Overview

- Least cost system operation
  - Economic dispatch
  - Recap: Generator cost characteristics
- Constrained optimization
  - Linear programming
  - Formulating and solving the Lagrangian
- Interpretation of economic dispatch results
  - Interpreting marginal cost and ‘System λ’

Linear Programming Definition

- Constrained optimization
  - Minimize an objective function, subject to given constraints
- Linear programming implies...
  - Linear constraints
  - Some binding, some non-binding
- Visualize via a ‘feasible region’
- Write the standard *economic dispatch* problem for power system operations

Formulating the Linear Programming Problem

- Objective function
  - Decision variables
- Constraints
  - Bounds (limits) on the variables
- Standard form
  - \( \min f(x) \)
  - s.t. \( Ax = b \)
  - \( x_{\text{min}} \leq x \leq x_{\text{max}} \)
Formulating the Linear Programming Problem

• For power systems:
  \[
  \min C_T = \Sigma C_i(P_{G_i})
  \]
  s.t. \( \Sigma (P_{G_i}) = P_L \)
  \[
  P_{G_{i\min}} \leq P_{G_i} \leq P_{G_{i\max}}
  \]

• Our “decision variables” are ______?

Some Background: the Theory from Calculus

• We minimize gradients of both multivariate equations
  \[ C_T \] & constraint ‘w’: \( \Sigma P_{G_i} = P_L \)

• For all equations to be at a minimum, find the linearly dependent shared minimum of the gradients
  \[ \nabla C_T = \lambda \nabla w = 0 \]
  \[ \text{with } w = \Sigma P_{G_i} - P_L = 0 \]

• The “Lagrange multiplier,” \( \lambda \)
  \[ \text{– } \lambda \text{ is defined to be the scaling variable that brings } \nabla C_T \text{ and } \nabla w \text{ into linear alignment} \]

To Solve: Formulate the “Lagrangian”

• Rewrite the constrained optimization problem as an unconstrained optimization problem
  – Then we can use the simple derivative (unconstrained optimization) to solve
  – Need to introduce a new variable – the “Lagrange multiplier” lambda, \( \lambda \)

• The task is to interpret the results correctly

Lagrangian Practice Example

\[
\max g(x) = 5x_1^2 + x_2^2
\]
\[
\text{s.t. } h(x) = x_1 + x_2 = 6 \quad \text{or} \quad x_1 + x_2 - 6 = 0
\]

Formulate \( L = g(x) - \lambda h(x) \)
Find \( ? \)
\[
dL/dx_1, \ dL/dx_2, \ dL/d\lambda \\
x_1 = 1, \ x_2 = 5, \ \lambda = 10
\]
Formulating the Linear Programming Problem

• For power systems we have:
  \[
  \min C_T = \Sigma C_i(P_{G_i}) \\
  \text{s.t. } \Sigma (P_{G_i}) = P_L \\
  P_{G_i \text{min}} \leq P_{G_i} \leq P_{G_i \text{max}}
  \]

• Identify the
  – Objective function
  – Constraints
  – Decision variables

Economic Dispatch Example

• What is the economic dispatch for a two generator problem with:
  (The cost curve units are $/hr)
  \[
  C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \\
  C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \\
  P_{G1} + P_{G2} = P_L = 500 \text{ MW}
  \]

Formulate the ED Problem Using the Lagrangian

\[
\min C_T = \Sigma C_i(P_{G_i}) \\
\text{s.t. } \Sigma (P_{G_i}) = P_L \\
P_{G_i \text{min}} \leq P_{G_i} \leq P_{G_i \text{max}}
\]

Then \(L = ?\)

\[
L = C_T - \lambda \left( \Sigma P_{G_i} - P_L \right)
\]

Economic Dispatch Example

• Formulate the Lagrangian
• Take derivatives
• Solve

\[
\frac{\partial C_T(P_{G1})}{\partial P_{G1}} = 20 + 0.02 P_{G1} - \lambda = 0
\]

\[
\frac{\partial C_T(P_{G2})}{\partial P_{G2}} = 15 + 0.06 P_{G2} - \lambda = 0
\]

\[
500 - P_{G1} - P_{G2} = 0
\]
Economic Dispatch Example, cont’d

- Solve these three linear equations

\[ 20 + 0.02 P_{G1} - \lambda = 0 \]
\[ 15 + 0.06 P_{G2} - \lambda = 0 \]
\[ 500 - P_{G1} - P_{G2} = 0 \]

\[
\begin{bmatrix}
0.02 & 0 & -1 \\
0 & 0.06 & -1 \\
-1 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
P_{G1} \\
P_{G2} \\
\lambda
\end{bmatrix}
= 
\begin{bmatrix}
-20 \\
-15 \\
-500
\end{bmatrix}
\]

\[
\begin{bmatrix}
P_{G1} \\
P_{G2} \\
\lambda
\end{bmatrix}
= 
\begin{bmatrix}
312.5 MW \\
187.5 MW \\
$26.25 / MWh
\end{bmatrix}
\]

Economic Dispatch: Formulation

- For our problem, we find that
  \[ P_{G1} = 312.5 MW \]
  \[ P_{G2} = 187.5 MW \]

- \( \lambda = $26.25 / MWh \)

- **Question**: What happens if we dispatch the generators at different output levels?

- **Note**: The values of the decision variables change with each value for \( P_L \)

Discussion

- Key results for Economic Dispatch?
  - The ‘marginal’ or ‘incremental’ cost of all generating units is equal
  - This incremental cost is the Lagrangian multiplier, \( \lambda \)
  - \( \lambda \)' is called the ‘System \( \lambda \)' and is the system-wide cost of generating electricity
    - This is the price charged to customers
    - On ISO websites, this is called either ‘System \( \lambda \)' or LMP (locational marginal price)

Economic Dispatch Ignores...

- Economic dispatch determines the best way to minimize the generator operating costs
  - It is **not** concerned with determining which units to turn on/off (this is the unit commitment problem)
  - It **ignores** the transmission system limitations
Summary

• Develop the mathematical origin for generator costs
  – Defined heat rate
• Formulate the economic dispatch problem conceptually
• Develop mathematical formulation for solving the economic dispatch problem
• Interpret the results, including the Lagrange multiplier and slack variables