Least Cost System Operation: Economic Dispatch 1

Smith College, EGR 325
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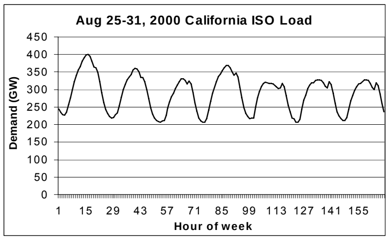
Overview

- Complex system time scale separation
- Least cost system operation
  - – Economic dispatch first view
  - – Generator cost characteristics
    - Four curves of generator performance
- Constrained optimization
  - – Linear programming
  - – Economic dispatch completed

- Long term system planning: <u>Production cost</u>
  - – Understand generator and load characteristics, and decide what to build to serve load
- Hourly to monthly decisions: <u>Unit commitment</u>
  - – Decide which plants to have warmed up and ready to go
  - – Different technologies have different requirements
- Minutes to Hour: <u>Economic dispatch</u>
  - – Decide which plants to use to meet the expected load now
  - – 5 minutes to 1 hour
- Cycles to Minutes: Short term system operations and <u>Load Flow Model</u>
  - – Maintain supply and demand balance moment to moment
  - – ~17msec per cycle up to 5 minute control functions

Power System Economic Operation

- The installed generating capacity is greater than the load at any specific moment
- This give us a lot of flexibility in deciding which generators to use to meet the load at any moment



What is "Economic Dispatch?"

- Economic dispatch (ED) determines the <u>least cost dispatch</u> of generation for a system.
  - – To dispatch ≡ To control generators to generate (more or less) energy
- Economic Dispatch (from EPACT 1992)
  - – The operation of generation facilities to produce energy at the lowest cost to reliably serve consumers, recognizing any operational limits of generation and transmission facilities.

Economic Dispatch Formulation

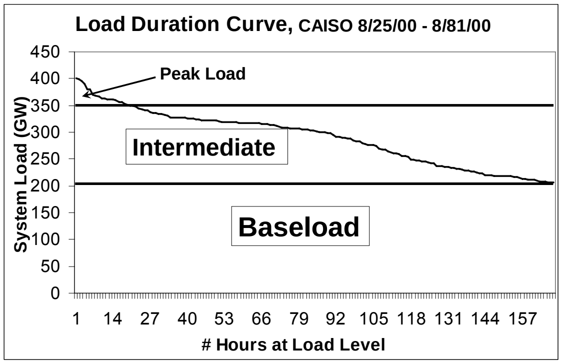
- Focusing on our objective
  - – How do we represent our objective mathematically?
  - – What mathematical tool do we use to obtain this objective?
- What does solving our (to be developed) set of equations help us to decide?

Economic Dispatch Formulation

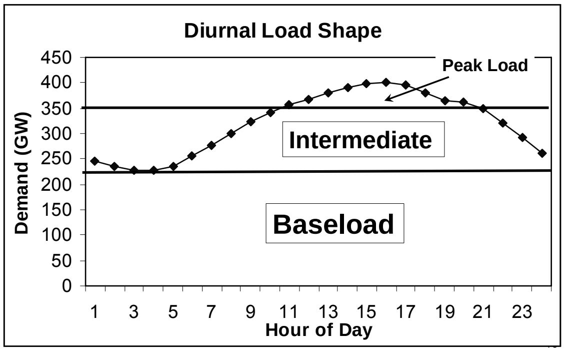
- Therefore we need to understand
  - How to represent system generating costs mathematically
    - Costs of operating (dispatching) generators
  - How to find the minimum system cost given
    - Generator costs and
    - System constraints
      - Such as: total generation must equal total demand
  - Constrained optimization via linear programming

Generator Cost Characteristics

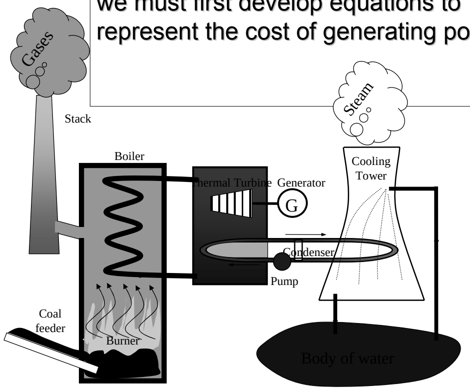
Generator Loading



Generator Loading



To minimize total system generating costs we must first develop equations to represent the cost of generating power

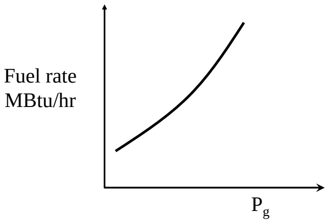


Generator Cost Curves

- Generator costs are determined by fuel costs and generator efficiency
  - We typically use a quadratic equation to model generator costs
- These costs are represented by four graphs defining unit performance
  - input/output (I/O) curve
  - fuel-cost curve
  - heat-rate curve
  - unit generating cost curve (and incremental cost curve)

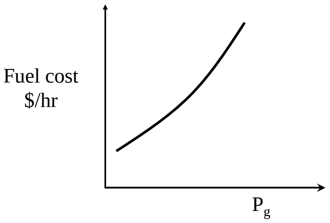
Input/Output Curve

- The I/O curve plots fuel input (in MBtu/hr) versus net MW output.



Fuel-cost Curve

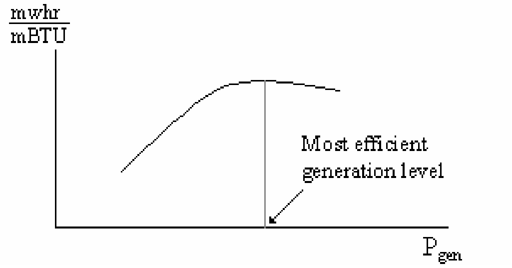
- The fuel-cost curve is the I/O curve scaled by fuel cost



An Efficiency Curve

- Efficiency = Output vs. Input
- Interpret this curve...

*** Efficiency changes with output level ***

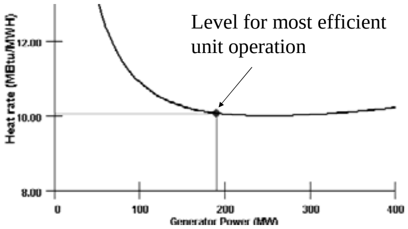


Most efficient generation level

The Heat Rate Curve

- Plots the average number of MBtu/MWhr of fuel input per MW of output
  - A version of efficiency
- Heat-rate curve is the I/O curve scaled by MW

** and is not constant **



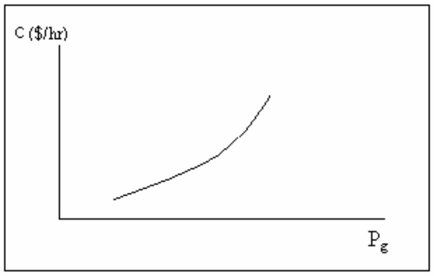
Level for most efficient unit operation

Heat Rates

- What is a heat rate?
  - Is a large or a small value preferable?
  - What are the units for a heat rate?
- Typical heat rate values
  - Coal plant is 10 mmBtu/MWh
  - Modern combustion turbine is 10 mmBtu/MWh
  - Combined cycle plant is 7 to 8 mmBtu/MWh
  - Older combustion turbine 15 mmBtu/MWh

Cost Curve

- Plots the \$/hr as a function of P_{gen} MW output
  - What are the units of each point on the graph?



Mathematical Formulation of Costs

- Typically curves can be approximated using
  - quadratic or cubic functions
  - piecewise linear functions
- Relying on the quadratic nature of HR, we will use a quadratic cost equation
- Standard quadratic representation is...?

$$C_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2 \quad \$/\text{hr}$$

Constrained Optimization

Linear Programming Definition

- Optimization is used to find the “best” value
  - “Best” defined by us, the analysts and designers
- Constrained optimization
  - Minimize/maximize an objective, subject to certain constraints
- Linear programming
  - Linear constraints
  - Some binding, some non-binding
- Visualize via a ‘feasible region’

Formulating the Linear Programming Problem

- Objective function
  - Decision variables
- Constraints
  - Bounds (limits) on the variables
- Standard form
  - min $c\mathbf{x}$
  - s.t. $A\mathbf{x} = \mathbf{b}$

    x_{min} <= x <= x_{max}

Economic Dispatch Recap

- Economic dispatch determines the best way to minimize the generator operating costs
  - Economic dispatch is not concerned with determining which units to turn on/off (this is the unit commitment problem)
  - Economic dispatch ignores the transmission system limitations

Formulating the Linear Programming Problem

- For power systems:

  min $C_T = \Sigma C_i(P_{Gi})$

  s.t. $\Sigma(P_{Gi}) = P_L$

  $P_{Gi\,min}$ <= P_{Gi} <= $P_{Gi\,max}$

- Our decision variables are ______?

Constrained Optimization & Economic Dispatch ➔The Lagrangean

Formulating the Lagrangean

- Rewrite the constrained optimization problem as an unconstrained optimization problem !
  - Then we can use the simple derivative (unconstrained optimization) to solve
- The task is to interpret the results correctly

Formulating the Lagrangean: The Theory from Calculus

- We are minimizing gradients of both multivariate equations
  - → C_T & $\Sigma P_{Gi} = P_L$
- For both equations to be at a minimum these gradients must be linearly dependent vectors
  - $\nabla C_T - \lambda \nabla w = 0$
  - with $w \equiv \Sigma P_G - P_L = 0$
- The "Lagrangean multiplier," λ
  - λ is defined to be the scaling variable that brings ∇C_T and ∇w into linear alignment

Lagrangean Example

max $g(x) = 5x_1^2 + x_2^2$

s.t. $h(x) = x_1 + x_2 = 6$

or $x_1 + x_2 - 6 = 0$

Formulate L =

$L = g(x) - \lambda h(x)$

Find ?

dL/dx_1, dL/dx_2, $dL/d\lambda$

$x_1 = 1$, $x_2 = 5$, $\lambda = 10$

Economic Dispatch & the Lagrangean

min $C_T = \Sigma C_i(P_{Gi})$

s.t. $\Sigma(P_{Gi}) = P_L$

$P_{Gi\ min} <= P_{Gi} <= P_{Gi\ max}$

Then *L* = ?

$$L = C_T - \lambda\left(\Sigma P_{Gi} - P_L\right)$$

Economic Dispatch Example

- What is the economic dispatch for a two generator problem with

  $P_{G1} + P_{G2} = P_L = 500MW$

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \quad \$/hr$$

$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \quad \$/hr$$

Economic Dispatch Example

- Formulate the Lagrangean
- Take derivatives
- Solve

$$\frac{dC_1(P_{G1})}{dP_{G1}} - \lambda \quad = 20 + 0.02P_{G1} - \lambda \quad = 0$$

$$\frac{dC_2(P_{G2})}{dP_{G2}} - \lambda \quad = 15 + 0.06P_{G2} - \lambda \quad = 0$$

$$500 - P_{G1} - P_{G2} = \quad 0$$

Economic Dispatch Example, cont'd

We therefore need to solve three linear equations

$$20 + 0.02P_{G1} - \lambda \quad = 0$$

$$15 + 0.06P_{G2} - \lambda \quad = 0$$

$$500 - P_{G1} - P_{G2} = \quad 0$$

$$\begin{bmatrix} 0.02 & 0 & -1 \\ 0 & 0.06 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -20 \\ -15 \\ -500 \end{bmatrix}$$

$$\begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} 312.5 \text{ MW} \\ 187.5 \text{ MW} \\ 26.2 \text{ \$/MWh} \end{bmatrix}$$

Economic Dispatch: Formulation

- We find that
  - – P_{G1} = 312.5MW;
  - – P_{G2} = 187.5MW

- λ = $26.2/MWh

Discussion

- Key results for Economic Dispatch?
  - – Incremental cost of all generating units is equal
  - – This incremental cost is the Lagrangean multiplier, λ
  - – 'λ' is called the 'System λ' and is the system-wide cost of generating electricity
    - This is the price charged to customers

Mathematical Formulation of Costs

- From total cost to marginal cost...
- The *marginal cost* is one of the most important quantities in operating a power system
  - – Marginal cost = incremental cost: ***the cost of producing the next increment of power (the next MWh)***

Summary

- Formulated the economic dispatch problem *conceptually*
- Examined the mathematical origin for generator costs
  - – Defined heat rate
- Developed mathematical formulation of the economic dispatch problem