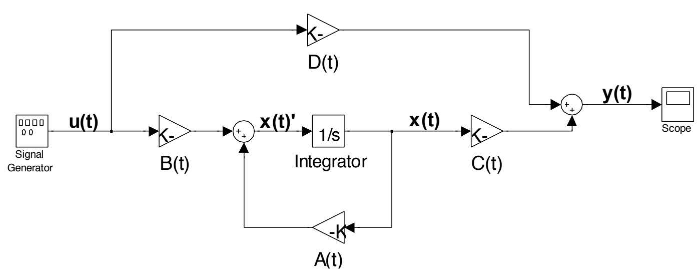
Standard Forms: Simulation Diagrams and State Space Representation

EGR 326
Feb 15 & 17, 2012

Overview

- Techniques to convert any dynamic model into state-space format
  - Input/Output equations (no state variables, only I/O)
  - Transfer functions (frequency domain, only I/O)
    - Ability to include input derivatives
  - Block diagrams (picture of time domain equations)
- …In order to be able to use state-space analysis tools for analyzing:
  - Behavior and modes of behavior
  - Stability & Control
- Canonical forms for **A** matrix

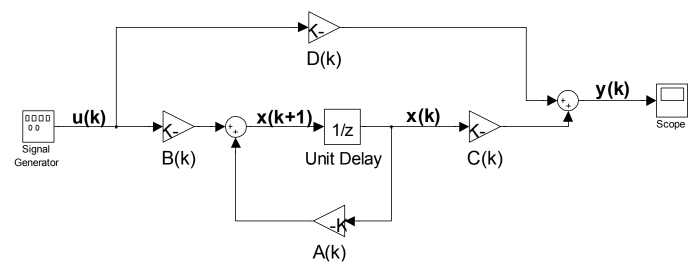
CT State Space Diagram



$$\dot{\mathbf{x}}(t_1) = \mathbf{g}(t_0, t_1, \mathbf{x}(t_0), \mathbf{u}_{[t_0,t_1]}) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t_1) = \mathbf{h}(\mathbf{x}(t_1), \mathbf{u}(t_1), t_1) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

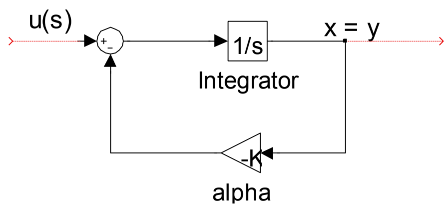
DT State Space Diagram



$$\mathbf{x}[k+1] = \mathbf{g}(\mathbf{x}[k], \mathbf{u}[k], k) = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k]$$

$$\mathbf{y}[k] = \mathbf{h}(\mathbf{x}[k], \mathbf{u}[k], k) = \mathbf{C}\mathbf{x}[k] + \mathbf{D}\mathbf{u}[k]$$

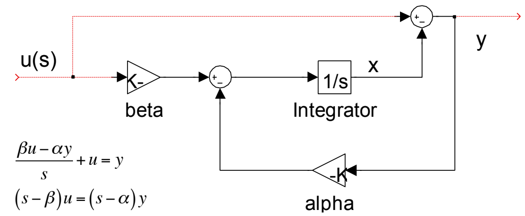
Basics ('poles' from EGR 220)



Derive the transfer function:

$$T(s) = \frac{y(s)}{u(s)} = \frac{1}{s+\alpha}$$

Basics ('zeros' from EGR 220)



$$\frac{\beta u - \alpha y}{s} + u = y$$

$$(s-\beta)u = (s-\alpha)y$$

Derive the transfer function:

$$T(s) = \frac{y(s)}{u(s)} = \frac{s-\beta}{s-\alpha}$$

Basics – Transform between 's' and 'time' domains

- Write this dynamic equation in transfer function form, and then back into I/O form, to be comfortable with the direct translation between the two forms

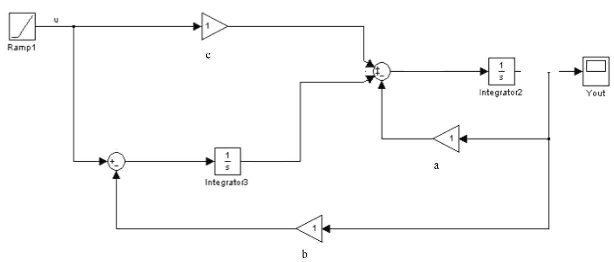
$$\ddot{y} + a\dot{y} + by = u + c\dot{u}$$

Canonical Forms

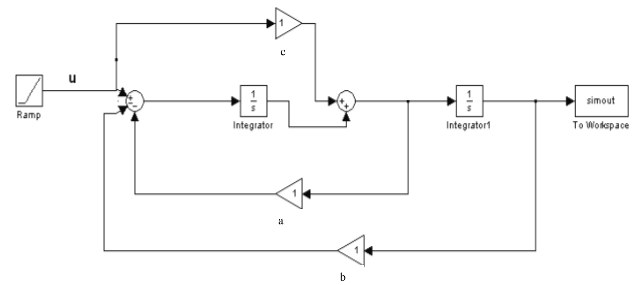
- Controllable canonical form and Observable canonical form
  - More on this in later chapters
- Example in handout demonstrates both canonical forms

$$\ddot{y} + a\dot{y} + by = u + c\dot{u}$$

Example 1



Example 2



Controllable Canonical Form

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 & \dots & -a_{n-1} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u(t) = \mathbf{A}\mathbf{x} + \mathbf{B}u(t)$$

$$y(t) = \begin{bmatrix} b_0 & b_1 & \dots & b_{n-1} \end{bmatrix} \mathbf{x}(t) = \mathbf{C}\mathbf{x}(t)$$

(Note form of C matrix allows for derivatives of the input.)

Observable Canonical Form

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & -a_{n-1} \\ 1 & 0 & 0 & \dots & 0 & -a_{n-2} \\ 0 & 1 & 0 & \dots & 0 & -a_{n-3} \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & \dots & 0 & -a_1 \\ 0 & 0 & 0 & \dots & 1 & -a_0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} b_{n-1} \\ b_{n-2} \\ b_{n-3} \\ \vdots \\ b_1 \\ b_0 \end{bmatrix} u(t) = \mathbf{A}\mathbf{x} + \mathbf{B}u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix} \mathbf{x}(t) + b_n u(t)$$

State Equations from Transfer Functions → 2 Options

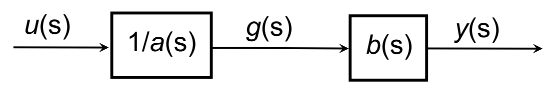
$$\frac{Y(s)}{u(s)} = T(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0} = \frac{b(s)}{a(s)}$$

- Y(s) = {u(s)*[1/a(s)]} * b(s) = g(s) * b(s)
  - → Controllable canonical form
  - (examples 1.5 & 1.6 in text)
- Y(s)*a(s) = u(s)*b(s)
  - → Observable canonical form
  - Details in later chapter; ‘nested integrator’ method

Controllable Canonical Form

$$\frac{Y(s)}{U(s)} = T(s) = \frac{b(s)}{a(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0}$$

Split *T(s)* and create “*g(s)*” as an intermediate stage between the denominator and numerator functions



$$\frac{y(s)}{u(s)} = \frac{b(s)}{a(s)} = \left(\frac{}{}\right)\cdot\left(\quad\right) \Rightarrow$$

Controllable Canonical Form

- This form results from
  - I/O equation using ‘phase variables’ for state variables
  - ‘Splitting’ the transfer function → Especially useful when there are ***derivatives of the input***
- Steps when “splitting” the transfer function
  1. Factor H(s) = ($1/_{a(s)}$) • (b(s)) = $^{Y(s)}/_{U(s)}$
  2. Introduce: g(s) = ($1/_{a(s)}$) • U(s)
  3. Write the state equations, using phase variables
     - x_1 = g; x_2 = g’ = x_1’; *etc.*
  4. Observe: Y(s) = b(s) • g(s)
  5. Write output equation, using the same phase variables

Practice: Develop Canonical Forms

$$T(s) = \frac{(s+3)}{(s^3 + 9s^2 + 24s + 20)}$$

Observable Canonical Form

- This form results from
  - 'Nested integrator' method, starting from I/O equation or transfer function
  - Captures the idea that integrators (and Σ) represent memory elements & the dynamic history of the system
- Steps for 'nested integrator' method
  1. Group terms with same order derivative.
  2. Write equation with highest order derivative term(s) on the left and all other terms on the right (still grouped).
  3. Insert 'integrators' in order to eliminate derivatives
  4. Assign a state variable to each nested integrand
     - x_n = innermost integrand, up to x_1 = RHS of equation (and thus also the LHS)

Observable Canonical Form

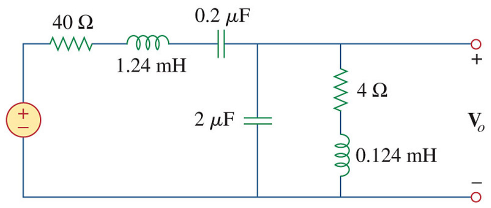
$$\ddot{y} + a\dot{y} + by = u + c\dot{u}$$

- *Task:* Using nested integrator method, obtain the observable canonical form of the state space model.

Practice: Develop Canonical Forms

$$T(s) = \frac{(s+3)}{(s^3 + 9s^2 + 24s + 20)}$$

Practice: Circuit Analysis



Practice: Circuit Analysis

$$T(s) = \frac{y(s)}{u(s)} = \frac{V_o}{V_i} =$$

Summary

- Techniques to convert any dynamic model into state-space format
- …In order to be able to use state-space analysis tools for analyzing
  - Behavior and modes of behavior
  - Stability
  - Control
- Canonical forms for **A** matrix