(1) To find the eq'n, use trial + error + some deduction

given $x(1)=1$, $x(2)=2$ $x(3)=5$ $x(4)=12$ ...

"Find a 2nd order eq'n" means we will have an eq'n of the form $x(k)=f(x(k-1), x(k-2))$

observe
$$\left.\begin{aligned} 5 &= 2(2)+1(1) \\ 12 &= 2(5)+1(2) \end{aligned}\right\} \quad x(k) = 2x(k-1) + x(k-2)$$

Initial conditions

The number of initial conditions required is equal to the order of the model (of the system)

When you develop a model based on a physical system this should be apparent in that

# of dynamic elements = order of system

and clearly each element needs an initial condition

In the Matlab program

- Include your name, date, title of program (this can simply be HW2 problem #1 : for this assignment)
- Comment sections for constants; for initial conditions
- Comment the "for" loop + the output statement (if you include one)
- Include
  - constants
  - initial conditions
  - a "for" loop

```
% EGR 326 HW 2, #1
%
% Name: J Cardell
% Date: Spring 2012

% constants
y_end = 143;

% initial conditions
y(1) = 1;
y(2) = 2;


% Dynamic equation for loop
for k = 1 : y_end
    y(k+2) = 2*y(k+1) + y(k);
end

% results
disp ('The answer is ')
disp (y(y_end))
```

(2) for $y(k+2) - 2a\,y(k+1) + a^2 y(k) = 0$

show that a^k, and ka^k are solutions

<u>SOLUTION</u>: simply plug in each proposed solution and demonstrate the equality is true

$\boxed{a^k}$

$$y(k+2) - 2a\,y(k+1) + a^2 y(k) = 0$$
$$a^{k+2} - 2a \cdot a^{k+1} + a^2 a^k = 0$$
$$a^{k+2} - 2a^{k+2} + a^{k+2} = 0$$
$$2a^{k+2} - 2a^{k+2} = 0$$
$$0 = 0 \quad \text{QED}$$

$\boxed{ka^k}$

$$(k+2)a^{k+2} - 2a \cdot (k+1)a^{k+1} + (a^2)(k)(a^k) = 0$$
$$(k+2)a^{k+2} - 2(k+1)\,a^{k+2} + ka^{k+2} = 0$$
$$(k-2k+k)a^{k+2} + (2-2)a^{k+2} = 0$$
$$(0) \cdot a^{k+2} + (0) \cdot a^{k+2} = 0$$
$$0 = 0 \quad \text{QED}$$

(3) Dynamic model of bank balance

let k = months

$d(k) = d \rightarrow$ same deposit each month $\equiv$ input

i = interest, so $(1+i)\cdot$(balance) = new principal

define state variable $x(k)$ = balance of bank account

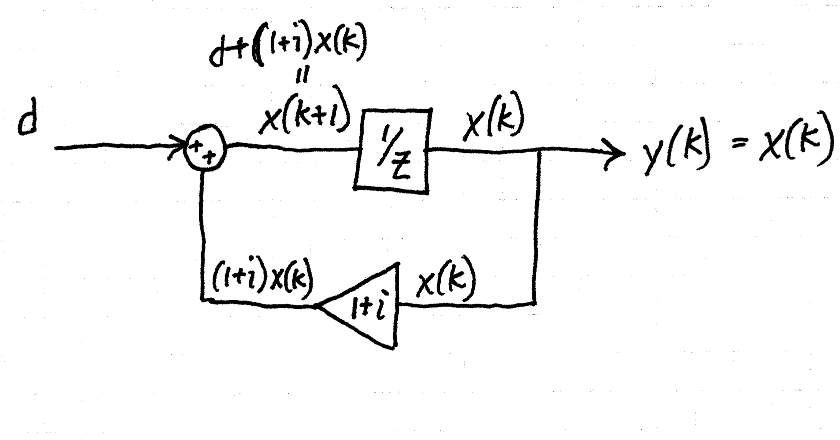
The new balance = $(1+i)\cdot$(prev. month balance) + (this month deposit)

$$x(k+1) = (1+i)\,x(k) + d$$

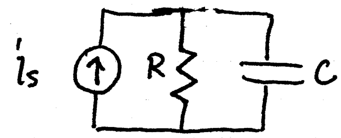
Select the output to be the balance : $y(k) = x(k)$

So the model is

$$x(k+1) = (1+i)\,x(k) + d$$
$$y(k) = x(k)$$



4 Dynamic model of // RC ckt



Element eq'ns

1. R: $V_R = i_R R$; $i_R = \frac{V_R}{R}$
2. C: $i_C = C\frac{dv_c}{dt}$

Interconnecting eq'ns, KCL, KVL

3. KCL: $i_s = i_R + i_C$
4. KVL $v_R = v_C$

State variable (1st order system = 1 state variable)

We know this is a 1st order system b/c there is only one dynamic element, the capacitor

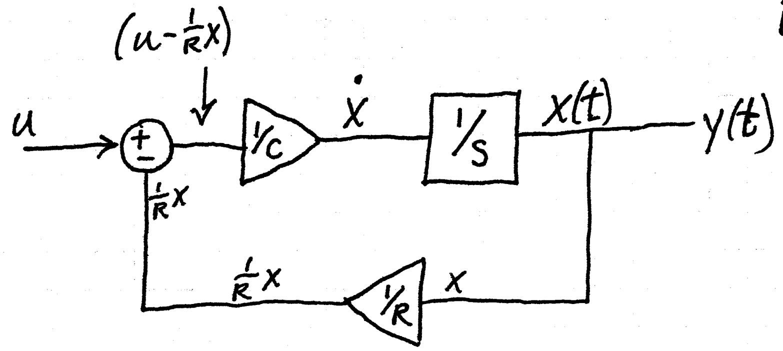
$x = v_c$

State eq'n $\dot{x} = f(x, u, t)$

from 1 & 4 we have $i_R = \frac{V_c}{R} = \frac{x}{R}$

subs into 3 $i_s = i_R + i_c$

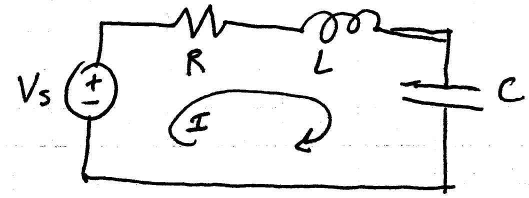
$$u = \frac{1}{R}x + C\dot{x} \Rightarrow \boxed{\begin{aligned}\dot{x} &= \frac{-1}{RC}x + \frac{1}{C}u \\ y &= x\end{aligned}}$$



block diagram for reference

HW2 EGR 326

(5) Series RLC ckt model



standard state variables

$x_1 = i_L$

$x_2 = v_C$

Note: using a current source makes this problem more difficult

element eq'ns R: $v_R = i_R R$, $i_R = \frac{v_R}{R}$ (1)

C: $i_c = C\frac{dv_c}{dt} = C\dot{x}_2$ (2)

L: $v_L = L\frac{di}{dt} = L\dot{x}_1$ (3)

KCL + KVL

KCL: $i_R = i_c = i_L \Rightarrow x_1$ (4)

KVL: $v_s = v_R + v_L + v_c \Rightarrow u = v_R + v_L + x_2$ (5)

Goal: 2 dynamic eq'ns of form $\dot{\mathbf{x}} = f(\mathbf{x}, u, t)$

$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

combine (1) + (4): $i_R = i_L = x_1 = \frac{v_R}{R}$ or $v_R = R x_1$ (6)

subs (6), (3) into (5): $u = Rx_1 + L\dot{x}_1 + x_2$

$L\dot{x}_1 = -Rx_1 - x_2 + u$

$\dot{x}_1 = -\frac{R}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}u$

combine (2) + (4): $\dot{x}_2 = \frac{1}{C}x_1$

Define $y = x_2$ for output

Write as matrix

$$\left.\begin{aligned} \dot{x}_1 &= -\frac{R}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}u \\ \dot{x}_2 &= \frac{1}{C}x_1 + 0 + 0 \\ y &= 0 + x_2 \end{aligned}\right\} \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

TOPS. 35502