priority levels the on-line OPF might have to work its way through. These considerations should be carefully taken into account in projecting total OPF computing effort, which is much greater than for a single OPF solution.

Alternative b) above starts by considering the least onerous subset of the OPF problem. In one version, only the highest priority constraints are initially included. If the problem is successfully solved, the problem is augmented with additional constraints, and so on, until infeasibility is encountered. At this point, the decision must be made whether to accept the solution, or continue to add the remaining constraints, with minimization of the violations.

J. Introduction to Optimization Methods for On-Line OPF

All OPF methods are based on general optimization principles and techniques [63], usually specially adapted to exploit the structure of the power system problem. The three chief components of any static optimization process are

- the minimum-seeking approach
- the approach for handling equality constraints
- the approach for handling inequality constraints.

There is a huge and ever-increasing number of specific methods in each of these categories. Many of these methods have been tried on OPF problems in different combinations over the last 25 years of research. Few implementations have been very successful. As a rule, the most powerful optimization methods are unacceptably time-consuming on problems as large as a power system network. Conversely, the faster methods tend to be less reliable in convergence, and/or require restrictive problem formulations and modeling assumptions. No practically usable methods guarantee to solve feasible problems or to find global optima.

The limitations of optimization technology, then, have been a major impediment to the development of production-quality OPF codes, their wider industrial exploitation, and the refinement of practical problem formulations. The situation has recently improved with the development of several promising methods, although there are still many deficiencies to be overcome. At this stage, it is uncertain whether the ideal of a single method that possesses the necessary speed, reliability, and flexibility for all on-line applications will ever be achieved.

K. Basic Optimization Approaches

The static optimization approaches to nonlinear OPF have traditionally been divided into those based on generalized nonlinear programming (GNLP) and separable nonlinear programming (SNLP). The following attempts to point out some major properties and differences of the approaches.

1) Generalized Nonlinear Programming Methods: Most methods require that the problem be smooth and, with some of the more powerful ones, it should be twice differentiable. Convergence is asymptotic, and is acceptable when the Kuhn–Tucker necessary conditions for optimality have been satisfied within tolerances. GNLP methods handle OPF formulations with objectives such as smooth generation cost curves or transmission losses. Control-variable discreteness is not handled directly. Where represented, it is imposed externally with varying success, usually after the smooth problem has initially been solved.

The minimum-seeking mechanism in most earlier OPF methods using GNLP was first-order descent [56]. These methods, employing first-derivative gradient information in determining the direction and length of the next step, were found to converge unpredictably. They had a tendency to zigzag or stop when the floor of a "valley" was reached. The trend in GNLP generally and in the OPF application in particular has moved firmly towards second-order methods, which use exact or approximate first- and second-derivative information about the problem at each step to find the direction and step length.

An important class of GNLP methods is based on successive quadratic programming (QP). At each OPF step the objective is approximated as quadratic and the constraints as linear. Then the resulting QP problem is solved. This is repeated successively until the nonlinear problem is converged. For well-posed problems the individual and overall solutions are likely to converge well, and with most QP methods infeasibility can be detected.

2) Separable Nonlinear Programming Methods: In the present context, a separable problem is one whose objective function is the sum of convex cost (input–output) curves and whose constraints are linear. Such a problem lends itself to very rapid and reliable solution by any one of a number of special separable techniques. Successive reapproximation and solution leads to convergence of the nonlinear OPF problem.

Most of the common OPF objectives can be handled by this approach. The choice of specific separable optimization technique is influenced by the kinds of convex cost curves to be accommodated. For instance, separable QP is efficient if the cost curves are all strictly quadratic (but this is rarely the case).

General smooth nonlinear curves such as piecewise quadratics, polynomials, and exponentials are handled by separable differential methods. Even greater flexibility is afforded by special versions of the linear programming (LP) approach. These accept *any* nonconcave piecewise cost curves, whether linear or nonlinear, and in any combination.

In the LP approach, smooth nonlinear curves are approximated as piecewise-linear by the algorithm. As the solution progresses, the sizes of the linear segments are gradually reduced, resulting in overall final accuracy equal to that obtainable by a true nonlinear method. Moreover, the LP-based OPF solution lies almost entirely at the segment break points in the objective function. This property can be used to promote discrete scheduling of controls such as transformer taps and shunts, and to minimize the number of controls moved.

In principle, at least, a separable method can also solve OPF problems with nonseparable objectives. It requires that the objective be approximated as separable at each iteration. However, if the objective is strongly nonseparable, as for instance in the case of transmission loss minimization, such an approximation is very poor and convergence tends to be slow and oscillatory.

L. Basic Constraint Techniques

This section gives a brief outline of the basic constraint techniques used in most recent OPF methods. The details of their application in constrained optimization tend to be much more complicated [64]. Referring to the problem definition in Section III-B, the equalities (5b) and inequalities (5c) must be imposed on the optimization problem. The basic approaches serve for both equality and inequality constraints: an unsatisfied inequality constraint becomes converted into an equality at the binding limit. A backoff mechanism is needed to free the inequality constraint from the limit if later on it should no longer be binding.

There is no restriction that the same constraint technique be used for all equalities and inequalities. The binding constraint set, comprising equalities $g(u, x)$ and/or enforced inequalities $h(u, x)$, is expressed in generalized form as $a(u, x)$.

1) Lagrange Multiplier Method: The constrained minimization becomes the problem of finding a stationary point of the unconstrained Lagrangian function

$$L(u, x, \lambda) = f(u, x) - \lambda^t \cdot a(u, x) \tag{6}$$

where λ_i is the Lagrange multiplier (dual variable) associated with the ith constraint. The extra variables λ increase the problem's dimensionality, but tend to preserve its linearity and sparsity. For an inequality, the sign of the multiplier (also called the Kuhn–Tucker variable) indicates when the limit should be backed off. At the solution, each multiplier represents the sensitivity (marginal cost) of the objective with respect to its constraint.

2) Penalty Function Method: The most commonly used penalties are quadratic. Then the constrained problem is converted to the unconstrained minimization of the augmented objective

$$F(u, x) = f(u, x) + \sum w_i \cdot a_i(u, x)^2 \tag{7}$$

where the w's are penalty weightings. The dimensionality is lower than in (6) due to the absence of the multipliers, but squaring the constraints can often increase problem nonlinearity and decrease sparsity. The weightings have to be adaptively modified to ensure that the constraint function values tend to zero within engineering tolerances. Unless it wants to back off the limit, a binding inequality is always violated at the solution. Apart from penalty weight control, another scheme to help the penalty to enforce a limit within tolerance is to adaptively shift the limit itself. Marginal cost information can be extracted from quadratic penalties of the above type.

3) Simplex-Based Methods: These methods are characteristic of linear programming and related methods such as quadratic programming and reduced gradients. A tableau containing the coefficients of the linearized equality and inequality constraints is maintained. The mechanisms vary, but the principle is as follows. At any stage during the solution, variables are nonbasic (fixed on limits) or basic (free to be evaluated by solving a "basis matrix" equation). This matrix represents the current active (enforced, or binding) constraints in the tableau. When the optimum-seeking process enforces a new limit, the relevant inequality constraint is converted to an equality. Now that it is a member of the active set, its coefficients enter the basis matrix in exchange for those of a previously enforced limit that is consequently freed. The factors or inverse of the basis matrix have to be updated. Marginal cost information is usually a by-product of the approach.

M. Reduced Techniques

At each step of a reduced or "compact" method, the equality constraints are linearized about the current values u^0 and x^0 of the control and dependent variables, respectively. The changes Δx are then expressed as linear functions of the control variable changes Δu, and substituted into the problem to produce an objective

$$f(u, x) = f(u^0 + \Delta u, x^0 + S \cdot \Delta u) = F(\Delta u) \tag{8}$$

where S is the nonsparse sensitivity matrix between x and u. This objective is now minimized, subject to the problem inequalities, linearized as functions of Δu. Having updated u, the corresponding new value of x is calculated by solving the nonlinear equations (5b).

In practice, it may be very difficult to derive an explicit algebraic expression in the form (8). Most reduced and other optimization methods, therefore, only require successive approximations (linear or quadratic) to $F(\Delta u)$. In the case of the cost function, the form of $F(\Delta u)$ remains very much simpler if one "power balance" equality constraint is retained. This constraint is quadratic or linear, depending on the optimization technique.

The dimensionality of the reduced approach is low, but loss of sparsity can be a severe problem, depending on the objective, system size, and optimization method employed. Simple limits on the variables x now become nonsparse linear functions of Δu.

N. Current OPF Approaches

It is not possible to review all of the many different OPF methods that have been proposed. The intention of this section is merely to highlight several of the most promising current approaches. It deals with the generalities of the methods, rather than with specific existing codes, most of which have proprietary aspects and will probably have been further developed by the time this paper is read. It should also be recognized that the precise implementation details of a method often have the greatest bearing on its ultimate success. Invariably, optimization algorithms that exploit the specific structure of the power system OPF problem are many times faster than their general-purpose counterparts.

1) Successive Sparse Quadratic Programming [62], [65]: This OPF approach is relatively straightforward. At each iteration, the OPF problem is approximated by a sparse quadratic objective and sparse linearized constraints (expressed in terms of deviations from the current state). The approximated problem is solved for the corrections to the variables by quadratic programming. Then the current point is updated. This approximation–solution–update process is iterated to convergence.

At present, there are no customized QP methods that can efficiently exploit the special sparsity structure of the power system problem. Therefore, general-purpose sparsity-oriented QP packages have tended to be used. When the QP package is treated as a plug-in "black box" optimizer, the main development effort for the central OPF solution is the power system problem approximation. Then the approach has the appeal of relative ease and simplicity. It can handle classical smooth OPF formulations, including any linearizable constraints.

The efficiency of the specific sparse QP process used is critical. In general, the computing effort per iteration is very high, but convergence of the basic smooth problem is normally obtained in a few iterations. Nonanalytic constraints and program features may be difficult to incorporate efficiently. They are likely to require the expenditure of a few or even many extra iterations. Computational effort tends to rise rapidly with the number of controls, and the number of limit enforcements [66].

2) Successive Nonsparse Quadratic Programming [67], [68]: This method is related in concept and performance to Carpentier's much earlier method using Generalized Reduced Gradients (GRG) [69]. The QP version is presented here since QP is a much better known optimization approach than GRG.

A reduced or "compact" approach as per Section III-M is adopted. At each iteration, the objective is approximated as a nonsparse quadratic function of Δu: the changes in the system control variables about the current point. The system inequality constraints are expressed by sensitivity analysis as linear nonsparse functions of Δu. Then this reduced problem (from which the equality constraints have been eliminated) is inserted into a general-purpose nonsparse QP package.

The QP solution provides an update to the control variables u. A conventional power flow solution is now performed to find the new dependent variables x. The entire process is repeated iteratively, generally with convergence in a few iterations.

Unlike the sparse version, it is quite impractical to present the QP process with all inequality constraints at the same time. Instead, a critical constraint set (inequalities with actual or near violations) is chosen. After solving the QP problem, the linearized inequalities are tested (preferably sparsely), to identify any new violations. At this point, it is very advantageous if the QP package has the ability to update its optimal solution rapidly when presented with the additional inequalities. This makes it possible to obtain a complete solution of the approximated problem, before re-approximating and starting a new QP process, thereby saving iterations.

The overall efficiency of this kind of method depends very much on problem type and size, as well as on the performance of the specific nonsparse QP package. Computing effort rises rapidly with problem size. For instance, if the number of control variables is n, the compact transmission loss objective will have n^2 coefficients, and for large n the generation of these coefficients and the QP solution will be extremely time-consuming.

In Carpentier's version, the approximation to the OPF problem does not necessarily have to be in exact QP form, since a special GRG optimization package is used instead. For instance, a nonlinear power-balance equality constraint can be accommodated. Again, however, the number of coefficients in this equation may be n^2. Versions of the method that avoid the explicit computation of these coefficients are under development [89].

The approach has similar advantages to Sparse Quadratic Programming, in being able to utilize a general well-developed optimizing package. Overall, however, it is less easy to implement by virtue of the sensitivity analysis needed for reduction.

3) The Newton Approach [70]–[72], [88]: Newton-based OPF solves classical smooth twice-differentiable problem formulations. It has two very unique characteristics:

i) for the basic OPF problem, it truly preserves power system network sparsity, and
ii) it is not based on any well-developed general-purpose optimization method or package.

The first feature is a major breakthrough, since it offers freedom from the "curse of dimensionality." Computational effort now rises barely more than linearly with problem size, just as in conventional power flow. There is now no computational barrier to performing OPF on very large power system problems. This linearity property deteriorates slightly as more advanced constraints, such as interchanges and reserves, are incorporated.

The second feature is an unfortunate but apparently unavoidable consequence of the first. Newton OPF is very much an "approach" rather than a definitive "method." An intricate special-purpose optimization scheme, accommodating inequality constraints, needs to be constructed around the core idea. There are no standard procedures for accomplishing this. The optimization and computation techniques used in different versions are likely to be quite different from each other.

Equality constraints are imposed by the Lagrange multiplier method of (6). In the original version of [70], the penalty functions of (7) are used to incorporate sparsely some of the binding inequality constraints, particularly the limits on problem variables u and x. The remaining binding inequalities are handled by Lagrange multipliers.

The net result is a Lagrangian function $L(u, x, \lambda)$ that is a composite of (6) and (7). The optimum point must satisfy the zero-gradient necessary conditions, which are expressed as the following set of nonlinear equations

$$\begin{aligned} \partial L/\partial u &= 0 \\ \partial L/\partial x &= 0 \\ \partial L/\partial \lambda &= 0. \end{aligned}$$

These equations are solved iteratively by Newton's method, successively constructing and solving the "Hessian" matrix equation

$$\begin{array}{|c|c|} \hline H & -J^t \\ \hline -J & 0 \\ \hline \end{array} \quad \begin{array}{|c|} \hline \Delta z \\ \hline \Delta \lambda \\ \hline \end{array} = - \begin{array}{|c|} \hline \partial L/\partial z \\ \hline \partial L/\partial \lambda \\ \hline \end{array} \tag{9}$$

where $z = (u, x)$. Each Newton iteration is equivalent to the exact solution of an equality-constrained QP problem. The objective of this problem is the current quadratic approximation of $f(u, x)$ plus the limit penalties. It may be augmented by additional terms that strengthen the positive definiteness of the problem without altering the solution point. The constraints are the power flow and any other equations, linearized about the current point. An important property is the fact that penalties on the variables u and x are already quadratic, and incur no approximation or fill-in during matrix factorization. In particular, there is little computational cost associated with imposing limits on any number of controls and state variables such as bus voltages.

If the correct binding inequalities are known and the penalty weightings are adequate and fixed, the necessary-condition equations do not change from iteration to iteration, and their Newton solution will converge quadratically. However, the binding inequality set is *not* known *a priori*. Therefore, heuristic trial techniques have been used in between main Newton iterations in an attempt to identify this binding set.

The main network portion of the Hessian matrix in (9) has Y_{bus} sparsity in 4×4 block form. Large numbers of extra rows and columns, corresponding to controls and certain constraints, pre-border and post-border this block matrix, or are inserted inside it. Each main iteration of the Newton OPF method involves constructing, factorizing, and solving (9), at around four times the effort of a conventional Newton power flow iteration. It is essential to employ sophisticated mixed-block sparsity methods.

By decoupling the Hessian matrix [70], the work per iteration can be reduced by a factor of nearly four, and the overall solution effort can be reduced. As in decoupled power flow, convergence is now obtainable only to practical engineering accuracies, since the Newton quadratic convergence property is lost. There is some indication that OPF problems are more sensitive to decoupling than are ordinary power flows.

4) Successive Nonsparse Separable Programming [16], [73]–[81]: Approaches 1) to 3) above all fall into the category of general nonlinear programming (GNLP) methods as described in Section III-K1. However, for problems formulated with separable objectives, specially fast solutions are available using the methods of Section III-K2. All the objectives in Section III-F except loss minimization are amenable to solution by the separable approach.

A nonsparse separable method has a great deal in common with the nonsparse QP method 2) above. It follows the same principles of successive approximation and iteration with power flow. It requires the same sensitivity analysis to express the constraints as linear functions of the control variables. Both primal and dual versions have been used, with the latter appearing to be more efficient.

The dual approach starts by optimizing the problem with limits only on control variables. In the case of cost minimization, this solution is the same as conventional loss-coordinated economic dispatch. Then the operating limit violations (if any) are introduced into the problem one at a time. To correct each violation, the optimizer reschedules the controls at least extra cost, permitting backoff of previously enforced limits as dictated by optimality. Eventually, all violations are removed and the overall optimal secure solution is reached.

The specific optimizer of current choice appears to be one centered on a special dual LP solution with upper and lower bounding, constraint relaxation, and efficient handling of the separable piecewise-linear convex cost curves [75]. At each iteration, piecewise linearization (with automatic successive refinement) represents the cost curve shape more accurate than in most GNLP methods, which merely approximate them as quadratic. Indeed, in some implementations, piecewise linearization is more conceptual than explicit: the distinction between separable LP and NLP becomes rather fine.

Computationally, the approach is very favorable. The size of the nonsparse tableau is $m \times n$, where n is the number of controls and m is the varying number of binding operating limits (not including limits on controls, which are handled separately by the upper and lower bounding). Since the number of operating limits m enforced in a solution is typically small, the method remains efficient, however large the power system. In fact, on problems such as cost minimization with active-power controls, or minimum deviation with active or reactive controls, the overall speed of the approach tends to be considerably faster than any GNLP method.

Apart from speed, the approach has other very desirable properties. It handles the minimum-number-of-controls objective well: unlike methods that prefer or require twice-differentiable functions, it has no difficulties with the "V" shaped cost curves referred to in Section III-F4. The version is well-suited to inhibition of ineffective rescheduling as described in Section III-G. Importantly, it detects problem infeasibility at any stage in a rapid and clear-cut manner, as needed for the successful implementation of a control and constraint priority system (Section III-H). Special extensions to the algorithm provide a very reliable "end game," where an infeasible problem is immediately and automatically solved with least squares violations of operating limits.

Some LP-based separable programming and other methods designate generator reactive powers rather than voltages as the control variables. Such formulations are normally unsuitable for on-line applications, because the rescheduling of even a single generator reactive output then reschedules all controllable system voltages.

Transmission loss minimization is not handled efficiently because its objective function is strongly nonseparable. Many versions claiming to minimize losses have nevertheless been reported, e.g., [78]–[81]. It is quite unlikely that these versions will have good convergence to accurate minimum-loss solutions. If, however, they could guarantee to give reliably reduced losses, they would be attractive because of their other highly desirable computational properties.

O. Contingency-Constrained OPF Solutions

Contingency constraints are a fundamental element of economy–security control. They are intrinsic to security levels 1 and 2 in Fig. 1, one or a combination of which represents the operational goal of virtually every power system. However, only a small proportion of the work on OPF has considered the special problems of including these constraints. The most successful applications have been to the security-constrained MW dispatch OPF subproblem. In some power systems today, the need to represent contingent voltage constraints is becoming acute.

The total number of contingency constraints to be imposed on the OPF calculation is enormous. Depending on the power system size, each contingency case may involve hundreds or even thousands of inequalities. Since a typical contingency list is large, the total number of individual contingency constraints can reach even millions. Fortunately, very few of these constraints will be binding in the solution—otherwise contingency-constrained scheduling would be impractical from the outset. Relaxation techniques, as used in all other OPF work (i.e., where nonbinding constraints become ignored), become particularly effective.

Contingency-constrained OPF may be undertaken either with or without first optimizing with respect to the base-case (pre-contingency) constraints. The general approach is as follows:

a) Contingency selection (or complete analysis) is performed at the current operating point, to identify those N_c contingency cases with violations or near violations.

b) These N_c selected cases, which ought to be a small proportion of the total, are incorporated into the OPF problem.

c) The OPF problem is solved subject to both the pre-contingency (base-case) constraints, and to the post-contingency constraints of the selected cases.

d) The rescheduling may now have created new power system insecurities. Therefore, the entire process must be cycled through step a) until no violations remain.

The post-contingency constraints in step c) may add very considerably to the normal OPF solution effort. Much worse, however, is the need to cycle the entire process. It is onerous to have to repeat step a) even once. In a real-time application, cycling might be dispensed with, provided that the contingency-constrained scheduling is performed often enough, in the MW case perhaps even as frequently as ordinary economic dispatch [76]. No such corner cutting is easily justifiable in the study mode.

It is therefore seen that contingency-constrained scheduling represents potentially massive further computing effort, an effort that is already very large for contingency analysis and pre-contingency OPF on their own.

P. Contingency-Constrained OPF Formulation

The original OPF formulation in (5) can be expanded to include contingency constraints, thus

Minimize

$$f(u^0, x^0) \tag{10a}$$

subject to

$$g^k(u^k, x^k) = 0, \quad \text{for} \quad k = 0, 1, \cdots, N_c \tag{10b}$$

and

$$h^k(u^k, x^k) \geq 0, \quad \text{for} \quad k = 0, 1, \cdots, N_c \tag{10c}$$

where superscript "0" represents the pre-contingency (base-case) state being optimized, and superscript "k" ($k > 0$) represents the post-contingency states for the N_c contingency cases selected in step a) of Section III-O. Inequalities h^k ($k > 0$) need have no relation to h^0, reflecting different monitored quantities and/or limit values.

Each post-contingency state differs from the pre-contingency state as follows:

a) Equality constraints g^0 change to g^k to reflect the outaged equipment.

b) Control variables u^0 respond by changing to u^k.

The difference between security levels 1 and 2 in Fig. 1 lies in item b). Security level 1 represents the traditional conservative approach, where each post-contingency state should be violation-free without any post-contingency EMS-directed corrective control action. The control variable set for each such state is given by

$$u^k = u^0 + \Delta u^k \tag{11a}$$

where Δu^k is the response of the power system spontaneous automatic controls. One of the principal examples is generator MW response based on inertia, governor droop, or AGC participation. It is essential to have an analytical expression for Δu^k as a function of (u^0, x^0). For controls such as generator voltages, u^k is usually equal to u^0.

Security level 2 offers an operation cost that is potentially considerably lower. This is obtained by relying on the EMS to perform post-contingency corrective rescheduling δu^k, to remove any contingent limit violations. The EMS corrective control action becomes an integral part of the power-system model

$$u^k = u^0 + \Delta u^k + \delta u^k. \tag{11b}$$

The objective of the corrective rescheduling can be entirely different from that in (10a).

Q. Contingency-Constrained OPF (Security Level 1)

The post-contingency control variables u^k ($k > 0$) are as in (11a). This section outlines the two basic approaches to the contingency-constrained solution in step c) of Section III-O above. There are many possible variants.

1) The Nondecomposed Approach [82], [83]: In this approach, the problem of minimizing objective (10a) subject to the relevant equalities and inequalities in (10b) and (10c) is handled as one single very large multivariable, multi-constraint optimization solution. Either sparse or reduced (compact) formulations can be used.

The organization of the solution was relatively easy when using a first-order gradient (obsolete) OPF method, because each contingency case could be handled separately [82], virtually in the manner of a decomposed approach. Otherwise, the optimizing process has to solve the combined base and contingency cases as one structurally unique problem, whose size is (N_c + 1) times as large as the original base-case problem, and whose computing effort could easily be much more still. Relatively little work has yet been done on this approach. A reduced formulation has a predictable nonsparse structure. The dual separable versions seems to be relatively efficient. In a sparse approach, efficient exploitation of structure is critical and difficult.

2) The Decomposed Approach [69], [73], [76], [90]: This is the traditional approach that has been used in almost all contingency-constrained scheduling since the 1960s. OPF is performed only for the base case (the Master Problem). This problem is augmented by a small number of individual post-contingency inequality constraints, expressed in terms of the base-case variables via large-perturbation linearized sensitivity analysis. The OPF solution is iterated with contingency constraint linearization until exact convergence is reached.

The base-case problem is usually optimized first on its own. The main idea is outlined by the following steps.

a) Contingency analysis is performed for each selected contingency case, and its current post-contingency state (u^k, x^k) is obtained.

b) Each selected post-contingency constraint is now linearized about the relevant state (u^k, x^k), and transformed into a function of the base-case variables (u^0, x^0). This is accomplished using large-perturbation sensitivity analysis, normally employing the Inverse Matrix Modification Lemma (IMML), or equivalently, compensation [32]. The constraint can be expressed sparsely in terms of the base-case state variables, or nonsparsely in terms of the base-case control variables, depending on whether or not the OPF method uses the reduced approach.

c) The base-case OPF problem is now resolved, augmented by the transformed contingency constraints. The entire process is converged by cycling through step a).

Steps a) to c) are embedded within steps a) to d) of Section III-O. Some of the transformations in step b) are relatively easy, paricularly when invoking suitable approximations. A good example is the well-known distribution factor method, which is a direct result of applying the IMML to large-perturbation branch-outage sensitivity analysis on the decoupled active-power model of (2).

Many specific details of methods and implementations affect the comparison between the nondecomposed and decomposed approaches 1) and 2), respectively. As a trend, however, approach 2) becomes much more economical than approach 1) under any of the following circumstances:

- If the number of contingency cases N_c, with potentially binding limits, is not very small. This is obviously dependent upon the individual power system characteristics, and varies according to topology and loading.
- If the contingency-constraint linearizations used in approach 2) are good (i.e., little or no need for iteration with contingency analysis in order to converge the constraint enforcement).
- If the optimizing process's effort in approach 1) increases nonlinearly with overall problem size, since approach 2) limits the optimization to the augmented base-case problem.

Some economy might be obtained by using more extensive network reduction in the contingency cases than in the base case.

R. Contingency-Constrained OPF (Security Level 2)

The post-contingency control variables u^k are now free to be scheduled to correct contingent violations, as in (11b). The amount δu^k by which they can be scheduled is governed by the time allowed for correction, in conjunction with their rates of changes [73]. Therefore, from (11b), this translates into the restrictions

$$b^{\min} \leq u^k - u^0 - \Delta u^k \leq b^{\max}. \tag{12}$$

The entire approach is substantially generalized. The time allowed for correction dictates the degree of level 2 security. With $b^{\min} = b^{\max} = 0$, security becomes that of level 1.

As before, the nondecomposed and decomposed approaches are available. The former accommodates this formulation without too much change in problem structure or computing effort. The objective function (10a) is now augmented by the contingency-case corrective-rescheduling objectives as functions of (u^k, x^k). Inequalities (12) are included.

In the case of the decomposed approach, there is a much greater computational penalty in going from level 1 to level 2 security. At the time of writing, the most promising method uses Benders decomposition [84]. The base case (Master Problem) is first optimized, after which contingency analysis is run to identify cases with violations. Corrective rescheduling is then performed separately for each such contingency case. If the case is feasible, nothing further needs to be done. Otherwise, the case is modified to generate linear inequality constraints known as Benders cuts, which provide information about the nature and amount of the infeasibility. Now the base-case OPF is resolved, augmented by these extra linear constraints.

The Master Problem optimization takes care of the linearized contingent constraints imposed on it. However, both linearization error and the possible introduction of new contingency violations necessitates iteration between Master Problem optimization, contingency case optimization, and contingency analysis. The overall computing effort can be considerable.

S. Comments on OPF Problem Formulations

A mistake in analytically formulating OPF problems is to regard them as simple extensions to conventional power flow. Once the power flow problem has optimizing degrees of freedom, superficially normal problems can turn out to be badly posed. The optimization problem has its own principles of solvability analogous to the observability rules of state estimation. For OPF, these rules are not yet well formalized, and vary from firm to tentative. Such considerations have hardly if ever been mentioned in the vast literature on OPF.

This section illustrates the issue by describing a number of problematic cases. Unless otherwise stated, they apply to the two most common objectives of cost and active-power loss minimization. It is interesting to note that the majority of items are associated with voltage/VAR scheduling.

Some cases involve lack of uniqueness, which can manifest itself in problem singularity or ill-conditioning. This is due to the absence of some important information about the desired operation of the power system. In appropriate cases, combining parallel apparatus models can sometimes by-pass the problem. In general, unless the particular optimization mechanism happens to provide acceptable resolution of the arbitrariness, the formulation of all such cases should either be improved or avoided completely.

a) A radial transformer with optimized taps on the radial bus side (either load or generator) is an extremely common configuration. However, there is an infinite number of optimal solutions for the tap and radial bus voltage.

b) The above problem persists when the transformer is modeled with no resistance and the tap is on the other side of the transformer. Including transformer resistance makes it solvable but like in case a), information is missing about what constitutes a good engineering solution. Is it preferred that the tap be rescheduled, or that the radial bus voltage change? In what proportions, if any?

c) Generators and their step-up transformers are usually connected in parallel to a common bus. In the absence of transformer resistances, there is an infinite number of optimal solutions for the generator terminal voltages. There is no such difficulty if resistances are present, but on their own, these do not lead to a good engineering solution. Extra constraints need to be explicitly added to share the VARs between the generators.

d) If a radial generator is connected to another generator through a purely reactive branch, there is no unique optimal solution for its voltage. The situation and its remedy are similar to that in c).

e) Apart from adding resistance, another fix to make problems solvable is to add a small term to the objective that mildly discourages the relevant (or all) variables from moving away from their initial or current points. Again, this term can introduce weighting-dependent arbitrariness, not achieving any very specific sensible engineering solution, and often preventing a true optimum from being reached.

f) When controlled shunts are separately represented in parallel, they have no unique solution unless their VAR sharing is defined and represented.

g) As previously mentioned in Section III-F3c) in connection with minimum deviation, mixed objectives can be dangerously unpredictable. Much of this stems from the arbitrariness of assigning relative weights to dissimilar quantities.

h) Load modeling has a major effect on the optimal solution, and carries the danger of obtaining totally unwanted engineering results. For instance, consider the classical "full OPF" objective of generating cost minimization via active and reactive power scheduling. If the MW load is modeled as predominantly decreasing with voltage, the bus voltages will tend to become as low as possible in order to reduce the total power delivered to the consumer.

i) Network equivalents do not exhibit correct transmission loss characteristics. Therefore, the use of reduced models electrically close to the optimized portion of the system is not compatible with voltage/VAR scheduling for cost or loss minimization [93].

j) There is an inherent lack of coordination between optimized controls and electrically close "local" conventional power-flow controls. This can lead to slower convergence or hunting. As an example, an LTC transformer operates to maintain a constant target voltage, while a nearby generator voltage tries to raise this and other voltages to minimize losses. Where severe conflicts can arise, it is better to redesignate a "local" control as optimized, with a narrow range on the target value.

k) Incompatibility between objective, controls, and constraints has to be avoided in formulating well-posed OPF problems. Some examples were given in Section III-B. Branch-flow limit enforcement with only voltage/VAR controls is very often unfeasible. Imposing limits on electrically remote internal and particularly external quantities causes difficulties.

T. Comments on On-Line Implementation

There are many problems associated with on-line OPF implementation that cannot be covered in this paper. This section mentions several of them. One obvious major area is interactivity with the operator, for which artificial intelligence [85] offers considerable future scope.

Great attention needs to be given to the entire field of closed-loop, operator-free OPF application. Both active and reactive power controls pose many problems. One issue is whether to dispatch calculated control-variable base points, or raise/lower signals. Another is how to schedule locally controlled apparatus such as LTC transformers and capacitors (e.g., should the controlled or controlling quantity be optimized and set?).

Another very important and difficult topic is the sequence in which control changes should be implemented, in order not to cause or exacerbate violations. That is, the power system needs to be steered from a bad operating point to a calculated good one without making things even worse in between. The factors involved include the response rates of the different controls. The problem seems to be much more difficult for reactive devices. Like many other problems, this requires further research and analytical innovations, as well as field experience.

Among the most pressing implementation questions is the interfacing and integration of real-time OPF with the other EMS application functions. A particular difficulty is that state estimation, contingency analysis, OPF, economic dispatch (ED), and automatic generation control (AGC) are not all executed with the same frequency. Therefore, even apart from the inherent time skews due to communication and calculation delays, the more frequent functions have to work with outdated information from the less frequent ones.

The most immediate problem is to impose the security-constrained generation schedules from OPF onto the ED function and thence onto AGC. The industry is examining a range of different possibilities. Programs must possess certain degrees of compatibility before they can be interfaced with each other. For instance, to have meaningful communication between OPF and ED, it seems necessary for the cost curve, limit, loss, and other models in the latter to be a subset of those in the former.

A promising approach is to install a security-constrained economic dispatch (SCED), whose core calculation *is* the conventional ED, or gives identical answers. That is, the conventional ED solution stands if no binding security limits are encountered. The SCED function fulfills the combined roles of OPF and ED in the active-power subproblem. It could alternatively accept security constraints from a separate OPF calculation. In conjunction, operating limits can be slightly adjusted according to whether the load is increasing or decreasing.

Another approach is to substitute a so-called parametric OPF for the ED function. This is a form of tracking OPF, that simply updates the previous OPF solution rapidly, respecting the same set of previously binding security constraints, but reflecting the change in system load. Reduced LP [87] and QP [22] versions have been described. Once again, considerable research and development is needed.

Other less sophisticated designs seek to retain the traditional ED function and merely alter its generation limits to reflect the current security constraints identified by the OPF. The general idea is to compare the security-constrained schedule with the corresponding conventional ED schedule. This establishes the constrained MW output of each generator either as an upper or lower limit. The "security limit" idea for controls, as well as for constraints, seems to have potential pitfalls, and must be scrutinized very carefully.

Communication of security constraints to AGC faces similar problems. If fresh security-constrained base points are calculated and supplied to AGC often enough, little or no special provisions may need to be made. The power system will simply not be given enough time to stray much into the infeasible region. Otherwise, some of the same ideas as above have been proposed: equivalent generating unit limits, and parametric security-constrained solutions. Another scheme is constrained AGC participation factors. The idea here is that participation factors can be calculated from a particular secure solution point, such that generation changes in response to natural load-level changes will not violate already-binding operating limits. Like equivalent limits, such a scheme seems to merit close examination.

The control hierarchy problem in security-constrained scheduling is extensive. No general practically usable methodology appears to exist. The OPF–ED–AGC issues represent a part of the time-related aspect. The other main aspect is geographical. The communication and coordination of optimal secure solutions between higher control and lower levels (e.g., power pool, company main control center, company subarea control, substation, distribution level, etc.), and between equal levels (power pools, company EMSs), poses formidable challenges.

The problems referred to in Sections III-F4d) and III-G about restricting the number of controls scheduled require further research, particularly for voltage/VAR control. Pronounced discreteness, as in switched capacitors, reactors, and series lines, is still difficult to handle.

U. Comments on Optimality of Security Control

Rigorous optimality of power system control, as measured by the mathematical solution of the classical security-constrained smooth OPF formulation, appears to be a rather fictitious and illusory goal. As we have tried to show, such a formulation is highly simplistic, and neglects many practical engineering requirements. These requirements can at best only be catered to in a near-optimal manner. This section discusses some additional related issues.

Electric power systems and their apparatus have always been designed and operated on the principle that the active- and reactive-power subproblems are weakly coupled to each other. This is reflected in the vast majority of conventional controls, such as economic dispatch, AGC, generator voltages/VARs, LTC transformers, and so on. It is also reflected in the widespread current trend towards developing and implementing on-line OPF for the active- and reactive-power subproblems separately from each other. Studies have been made, for example, showing that there is little extra cost benefit to be gained by performing voltage/VAR scheduling for loss minimization more frequently than every hour [92].

Advantage is often taken of the simpler and more economical models and algorithms afforded by active–reactive decoupling. Cross-coupling features, such as active-power scheduling for voltage constraints, are incorporated only when strictly necessary, as dictated by the given power system's characteristics.

At the same time, classical OPF theoretically promises the possibility for on-line control (scheduling, dispatch) in an *overall* optimal manner. That is, the global objective, normally cost, is minimized by simultaneously scheduling all system active and reactive power controls, subject to all constraints. We will refer to this as "full OPF."

The intuitive appeal of the full OPF solution is that it represents the best possible secure utilization of the power system network resources to supply the load. It also, of course, inherently caters for those network bottlenecks where the traditional separation into active and reactive subproblems is inadequate. A question to be answered by the industry is whether on-line full OPF is at all practically possible, and if so, whether its advantages offset the additional problems in applying it.

With current technology, it is already very difficult to resolve all the problems of providing even separate scheduling of active and reactive power in a reasonably reliable, comprehensive, rapid, and near-optimal manner. These difficulties include formulations, algorithms, computing effort, interactivity, real-time interfacing, need to schedule few voltage/VAR controls at a time, and coordination with the well-established active and reactive subproblem power system automatic controls (AGC, LTC transformers, etc.). Computing effort becomes a particularly severe problem when contingency constraints are included, and when not one but several or many OPF solutions need to be performed iteratively in order to incorporate control and constraint priorities, discretization, control-movement limitation, startup/shutdown, switching, etc. All of the difficulties will tend to increase if full OPF is applied.

In discussing degrees of optimality, it is important to note that there are many other factors with great influence on the end result, and which deserve commensurate attention. Optimal operation is not only a moving target, never to be achieved, but the models used are subject to large unquantifiable errors. The generator cost curves are very inaccurate. The network parameters and models, including discreteness, are approximated. The metered, estimated, and forecasted quantities that form the basis for the optimization are uncertain. Aspects such as time skews and system dynamics are partially or completely ignored. Not all operational constraints are represented. Soft limit values in general have a huge degree of arbitrariness. For instance, small reassessments of MW-related limits may affect operating cost more than the entire exercise of on-line loss minimization.

One potential compromise between full and subproblem OPFs is to perform full OPF at extended intervals of time, to establish scheduling trajectories for economic secure operation. Then in the intervening periods, the task of coping with all operating limits rapidly and reliably is assigned to separate constrained active and reactive scheduling calculations. Provision has to be made, however, for cross-coupling, mainly where voltage limits impose constraints on MW scheduling.

V. Concluding Remarks on Security-Constrained Scheduling

Optimal power flow, in all its forms, has enormous scope and potential for on-line application. Extensions and improvements in the technology are expected throughout the foreseeable future. Parallel developments are occurring in the operation and planning fields. There are many similarities, but the different applications create very different emphases in the model, algorithm, and other capabilities required for the respective production tools.

At present, on-line OPF is by far the most complex EMS application-function area, and there is a tremendous amount of work to be done to translate existing concepts into fully routine field application.

The active-power subproblem, which of course has by far the most influence on economy of operation, is much the more tractable. Considerably greater challenges are encountered in the reactive-power subproblem, whose main goal is to ensure good voltage control, often with transmission loss reduction as a worthwhile by-product. In certain systems, voltage/VAR constraints are especially important because they restrict the economic transmission of active power.

Excessive amount of calculation is one of the big obstacles to on-line OPF. The situation is particularly acute in a sophisticated implementation involving contingency constraints and OPF solutions that need to be iterated many times in order to incorporate discreteness and decision-making such as control and constraint priorities, control movement sequencing, startup/shutdown, and switching.

In the real-time mode, an important related question is how often and how accurately to run security-constrained scheduling. Finite computing resources impose a direct tradeoff between accuracy and speed, and therefore frequency of execution. This tradeoff becomes most apparent when considering closed-loop implementations. Given the moving-target nature of the power system operating state, and its capacity for rapid change, it seems far better where possible to opt for fast and frequent scheduling, using models and solutions that are incrementally correct but not necessarily iterated to very high accuracies.

This leads to the subject of OPF methods. The most "powerful" methods that solve smooth classical formulations with quadratic convergence appear to be at a disadvantage, particularly for real-time application. Each iteration is very time-consuming, providing the un-needed accuracy overkill characteristic of these methods. Many such iterations are needed to represent the nonsmooth real-life features of the problem.

The attraction of satisfying all OPF requirements in an EMS using a single basic solution approach and module is well recognized. Yet such a method is still beyond the technology's grasp. The main computational goals of speed, reliability, and flexibility seem to have large areas of mutual incompatibility. Perhaps the most important attribute of an on-line OPF implementation is that it should reliably respect *all* the necessary operating constraints. It is easy to foresee situations where an OPF calculation incorporating many but not all operating limits, however accurately modeled and optimized, is worse than useless.

Finally, another major issue is how effectively security-constrained OPF can be applied to a limited part of the interconnected power system, such as an individual utility or a subarea of the utility. This reiterates questions concerning the need and the technology available for complicated security control hierarchies, that might encompass power-pool and even higher levels.

IV. Overall Conclusions

This paper has tried to outline many of the current trends and ideas in on-line security analysis and optimal power flow. These combined areas are vitally important for secure economic power system operation. The on-line applications have equally wide reaching implications for system planning, including cost saving through postponement of expansion and other means.

Contingency analysis, already a standard feature of the modern EMS, is a relatively well-understood component of the economy-security problem. However, it consumes excessive computation. In an attempt to overcome the computing-effort barrier, some techniques have been using modeling and solutions whose reliabiliy is suspect, particularly in respect to voltages. The development of new faster approaches should take these reliability considerations into account.

On-line optimal power flow can take many forms, and it is expected that the technology will continue to branch out in different directions. OPF functions are being specified and installed in most new EMSs, rarely if ever with concrete prior knowledge about how valuable they will be. Nevertheless, it seems inevitable that OPF will eventually become accepted and used just as much as conventional power flow is today, but not necessarily always in the presently envisaged ways.

Contingency-constrained OPF is a cornerstone of the power system static security concept, but here the computational effort is even more exorbitant. Barring unforeseeable major breakthroughs, it appears that nonlinear contingency analysis and contingency-constrained OPF for large power systems can only be run at satisfactory intervals with much faster processing power than is typical of present EMSs.

In terms of what is already conceptually possible, there is still great potential for further improvement in power system security control. Better problem formulations, theory, computer solution methods, and implementation techniques are needed. The scope for innovation is enormous. Feedback from the utility companies' experience with the relevant EMS tools will bring many valuable extra insights and refinements to the subject.

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