

AC Circuits, the Transmission System & Power Flow

Smith College, EGR 325
March 27, 2008

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Review (on own) Objectives

- EGR 220
 - Sinusoidal waveform representation
 - Root mean square
 - Phase shift
 - Phasors
 - Complex numbers
 - Complex impedance
- Electric Power
 - Complex: real & reactive power
 - Power factor and power factor correction

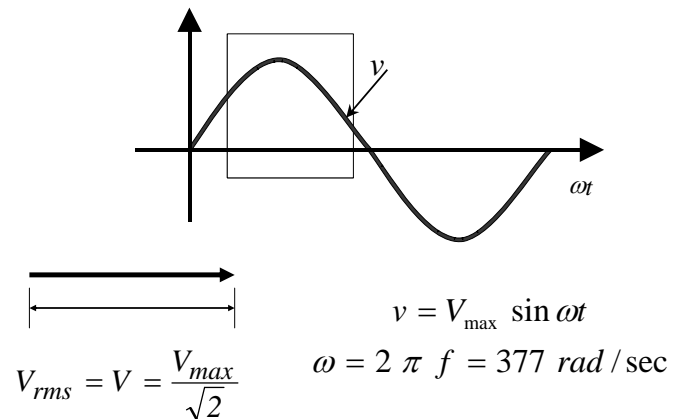
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Objectives

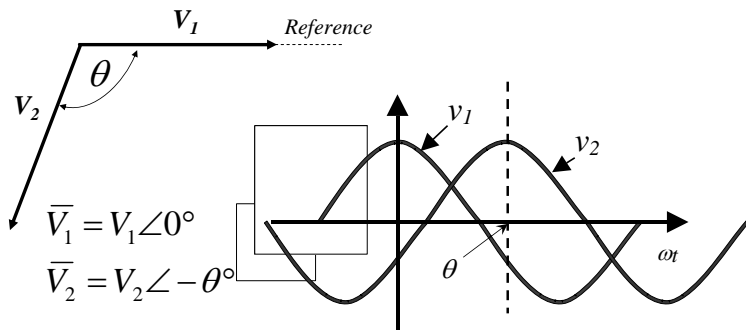
- E-M energy storage
 - Capacitance & inductance
- Power calculations and terminology
- Expand understanding of electrical power
 - from simple linear circuits to
 - a high voltage power system

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AC Phasor Representation



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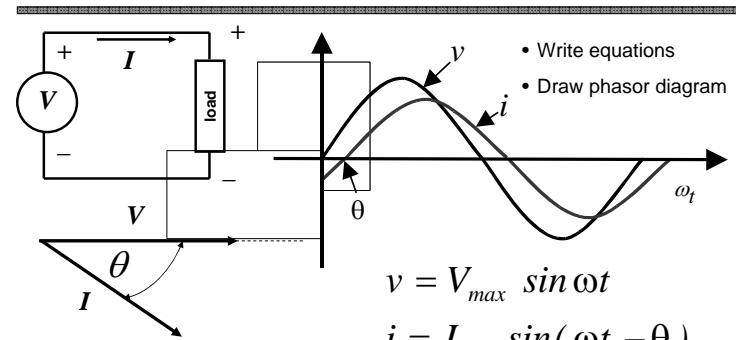


$$v_1 = V_{1\max} \cos \omega t$$

$$v_2 = V_{2\max} \cos(\omega t - \theta)$$

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Power Factor Angle



$$\bar{V} = V \angle 0^\circ$$

$$\bar{I} = I \angle -\theta^\circ \text{ also known as the power factor angle}$$

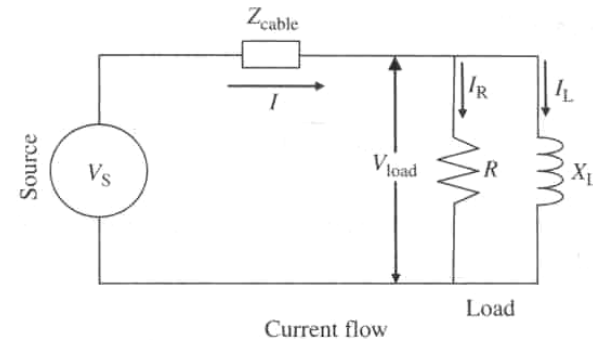
It is due to the presence of inductive and capacitive elements.

Example

- The circuit parameters are
 - $V_s = 120\text{V}$ at 60Hz
 - $R_L = 15\Omega$ and $L_L = 0.1\text{H}$
 - $R_{\text{cable}} = 0.5\Omega$ and $L_{\text{cable}} = 0.05\text{H}$
- Find
 - The load impedance
 - The line current
 - The load voltage
 - The complex current
 - The complex load power consumed
 - The complex power lost in the cable
 - The complex power supplied by the source

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Example

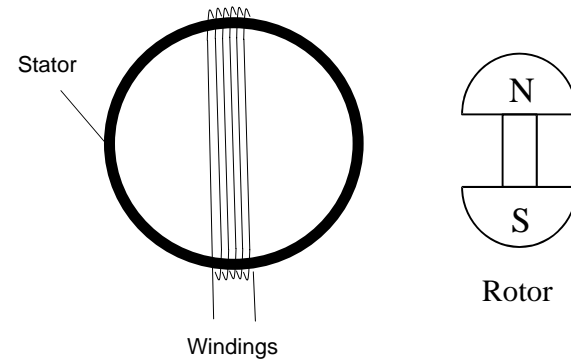


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Representing Power

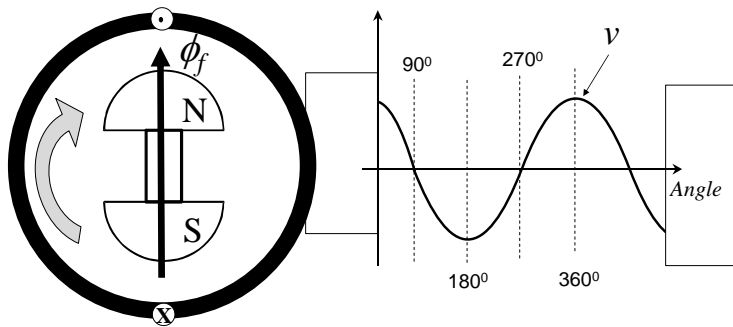
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How AC is Generated



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How AC is Generated



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Power Calculations

- $P = VI$
- $P = I^2R$
- $P = V^2/R$
- $S = VI^*$
- $S = I^2Z$
- $S = V^2/Z$

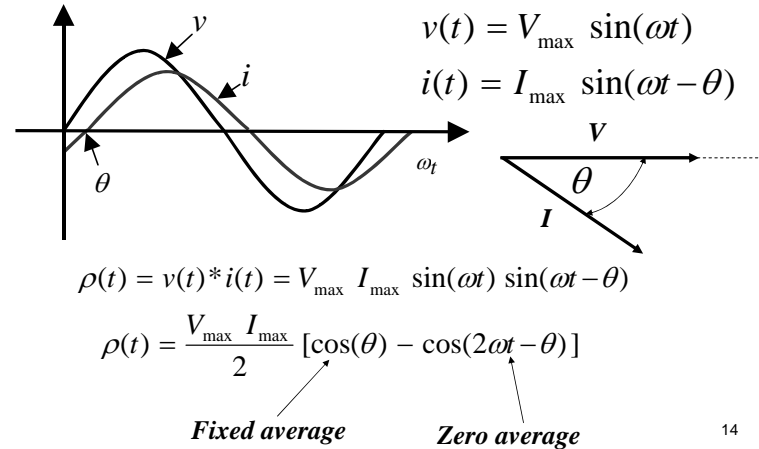
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Resistance → Impedance

- Complex power arises from complex impedance
- $Z = R + jX$
- $X_C = ?$
- $X_L = ?$
- $Y = ?$

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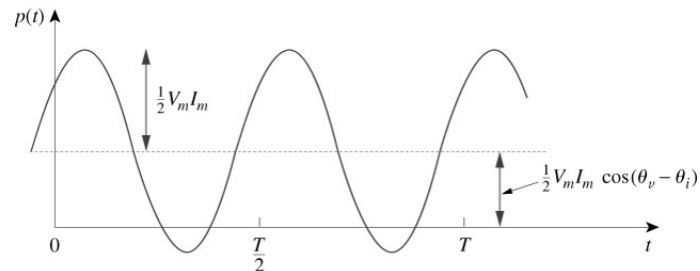
Instantaneous Electric Power [$p(t)$]



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Instantaneous vs. Average Power

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$



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Instantaneous vs. Average Power

- Instantaneous power is written as
- $$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$
- The average of this expression is

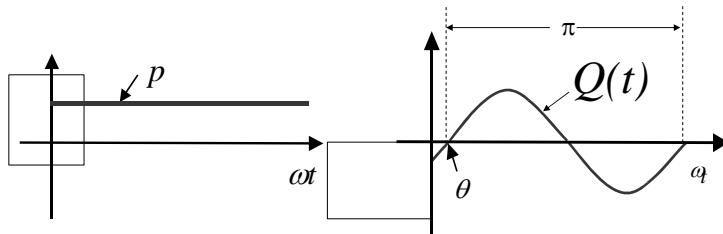
$$P = \frac{1}{2} V_m I_m \underline{\cos(\theta_v - \theta_i)}$$

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Real & Reactive Power – Time Domain

$$p(t) = \frac{V_{\max} I_{\max}}{2} [\cos(\theta_{vi}) - \cos(2\omega t - \theta_{vi})]$$

$$p(t) = P + Q(t)$$



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Power: Instantaneous vs. Average

- Power is equal to (voltage) x (current)
- Time Domain,
 - Instantaneous – at a given moment in time
- Phasor Domain
 - Power generated is a complex quantity
 - $\mathbf{S} = \mathbf{VI}^* = P + jQ$
 - Real, Average – over a specified time period
 - Reactive, Stored – oscillating with time

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Concept Check: Power & Phase Angle

- Writing average power in phasor notation:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$\mathbf{V} = V_m \angle \theta_v$$

$$\mathbf{I} = I_m \angle \theta_i$$

$$\frac{1}{2} \mathbf{VI} =$$

$$\frac{1}{2} \mathbf{VI}^* =$$

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Phasors: Instantaneous vs. Average Power

- Therefore

$$P = \frac{1}{2} \text{Re}[\mathbf{VI}^*] = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$
- What does P equal when
 - $\theta_v = \theta_i$
 - $\theta_v - \theta_i = \pm 90^\circ$
- What types of circuits do these situations represent?

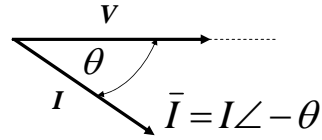
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Complex Power – Phasor Domain

IMPORTANT

θ is the power factor angle

$$\bar{S} \equiv \bar{V} \bar{I}^*$$



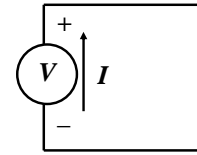
$$\bar{S} = \bar{V} \bar{I}^* = V \angle 0 \cdot I \angle -\theta = VI \angle \theta$$

$$\bar{S} = VI \cos \theta + j VI \sin \theta$$

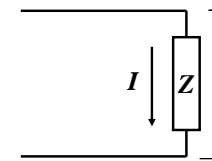
$$\bar{S} = \underset{\substack{\nearrow \\ \text{Real Power}}}{P} + j \underset{\substack{\nwarrow \\ \text{Reactive Power}}}{Q}$$

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Polarities



Source

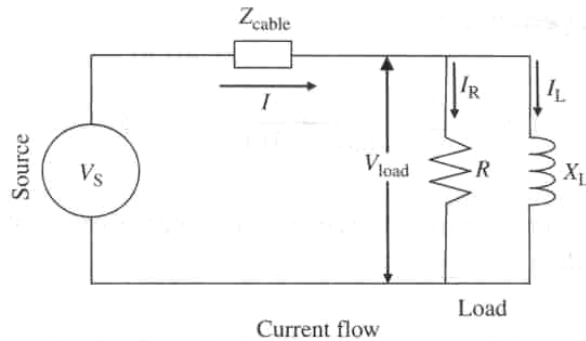


Load

- Positive sign convention from EGR 220
 $p = +vi$ if current enters the positive terminal
 $p = -vi$ if current enters the negative terminal

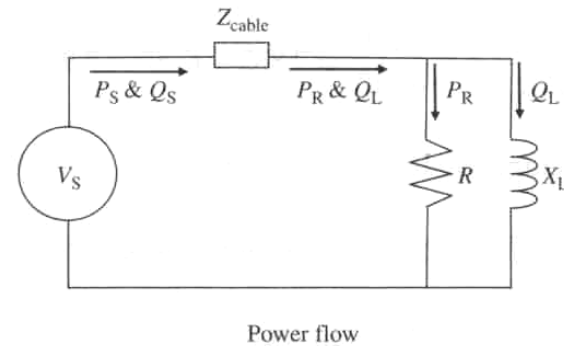
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Example: Current Flow



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Example: Power Flow



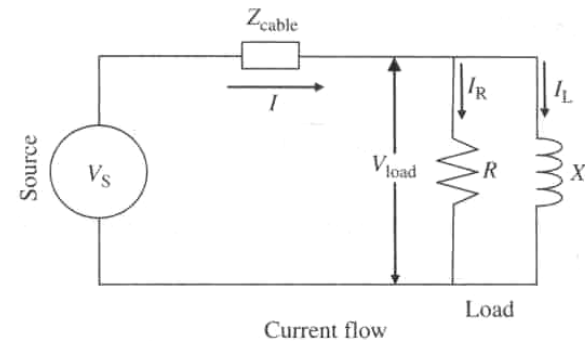
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Example: Power Calculations

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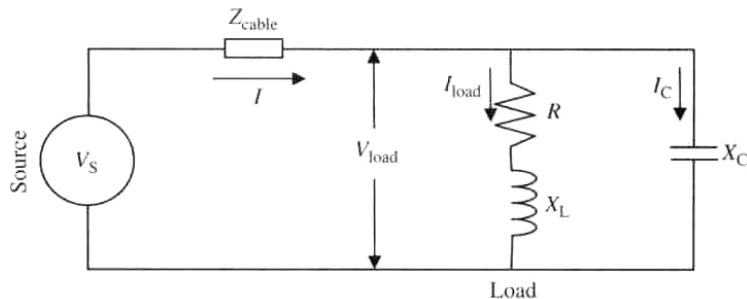
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Example



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Example: pf Compensation



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Summary

- AC circuits
 - Sinusoidal steady state
- Energy storage elements
 - C & L: Source of phase shift
- Phasors & phase angle
- Complex power
 - Real and reactive power
 - Significance of the *power factor*
- Power calculations

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