Chain Letter (discrete time) = Defined in class

define the following

dynamic variable $x[k]$ ≡ # of letters at k^{th} step

dynamic equation $x[k+1] = a\,x[k] = 5a[k]$

with $a=5$ ⇒ each iteration causes a growth by a factor of 5 in the number of letters

assumed solution for $x[k] = a^k$

so $x[k] = 5^k$

initial condition $x[0] = 1$, the letter you receive

output expression : $y[k] = 0.1\,x[k]$ with $k=6$

$k=1$ you send letters to 5 friends

$k=2$ each of your friends sends letters to 5 friends

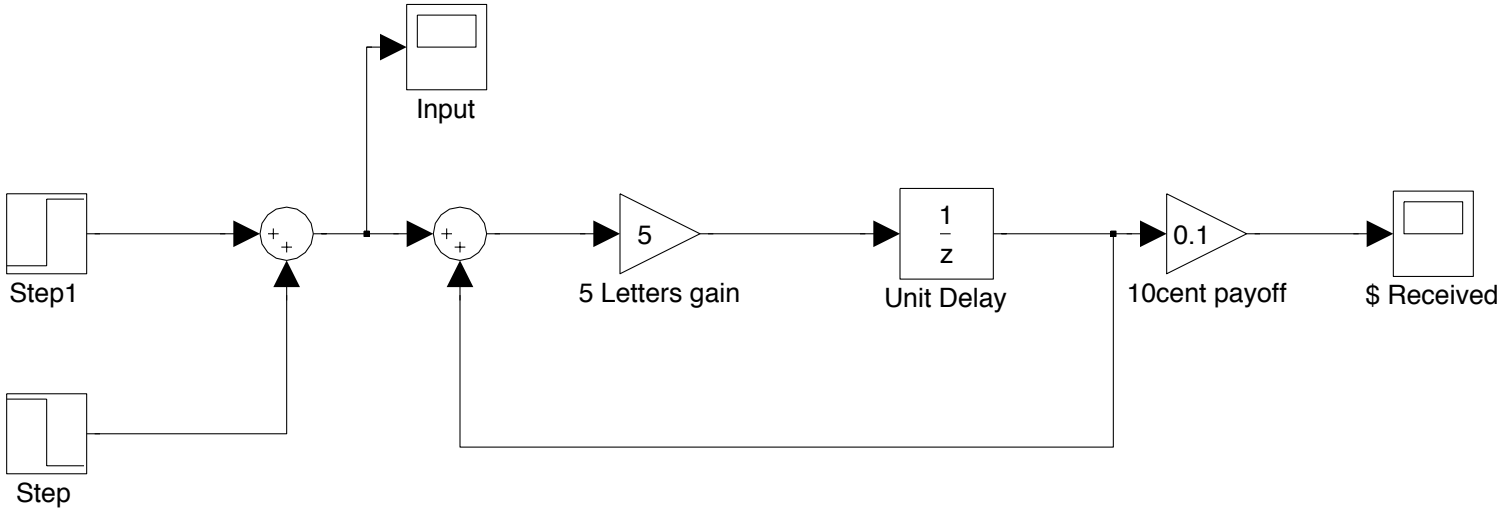
⋮

$k=5$ your name has reached the top of the list

$k=6$ your name is removed from the top of the list and all letter holders send you 10¢

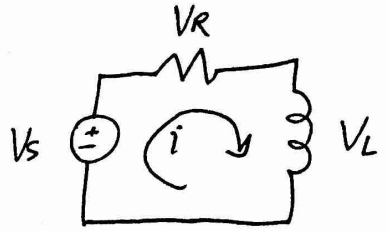
so $y[6] = (5^6)(0.1) =$ **$1562.5 you receive**

Chain Letter Dynamic Model



EGR 326 Class Example - circuit

RL circuit



KVL

$V_S = V_R + V_L$

$= iR + L\frac{di}{dt}$ let $V_S = 0V$, homogeneous eq'n

let $i(t) \equiv x(t)$

so $Rx + L\frac{dx}{dt} = 0$ or $Rx + L\dot{x} = 0$

Standard form is $\dot{x} = -\frac{R}{L}x$

---

from EGR 220

The known sol'n is of the form $x(t) = x_0 e^{t}$

let $\tau \equiv \frac{L}{R}$ then $x(t) = x_0 e^{-t/\tau}$

If we have $v_s \neq 0$ then we have

$Rx + L\dot{x} = v_s$ using "u" for the input variable and rearranging we have

$$\boxed{\dot{x} = -\frac{R}{L}x + \frac{1}{L}u} \Rightarrow \left\{\dot{x} = \left[-\frac{R}{L}\right]x + \left[\frac{1}{L}\right]u \text{ in matrix form}\right\}$$

And selecting the output of interest to you, you could select i, or v_R or anything

for output = i we have $y(t) = x(t)$

for output = v_R $y(t) = Rx(i)$