Least Cost System Operation 2: Interpreting Economic Dispatch

Smith College, EGR 325
March 4, 2008

Overview

- Complex system time scale separation
- Least cost system operation
  - – Economic dispatch first view
  - – Generator cost characteristics
- Constrained optimization
  - – Linear programming
  - – Economic dispatch completed
- Interpretation of economic dispatch results
- HW feedback

Power System Analysis

1. Decide what to build
2. Given the plants that are built ⇨ decide which plants to have warmed up and ready to generate…
3. Given the plants that are ready to generate ⇨ decide which plants to use to meet the expected load today, the next hour…
4. Given the plants that are generating ⇨ Decide how to maintain the supply and demand balance cycle to cycle

Expanding our understanding of generating plant behavior and cost curves – specific example: Efficiency of Coal Usage

Coal Usage Example

- A 500 MW (net) generator is 35% efficient
- It is supplied with coal
  - – Cost = $1.70/MBtu
  - – Energy content = 9000 Btu/lb

1. What is the coal usage in lbs/hr?
2. What is the cost of generating a MWh?
   - – Conversion factor of 0.2928MWh/MBtu might be useful (note mmBtu ≡ MBtu)

→ Set up the solution, keeping track of the units

Coal Usage & Costs Example

- With 35% efficiency, the fuel input is

$$\frac{500MWh}{h} x \frac{1}{0.35} x \frac{1mmBtu}{0.2928MWh} = 4879 mmBtu/h$$

$$\frac{4879mmBtu}{h} x \frac{1lb}{9000Btu} = 542{,}111 lb/h$$

- The cost is

$$\frac{4879mmBtu}{h} x \frac{\$1.70}{mmBtu} = \$8{,}294.30/h$$

$$\frac{\$8294.3}{h} x \frac{h}{500MW} = \$16.60/MWh$$

Coal Usage & Costs Example

- Assuming, first, a linear approximation
  - $/MWh = fuelcost * heatrate + variable O&M
- Re-examining the previous example
  - (With conv. factor 0.29MWh/MBtu)

$$\text{HR} = \frac{1}{0.35}\, x \,\frac{1 mmBtu}{0.2928 MWh} = 9.758 mmBtu / MWh$$

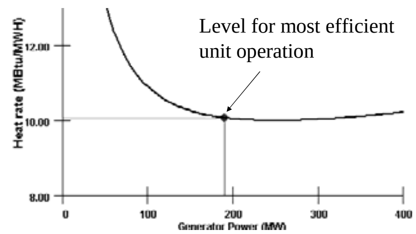
$$\text{Cost} = \$1.70 / mmBtu \; x \; 9.758 = \$16.60 / MWh$$

- But... the heat rate is not a constant!
- In HW, use quadratic HR curve

7

The Heat Rate Curve

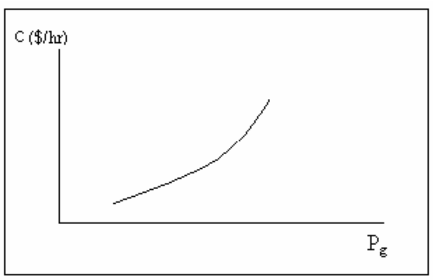
- Plots the average number of MBtu/MWhr of fuel input per MW of output
- Heat-rate curve is the I/O curve scaled by MW



8

Cost Curve

- Plots the $/hr as a function of P_{gen} MW output
  - What are the units of each point on the graph?



9

Mathematical Formulation of Costs

- Typically curves can be approximated using
  - quadratic or cubic functions
  - piecewise linear functions
- Relying on the quadratic nature of HR, we will end up with a quadratic cost equation
- Standard quadratic representation is

$$C_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2 \quad \$/\text{hr}$$

10

Constrained Optimization

11

Linear Programming Definition

- Optimization is used to find the “best” value
  - “Best” defined by us, the analysts and designers
- Constrained optimization
  - Minimize an objective function, subject to given constraints
- Linear programming
  - Linear constraints
  - Some binding, some non-binding
- Visualize via a ‘feasible region’

12

Formulating the Linear Programming Problem

- For power systems we have:

  min $C_T = \Sigma C_i(P_{Gi})$

  s.t. $\Sigma(P_{Gi}) = P_L$

  $P_{Gi\ min} <= P_{Gi} <= P_{Gi\ max}$
- Identify the
  - Objective function
  - Constraints
  - Decision variables

13

Formulate the ED Problem Using the Lagrangean

min $C_T = \Sigma C_i(P_{Gi})$

s.t. $\Sigma(P_{Gi}) = P_L$

$P_{Gi\ min} <= P_{Gi} <= P_{Gi\ max}$

Then *L* = ?

$$L = C_T - \lambda\left(\Sigma P_{Gi} - P_L\right)$$

14

Economic Dispatch Example

- What is the economic dispatch for a two generator problem with

  $P_{G1} + P_{G2} = P_L = 500\text{MW}$

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \quad \$/hr$$

$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \quad \$/hr$$

15

Economic Dispatch Example

- Formulate the Lagrangean
- Take derivatives
- Solve

$$\frac{dC_1(P_{G1})}{dP_{G1}} - \lambda = 20 + 0.02P_{G1} - \lambda = 0$$

$$\frac{dC_2(P_{G2})}{dP_{G2}} - \lambda = 15 + 0.06P_{G2} - \lambda = 0$$

$$500 - P_{G1} - P_{G2} = 0$$

16

Significance of Marginal Costs

- From total cost to marginal cost...
- The *<u>marginal cost</u>* is one of the most important quantities in operating a power system
  - Marginal cost = incremental cost: ***<u>the cost of producing the next increment of power (the next MWh)</u>***

17

Economic Dispatch: Formulation

- For our problem, we find that
  - ➢P_{G1} = 312.5MW
  - ➢P_{G2} = 187.5MW
- λ = $26.2/MWh
- **<u>Question</u>**: What happens if we dispatch the generators at different output levels?
- **<u>Note</u>**: The values of the decision variables change with each value for P_L

18

Super-Important Results

- At the least cost dispatch point, the incremental cost of all generating units is equal
- This incremental cost **is** the Lagrangean multiplier, λ
- For power systems, 'λ' is called the 'System λ' and is the system-wide cost of generating electricity
  - This is the price charged to customers
  - … given that we are ignoring power losses on the transmission lines
- In linear programming jargon, λ is the shadow price → what does this mean in terms of the objective function?

Now Include Inequality Constraints

$$\min C_T = \Sigma C_i(P_{Gi})$$
$$\text{s.t. } \Sigma(P_{Gi}) = P_L$$
$$P_{Gi\ min} <= P_{Gi} <= P_{Gi\ max}$$

- Rewrite the inequality constraints
- Add slack variables
- Solve via the Lagrangian

ED With Inequality Constraints

- What is the economic dispatch for a two generator problem with

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \quad \$/hr$$
$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \quad \$/hr$$

$P_{G1} + P_{G2} = P_L = 500$ MW

$P_{G1} <= 300$ MW

ED With Inequality Constraints

- For our problem, we previously found that
  - $P_{G1} = 312.5$MW;
  - $P_{G2} = 187.5$MW
  - λ = \$26.2/MWh
- But now P_{G1} output must be <= 300 MW, so what should we do?
  - Re-solve the Lagrangean and get stuck in recursive sets of constrain equations?
  - Set $P_{G1} = 300$ MW and observe that we then require $P_{G2} = 200$ MW

ED With Inequality Constraints

- Note, for the HW
  - If we have three generators, and you find that one of them must be set equal to a limiting value,
  - Then we have two generators remaining, and we cannot simply 'observe' the economic dispatch solution…
  - In this case, re-solve the Lagrangean problem with the remaining 2 generators, and the value P_L adjusted as is appropriate

Summary

- Develop the mathematical origin for generator costs
  - Defined heat rate
- Formulate the economic dispatch problem *conceptually*
- Develop mathematical formulation for solving the economic dispatch problem
- Interpret the results, including the Lagrangean multiplier and slack variables