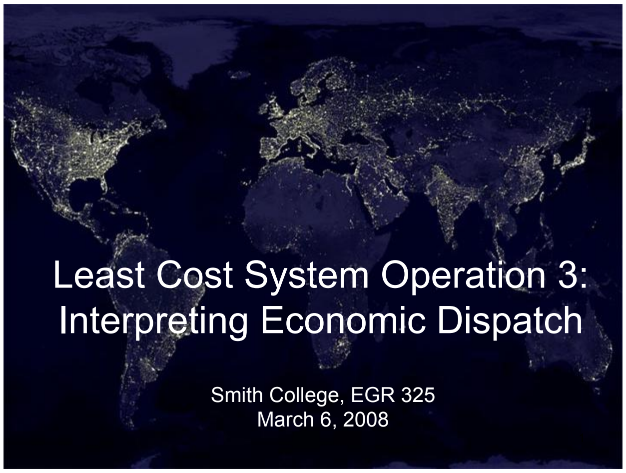


Formulating the Linear Programming Problem

- For power systems we have:
  - min $C_T = \Sigma C_i(P_{Gi})$
  - s.t. $\Sigma(P_{Gi}) = P_L$
  - $P_{Gi\ min} <= P_{Gi} <= P_{Gi\ max}$
- Identify the
  - Objective function
  - Constraints
  - Decision variables

Formulate the ED Problem Using the Lagrangean

min $C_T = \Sigma C_i(P_{Gi})$

s.t. $\Sigma(P_{Gi}) = P_L$

$P_{Gi\ min} <= P_{Gi} <= P_{Gi\ max}$

Then L = ?

$$L = C_T - \lambda\left(\Sigma P_{Gi} - P_L\right)$$

Economic Dispatch Example

- What is the economic dispatch for a two generator problem with
  - $P_{G1} + P_{G2} = P_L = 500\text{MW}$

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \quad \$/hr$$
$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \quad \$/hr$$

Economic Dispatch Example

- Formulate the Lagrangean
- Take derivatives
- Solve

$$\frac{dC_1(P_{G1})}{dP_{G1}} - \lambda = 20 + 0.02P_{G1} - \lambda = 0$$
$$\frac{dC_2(P_{G2})}{dP_{G2}} - \lambda = 15 + 0.06P_{G2} - \lambda = 0$$
$$500 - P_{G1} - P_{G2} = 0$$

Ex 2: Problem Statement

- A power system has four generators with the following cost characteristics
  - $C_1 = 1000 + 15P_1 + 0.05P_1^2$ \$/MWh
  - $C_2 = 1200 + 25P_2 + 0.12P_2^2$ \$/MWh
  - $C_3 = 2060 + 20P_3 + 0.01P_3^2$ \$/MWh
  - $C_4 = 2500 + 12P_4 + 0.03P_4^2$ \$/MWh
- Typical demand levels are:
  - → 750MW, 1000MW, 1500MW, and 2500MW

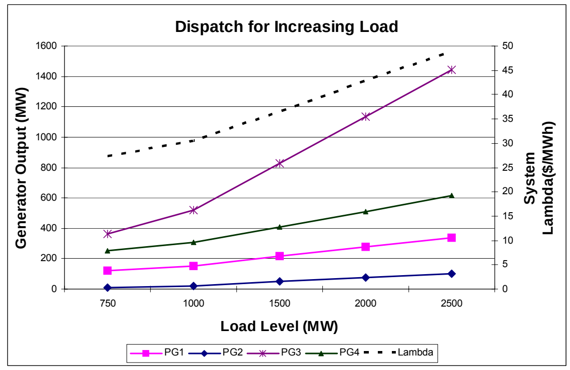
Ex 2: Matlab Results

- For changing load levels:

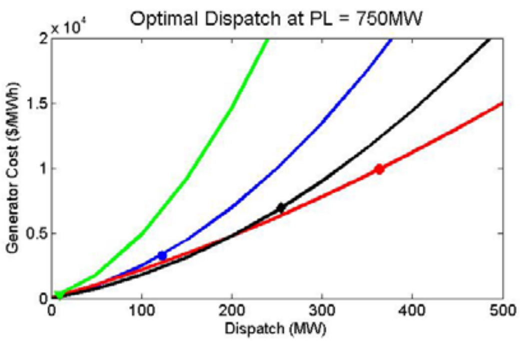
PL	PG1	PG2	PG3	PG4	Lambda
750	123	9	363	254	27
1000	154	22	518	306	30
1500	216	48	827	409	37
2000	277	74	1137	512	43
2500	339	100	1446	615	49

(the units are ___?)

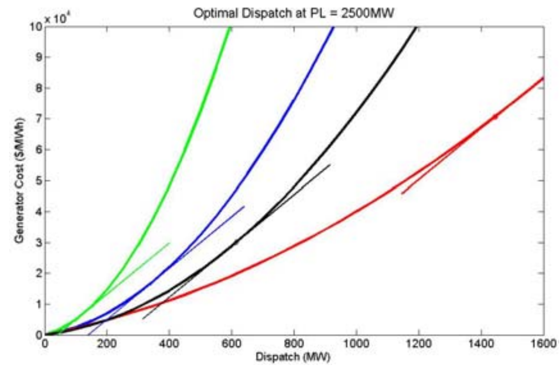
Dispatch and Price Results Plotted



Visualizing Equal Marginal Cost; Quadratic Cost Curves Graphed



Visualizing Equal Marginal Cost



Significance of Marginal Costs

- From total cost to marginal cost...
- The *marginal cost* is one of the most important quantities in operating a power system
  - Marginal cost = incremental cost: ***the cost of producing the next increment of power (the next MWh)***

Super-Important Results

- At the least cost dispatch point, the incremental cost of all generating units is equal
- This incremental cost **is** the Lagrangean multiplier, λ
- For power systems, 'λ' is called the 'System λ' and is the system-wide cost of generating electricity
  - This is the price charged to customers

Now Include Inequality Constraints

$\min C_T = \Sigma C_i(P_{Gi})$
s.t. $\Sigma(P_{Gi}) = P_L$
$P_{Gi\,min} <= P_{Gi} <= P_{Gi\,max}$

- Rewrite the inequality constraints
- Add slack variables
- Solve via the Lagrangian

ED With Inequality Constraints

- What is the economic dispatch for a two generator problem with

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \quad \$/hr$$
$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \quad \$/hr$$

$P_{G1} + P_{G2} = P_L = 500$ MW
$P_{G1} <= 300$ MW

- Write out the Lagrangean formulation with the inequality constraint

ED With Inequality Constraints

- Interpret our results
  - What is system-lambda now, and what does it represent?
  - What is the value of the slack variable associated with our inequality constraint and what does it represent?
  - What inefficiencies have been introduced into our solution as a result of the binding generator limit?

Summary

- Develop the mathematical origin for generator costs
  - Defined heat rate
- Formulate the economic dispatch problem *conceptually*
- Develop mathematical formulation for solving the economic dispatch problem
- Interpret the results, including the Lagrangean multiplier and slack variables