Overview

- Review complex power, real power, reactive power
- Introduce one-line diagram (circuit)
- Introduce the power flow problem
  - Must employ a numerical solution (not an analytic solution)
  - (Why?)

Memories of Circuit Theory

- Kirchhoff’s Laws
  - Equations
  - The underlying physics represented by the equations (conservation laws?)
- For circuits problems, typically we are…
  - Given what information?
  - Solving for what?

Power System Diagrams

- Circuit vs. one-line diagram
Power System Diagrams

- Circuit vs. one-line diagram

Generators are shown as circles
Transmission lines are shown as a single line
Arrows are used to show loads

New York + Northeast

Figure 2.4 Sample Map of Power Plant Sites and Transmission Lines in Honduras
(Source: FNFF)
Power Flow Analysis v. Circuits

- In circuit analysis, we know (or calculate)
  - V, I, Z; and ultimately the power, P
- In power system analysis we do not know
  - The complex voltages (magnitude & phase)
  - The complex current injections
- Instead, we do know
  - the complex power being demand at each load
  - estimates for the power ‘injected’ by generators
  - estimates for the voltage magnitudes at generators
- Therefore
  - we need to go beyond Ohm’s law
  - we must use ‘power flow’ equations

Complex Power $S = P + jQ$

- Instantaneous – at a given moment in time
- Average – over a specified time period
- Phasor domain (SSS)

\[ p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i) \]

\[ p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i) \]

Phasors: Power & Phase Angle

- Writing power in phasor notation:

\[ P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \]

\[ V = V_m \angle \theta_v = V_m e^{\theta_v} \]

\[ I = I_m \angle \theta_i = I_m e^{\theta_i} \]

- Moving to complex power (with RMS):

\[ VI = \]

\[ VI^* = \]

\[ S = Re\{VI^*\} + jIm\{VI^*\} = P + jQ \]
Concept: “Real” Power, P

\[ S = P + jQ \]

- \( P \) is what we buy from the power company
- Most customers do not pay for \( Q \)
- To maximize revenue, power companies would like to generate _____?
  - How would they operate or design the system to do this?
  - A role for the power factor?

Reactive (Imaginary?) Power

- Reactive power, \( Q \)
  - Oscillates within system
  - Energy stored in, and oscillating between, capacitance and inductance
- Voltage and (stored) reactive power allow real power to flow

Reactive Power Analogy

- Reactive power
  - Energy stored in capacitance and inductance
  - Supports the electromagnetic fields along transmission lines
  - Cannot be transmitted long distances
- Analogy
  - Inflatable firefighters’ water hoses
For Power Systems Analysis:

**Power Flow Equations**

- “Power flow” refers to how the power is moving through the system
- **Kirchhoff’s Current Law**
  – At all times in the actual power system, and in any simulation, the total power flowing into any bus (node) *must* equal zero (KCL)

### Power Flow Equations

- Using KCL, we can write the current flow into the power system from any node $i$
  \[ I_i = I_{Gi} - I_{Di} = \Sigma I_{ik} \]
- Since $I = YV$ (Ohm’s Law, where $Y = Z^{-1}$), we know: (subscripts in the summation?)
  \[ I_i = I_{Gi} - I_{Di} = \Sigma Y_{ik}V_k \]
- What do “$i$” and “$k$” represent?

### Complex Power – Phasor Domain

**IMPORTANT**

$\theta$ is the power factor angle

\[
\vec{S} = \vec{V} \vec{I}^* \quad \vec{I} = I \angle -\theta
\]

\[
\vec{S} = \vec{V} \vec{I}^* = V(\cos 0 + j \sin 0) \cdot I(\cos \theta + j \sin \theta)
\]

\[
\vec{S} = VI \cos \theta + j VI \sin \theta
\]

\[
\vec{S} = P + j Q
\]

**Real Power**  **Reactive Power**

### Power Flow Equations

- Turning to the power injection at node $i$, we can write
  \[ S_i = V_i I_i^* = V_i(\Sigma Y_{ik}V_k)^* = V_i \Sigma Y_{ik}^* V_k^* \]
- We want to be able to calculate $P_i$ and $Q_i$ separately, so we need to expand the expression for $S_i$ into real and reactive components.
Power Flow Equations

- Use the following expressions to expand the equation for complex power
  - \( Y_{ik} = G_{ik} + jB_{ik} \)
  - \( V_i = |V_i|<\theta_i = |V_i|e^{j\theta} \)
  - \( e^{j\theta} = \cos\theta + j\sin\theta \)
  - \( \theta_{ik} = \theta_i - \theta_k \)
  - \( S_i = P_i + jQ_i \)

- Finally, we can write
  \[
  S_i = V_i \sum Y_{ik}^* V_k = P_i + jQ_i
  = \sum |V_i||V_k| e^{j\theta_{ik}} (G_{ik} - jB_{ik})
  = \sum |V_i||V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})
  \]

- Separate this into real & reactive parts
  \[
  P_i = \sum |V_i||V_k| (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}) = P_G - P_D
  Q_i = \sum |V_i||V_k| (G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik}) = Q_G - Q_D
  \]

Belgian High Voltage Network

*“i” takes on each bus number in turn, 1 through 53.
If “i” is bus 41, for example, then “k” takes on numbers 35, 36, 37, 40, 46, and 47 → all buses directly connected to bus 41.
There will be a set of power flow equations, \( P_i \) and \( Q_i \), for each bus “i”.

Power Flow Analysis

- For power systems, we know
  - The system topology (the circuit diagram)
  - The impedance of each line, R, X, B
  - The load at each load bus (\( S = P + jQ \))
  - The capability of each generator (P, V)
- We want to know
  - The output of each generator (\( S = P + jQ \))
  - The voltage at each bus (\( V = V<\theta \))
  - The power flow on each line (\( P_{flow} \))
Real Power Flow Equations

- How many equations and how many unknowns?
- Numerical methods
  - Lack of convergence
  - Slack bus
    - Definition
    - Mathematical and physical role

Power Flow Summary

- What is the purpose of power flow analysis?
- What is the process?
  - What data do we have and seek?
  - How is our understanding improved by performing a power flow?

Power Factor Review

- What is the power factor?
  - In words and mathematically
  - Power factor = \( \cos(\theta_v - \theta_i) \)
    - The phase angle \( (\theta_v - \theta_i) \) is defined as the power factor angle
    - Note that this is also the impedance angle – do you see why?

Power Factor Review 2

- For inductive loads
  - The current lags the voltage, showing that \( \theta_i \) is less than (or more negative than) \( \theta_v \)
  - Therefore, \( (\theta_v - \theta_i) > 0 \)
  - The power factor is said to be lagging
  - The load is said to be lagging
  - Reactive power, \( Q > 0 \) → An inductor consumes \( Q \)
Power Factor Review 3

- For capacitive loads
  - The current leads the voltage, showing that $\theta_i$ is greater than $\theta_v$
  - Therefore, $(\theta_v - \theta_i) < 0$
  - The power factor is said to be leading
  - The load is said to be leading
  - Reactive power, $Q < 0 \rightarrow$ A capacitor generates $Q$

SIEPAC – Central America

Summary

- Introduction to the power flow problem
  - Review of complex power (and complex numbers)
  - Develop the power flow equations
  - Recap of power factor